AQA Physics A-level

Section 11: Engineering physics

Notes

3.11.1 Rotational dynamics

3.11.1.1 - Concept of moment of inertia

Any mass has **a resistance to a change in velocity when subjected to a force**, this is called its **inertia**. Therefore, the larger the inertia of a mass, the larger the force required to change its velocity by a certain amount.

The **moment of inertia (I)** of an object is a measure of its resistance to being **rotationally accelerated** about an axis. This value can be calculated for a **point mass** using the following equation:

$$I = mr^2$$

Where m is the mass of the object, and r is the distance from the axis of rotation.

For an object made of **more than one point mass** (known as an extended object), you can calculate its moment of inertia by finding the sum of all the individual moments of inertia of each point mass.

$$I = \Sigma mr^2$$

The **factors that affect an object's moment of inertia** are:

- The object's total mass.
- And how its mass is distributed about the axis of rotation, which varies as the distance from the axis of rotation (r) is varied.
  - ➔ For example, when someone does a backflip they may move their legs closer to their chest, this will decrease their moment of inertia as more of their mass is as a smaller distance from the axis of rotation, (making it easier for them to rotate).

You can find the **moment of inertia of a system** of objects by calculating the sum of their moments of inertia.

For example, if a rock of mass 10 g, gets stuck in the tread of a bike wheel of mass 500 g, with a radius of 40 cm. What is the new moment of inertia of the bike wheel, assuming the rock acts as a point mass and is 41 cm away from the axis of rotation.

The moment of inertia of a hollow ring (like a bike wheel) can be calculated using: $I = mr^2$

First, you must calculate the initial moment of inertia of the bike wheel.

$I = 0.5 \times 0.4^2 =$ 0.08 kgm^2

Next, calculate the moment of inertia of the rock.

$I = 0.01 \times 0.41^2 =$ 0.0017 kgm^2

Finally, calculate the sum of the two moments of inertia found above as this gives the moment of inertia of the system.

$I_{new} = 0.08 + 0.0017 =$ 0.0817 kgm^2

To find the **new moment of inertia of a system (I_{new})** when a **mass m**, is added, you can use the following equation:

$$I_{new} = I + mr^2$$

Where I is the initial moment of inertia and r is the distance of the mass from the axis of rotation.

3.11.1.2 - Rotational kinetic energy

Just like objects with linear motion, rotating objects have kinetic energy. This value of total kinetic energy can be found by summing the kinetic energies of all the individual particles making up the object:

- Each point mass (which is part of the object) has a kinetic energy:

$$E_k = \frac{1}{2}mv^2$$

- By using the fact that $v = \omega r$, you can rearrange the above equation:

$$E_k = \frac{1}{2}m(\omega r)^2$$

- To find the kinetic energy of the whole object you must find the sum of all the individual kinetic energies of each point mass:

$$E_k = \frac{1}{2}m_1\omega^2{r_1}^2 + \frac{1}{2}m_2\omega^2{r_2}^2 + ... + \frac{1}{2}m_n\omega^2{r_n}^2$$

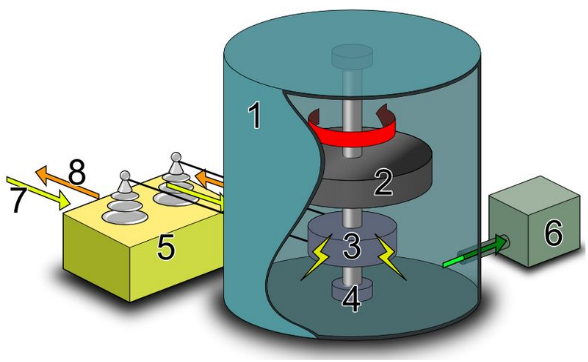
$$E_k = \frac{1}{2}\omega^2(m_1{r_1}^2 + m_2{r_2}^2 + ... + m_n{r_n}^2)$$

As $(m_1{r_1}^2 + m_2{r_2}^2 + ... + m_n{r_n}^2)$ = **sum of** mr^2 = $\Sigma mr^2 = I$ (As defined above)

$$E_k = \frac{1}{2}I\omega^2$$

Where I is the moment of inertia of the object and ω is the object's angular speed.

A **flywheel** is a heavy metal disc that spins on an axis and has a large moment moment of inertia, meaning it requires a large force in order to be rotationally accelerated, therefore once it begins spinning it will be difficult to stop. As a flywheel is spun, the input moment/torque that causes it to spin is **converted into rotational kinetic energy**, which is **stored** in the flywheel.
Flywheels optimised to store as much energy as possible are known as **flywheel batteries** (displayed on the image below).



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There are several factors which affect the amount of energy that can be stored in a flywheel:

- **Mass of the flywheel** - As its mass increases, its moment of inertia will increase. As rotational kinetic energy is directly proportional to the moment of inertia, this will also increase meaning more energy can be stored by the flywheel.
- **Angular speed of the flywheel** - As rotational kinetic energy is proportional to the square of the angular speed, if the flywheel's angular speed increases, its rotational kinetic energy increases meaning more energy can be stored.
- **Friction** - Even though they are very efficient, a flywheel will lose some of the energy that it stores to air resistance and friction between the wheel and its bearings, the effect of friction can be reduced by the following ways:
    - ➔ Lubricating the bearings.
    - ➔ Use bearings made of superconductors which would allow the flywheel to levitate. and have no contact with the bearing.
    - ➔ Use flywheels in vacuums or sealed containers to reduce air resistance.
- Using a **flywheel that is spoked** or **has more of its mass concentrated at its edges** will increase the amount of energy stored. This is because the moment of inertia of the object will be larger if most of its point masses are far from the axis of rotation.

The above factors can be varied to produce a flywheel which can store as much energy as possible, however a limitation of this is that the flywheel must still remain practical and have structural integrity.

Flywheels have many uses, examples include:

- **Regenerative braking in vehicles** - When the brakes are applied a flywheel is engaged, and it **will use the energy lost by braking to "charge up"**. Later, when the car needs to accelerate, the energy stored by the flywheel can be used to accelerate the vehicle.
- **Wind turbines** - flywheels are used to store excess power on very windy days or when electricity is not being used up as quickly as it is made. Then, when there is a reduction in electricity produced (due to there being no wind), the flywheel can be used to output power.
- **Smoothing torque and angular velocity** - some systems (such as vehicle engines) do not produce power continuously but rather in bursts, therefore the torque supplied will fluctuate. Flywheels will charge up with the burst of power and can output it smoothly to the system, therefore the fluctuations in torque and angular velocity are smoothed.
  If the **force that has to be exerted by a system varies**, flywheels are also used:
    - ➔ If the force required increases, the flywheel will **decelerate and output** some of its energy.
    - ➔ If the force required decreases, the flywheel will **accelerate and store** the excess energy until it is needed.
- **Production processes (e.g riveting machines)** - many of these processes rely on continuous, repeating actions where a large amount of power must be output in a short time. An **electric motor is used along with a flywheel** as the motor is used to charge up the flywheel, which can then transfer a short burst of energy (which can be used to press down and connect two materials together in a riveting machine, for example). This **prevents the motor from stalling** due to large changes of power moving through it, and also means **you can use a less powerful motor**.

3.11.1.3 - Rotational motion

Angular displacement (θ) - the **angle turned through** in any given direction **in radians**.
Angular speed (ω) - the **angle** an object moves through **per unit time** (has only magnitude). Units are **rads^{-1}**.
Angular velocity (ω) - the **angle** an object moves through **per unit time** (has magnitude and direction, which can be either clockwise or anticlockwise). Units are **rads^{-1}**.
Angular acceleration (α) - the **change in angular velocity over time** taken. Units are **rads^{-2}**.

Rotational equations are set up in the same way as linear equations and many linear variables have a rotational counterpart so they are found in a similar fashion to linear equations as shown below:

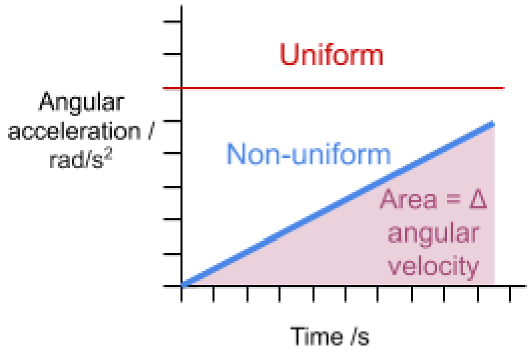
$$\textit{Linear velocity} = \frac{\textit{displacement}}{\textit{time}} \qquad \textit{Angular velocity}\ (\omega) = \frac{\textit{angular displacement}}{\textit{time}}$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

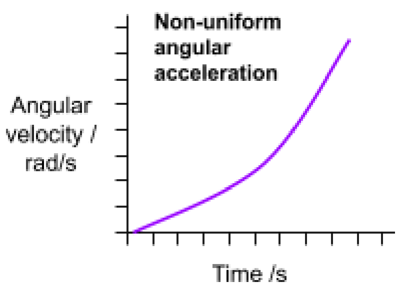
$$\textit{Linear acceleration} = \frac{\textit{velocity}}{\textit{time}} \qquad \textit{Angular acceleration}\ (\alpha) = \frac{\textit{angular velocity}}{\textit{time}}$$

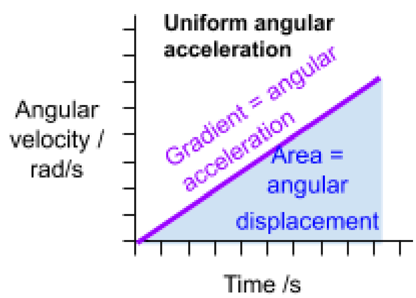
$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Like linear acceleration, angular acceleration can be **uniform**, which is where the angular acceleration is **constant**, as shown on the diagram below:

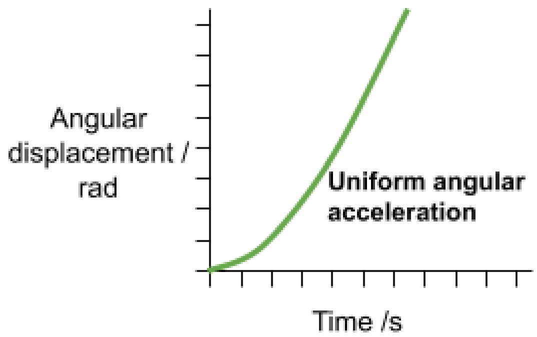


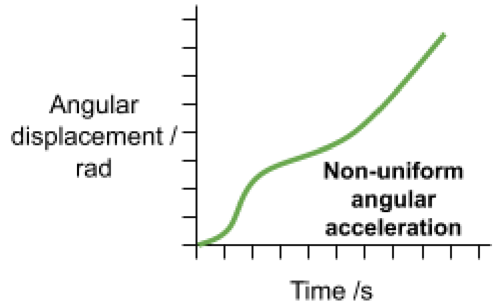
When angular acceleration is **uniform**, a graph of angular velocity against time will be a **straight line** graph. To find the angular acceleration from this type of graph you can simply find the gradient if the acceleration is uniform, if it isn't you can find the acceleration at a certain point by finding the gradient of a tangent to the graph at that point.





When angular acceleration is **uniform**, a graph of angular displacement against time will show that **displacement is proportional to t^2** :





Similarly to linear motion, rotational motion has several **equations for uniform angular acceleration**, which are found below:

Linear equation	Rotational equation
$v = u + at$	$\omega_2 = \omega_1 + \alpha t$
$s = \frac{1}{2}(u + v)t$	$\theta = \frac{1}{2}(\omega_1 + \omega_2)t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta$

Where ω_1 is initial angular velocity and ω_2 is final angular velocity.

3.11.1.4 - Torque and angular acceleration

Torque (T) is the **product of a force and its distance from its axis of rotation**. Torque often causes rotation and has the units **Nm**.

$$T = Fr$$

Where F is the force and r is the distance from the axis of rotation.

The effect of torque can be demonstrated by a wheel and axle as shown in the diagram to the right. The mass attached to the axle **causes a torque**, and so will cause an angular acceleration in the wheel because there is a resultant force acting on it.

The angular acceleration can be increased by:

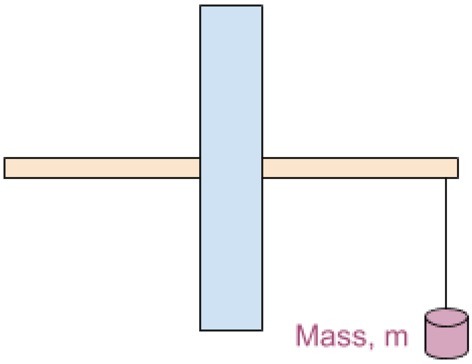
- **Increasing m** and so increasing the torque
- Using a **lighter wheel**, which will decrease the moment of inertia of the wheel

Therefore the torque, inertia and angular acceleration are related by the equation below:

$$T = I\alpha$$

Where I is the moment of inertia.

The above equation calculates the value of the **resultant torque** in much the same way as Newton's second law (F = ma) finds the value of resultant force.

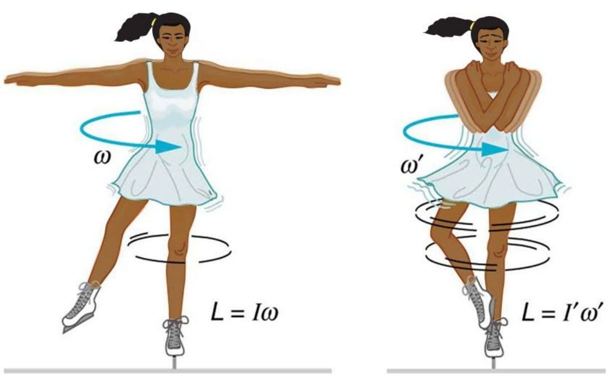


3.11.1.5 - Angular momentum

Angular momentum is the **product of the moment of inertia and angular velocity of an object**, and its units are **Nms**.

$$Angular\ momentum\ = I\omega$$

The **law of conservation of angular momentum** states that when no external torque acts, the angular momentum of a system remains constant. This can be demonstrated by looking at the example of an ice skater spinning about a vertical axis on the tip of her skate, with her arms extended. As she moved her arms into her chest, she will begin moving faster - her angular velocity increases. This occurs because by moving her arms inwards she has decreased her moment of inertia, however angular momentum is conserved (as no external torque acts) so her angular velocity must increase.



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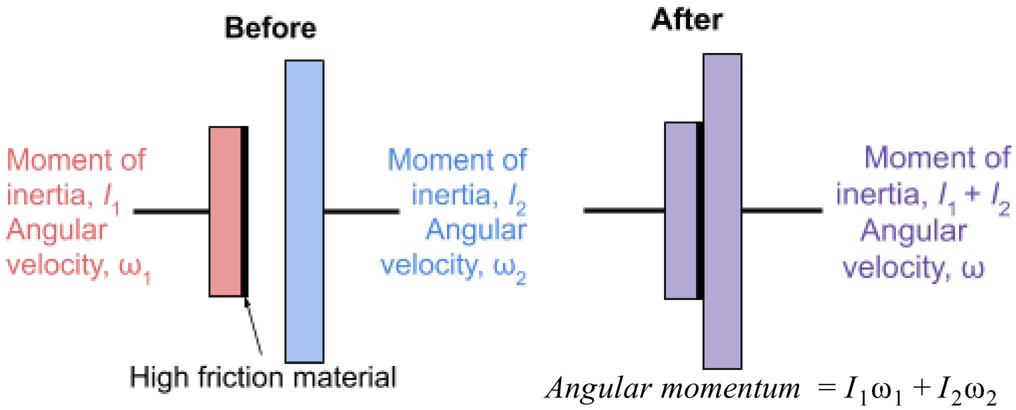
The law of conservation of momentum is also demonstrated in **simple clutches** where two discs of different moments of inertia are made to move at the same angular speed. Similarly to a linear collision, where momentum is conserved, a clutch causes a rotational dynamics "collision". Initially, the two discs with moments of inertia, I_1 and I_2 move at angular velocities ω_1 and ω_2. Once they collide, the discs act as one object so their moment of inertia becomes $I_1 + I_2$ and they moved at the common speed ω.



$$Angular\ momentum = I_1\omega_1 + I_2\omega_2$$

$$Angular\ momentum = (I_1 + I_2)\omega$$

Due to the law of conservation of momentum, **momentum before = momentum after**.

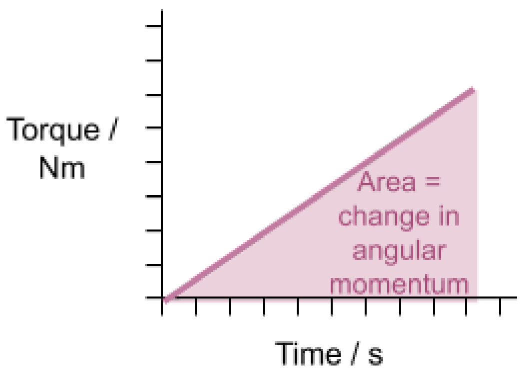
$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$$

Angular impulse is the **product of torque and its duration** where the applied torque is constant, and is equal to the **change in angular momentum**.

$$Angular\ impulse = change\ in\ angular\ momentum$$

$$T\Delta t = \Delta(I\omega)$$

Angular impulse can be found by calculating the area beneath a torque-time graph.



3.11.1.6 - Work and power

Work done (W) is defined as the force causing a motion multiplied by the distance travelled. Work must be done on an object in order to make it rotate, therefore to calculate work done on a rotating object you must find the **product of the torque and angular displacement**.

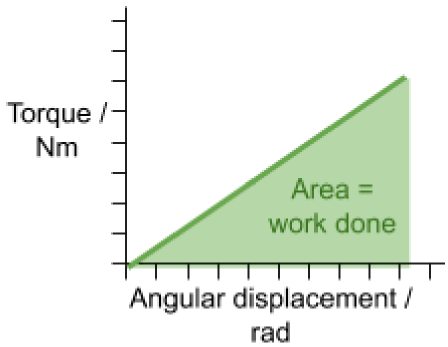
$$W = T\theta$$

Similarly to linear dynamics, an increase in work done can increase the rotational kinetic energy of an object, however it must first overcome the **frictional torque** that may be present.
You can **calculate the frictional torque** on a wheel experimentally:

1. Apply an accelerating torque to the wheel to bring up to a certain velocity.
2. Remove the accelerating torque, and measure the time taken for the wheel to come to rest.
3. Calculate the average deceleration (this assumes the frictional force is constant), by using the equation: $\alpha = \frac{\omega_2 - \omega_1}{t}$.
4. Finally, calculate frictional torque using the equation $T = I\alpha$ (given that the moment of inertia is known).

It is important to note that in most cases especially in rotating machinery, **frictional torque is minimised** in order to minimise the energy losses due to kinetic energy being transferred to heat and sound energy, however in some cases **frictional torque can be useful**. For example, when using a screwdriver you apply a frictional torque which increases the rotational kinetic energy.

Work done can also be calculated by finding the area under a torque-angular displacement graph.



Power (P) is the **rate of energy transfer** and as work is a measure of energy transfer, it is also the **rate of doing work**. You can calculate power by dividing the amount of work done by the time passed, which can be used to derive an equation for work done which is the **product of torque and angular velocity (ω)**.

$$P = \frac{W}{t} = \frac{T\theta}{t} \qquad \text{As } \frac{\theta}{t} = \omega$$

$$P = T\omega$$

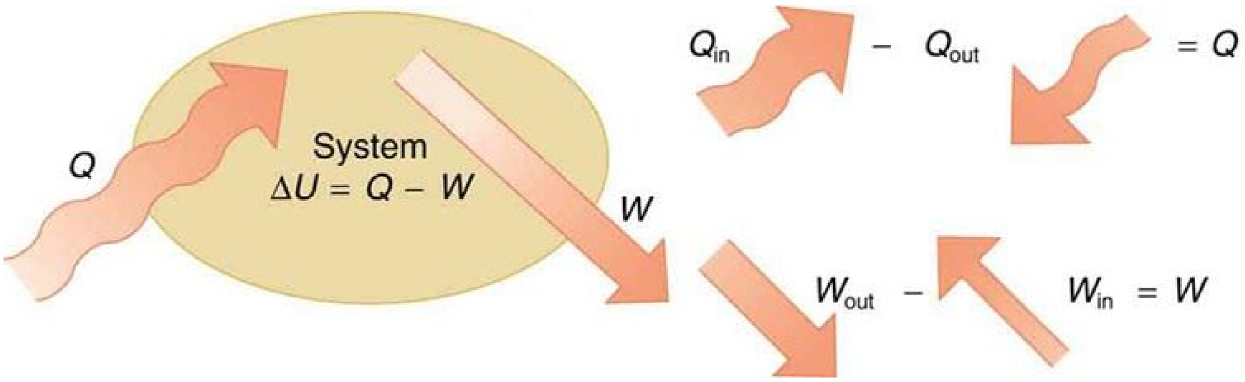
3.11.2 Thermodynamics and engines

3.11.2.1 - First law of thermodynamics

The **first law of thermodynamics** describes the conservation of energy in a system where energy can be transferred through doing work or heating. It states that the **energy transferred** to a system **through heating** is equal to the **sum** of the **increase in internal energy** and **work done by** the system. It is given by the following equation:

$$Q = \Delta U + W$$

Where Q is the energy transferred to the system by heating/cooling, ΔU is the **increase** in internal energy and W is the work done **by** the system.



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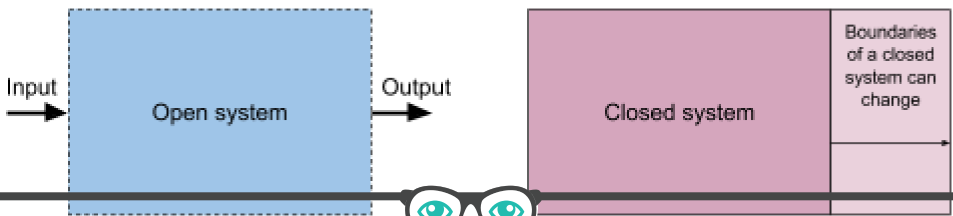
It is important to note that Q is the energy transferred **to** the system through heating, therefore if **Q is negative**, **energy is transferred away** from the system through cooling.
Similarly, W is the work done **by** the system, this occurs when a gas expands, therefore if **W is negative**, this value represents the **work done on the system,** this occurs when a gas is compressed.

The **internal energy (U)** of a system is equal to the sum of all of the kinetic energies and potential energies of **all** its particles. As ΔU represents the **increase** in internal energy, if **ΔU is negative**, the **internal energy will decrease**.

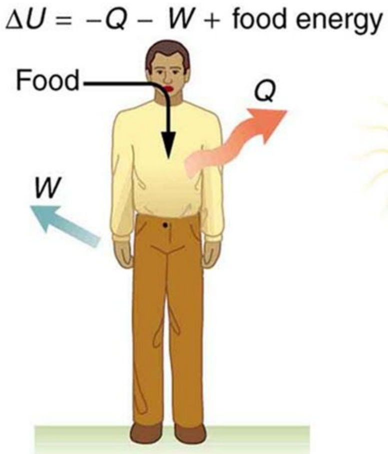
A system is a region containing a body of gas, and it can be either:

- **Open** - this is where gas can flow in, out or through the system, therefore gas can cross the boundaries of the system. An example of an open system is an aerosol can.
- **Closed** - this is where no gas can leave or enter the system, however the boundaries of the system may change when the gas changes volume. An example of a closed system is air inside a balloon.



One application of the first law of thermodynamics is the human metabolism, where Q is negative as the human body transfers heat to its surroundings, and W is positive as work is being done by the body. In the situation described you can see that the **internal energy of the body must decrease** as ΔU must be negative to make the equation below hold true.

$$-Q = \Delta U + W$$



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3.11.2.2 - Non-flow processes

Non-flow processes are changes which occur in closed systems (as the gas does not flow across a boundary). In order to apply the first law of thermodynamics to non-flow processes **you have to assume the gas in the system is ideal**. An **ideal gas** follows the gas laws perfectly, meaning that there is no other interaction other than **perfectly elastic collisions** between molecules, which means that no intermolecular forces act between molecules. As potential energy is associated with intermolecular forces, **an ideal gas has no potential energy**, therefore its internal energy is equal to the sum of the kinetic energies of all of its particles.

By assuming the gas is ideal, you can use the ideal gas equation: $pV = nRT$

Where p is the pressure, V is the volume, n is the number of moles of gas, R is the molar gas constant and T is the temperature in kelvin.

As the system is **closed** in non-flow processes, n will be constant so $\frac{pV}{T} = constant$.

This could be rewritten as $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$, which is useful to remember.

An **adiabatic process** is where **no heat leaves or enters the system**, therefore $Q = 0$.

Using the first law of thermodynamics you can see that the increase in internal energy of the system is equal to the work done by/on the system (depending on its sign): $\Delta U = \ -W$.

As the gas is assumed to be ideal, **internal energy is only dependent on temperature**.

- ➔ If the gas expands, such as when a balloon bursts, work is done **by** the system (so W is positive) so the temperature of the gas will decrease.
- ➔ If the gas is compressed, work is done **on** the system (so W is negative) so the temperature of the gas will increase.

For an **adiabatic change**, the product of pressure and volume to the power of the adiabatic constant (γ) is constant. The adiabatic constant depends on the gas in the system.

$$pV^{\gamma} = constant$$

So it follows that:

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

An **isothermal process** is where **the temperature of the system remains constant**, therefore $\Delta U = 0$. Using the first law of thermodynamics you can see that the energy transferred to the system through heating is equal to the work done by the system: $Q = W$.

Such processes can occur when the container is a very good conductor, and the process is very slow.

- ➔ If the system is heated (Q is positive), the work done **by** the system will be equal to the energy transferred to the system.
- ➔ Whereas, if the system is cooled (Q is negative), the work done **on** the system will be equal to the energy transferred from the system.

For an **isothermal change**, the product of pressure and volume is constant, therefore this process obeys Boyle's law, which is that pressure is inversely proportional to volume at constant temperature.

$$pV = constant$$

So it follows that:

$$p_1 V_1 = p_2 V_2$$

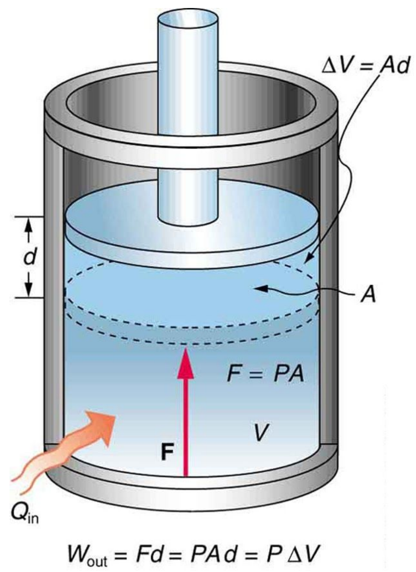
A **constant pressure change** is where the **pressure of the system remains constant** and so you can calculate the work done by using the following formula:

$$W = p\Delta V$$

Where W is the work done, p is the pressure and ΔV is the change in volume.

You can derive the above equation by considering a constant force, F being applied to a gas through a piston as shown in the diagram to the right. A constant applied force means that the pressure will also be constant, as force is the product of pressure and area.

$$W = F \times d \qquad F = pA$$
$$W = pA \times d \qquad Ad = \Delta V$$
$$W = p\Delta V$$



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Heating a gas at constant pressure will cause it to expand, the change in volume and work done by the system are **positive**.
Cooling a gas at constant pressure will cause it to be compressed, the change in volume and work done by the system are **negative**.

If pressure is kept constant, it follows that:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

A **constant volume change** is where the **volume of the system remains constant**, therefore $W = 0$ as **no work is done by or on the system**. Using the first law of thermodynamics you can see that the energy transferred to the system is equal to the increase in internal energy: $Q = \Delta U$.

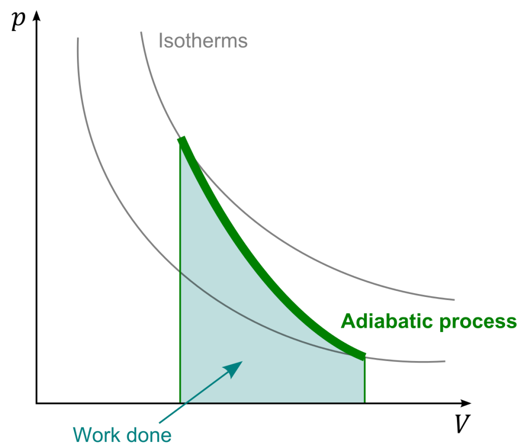
If the system is **heated**, the temperature will increase, whereas if the system is **cooled**, the temperature will decrease. The **entire value** of energy transferred is used to either heat or cool the system.

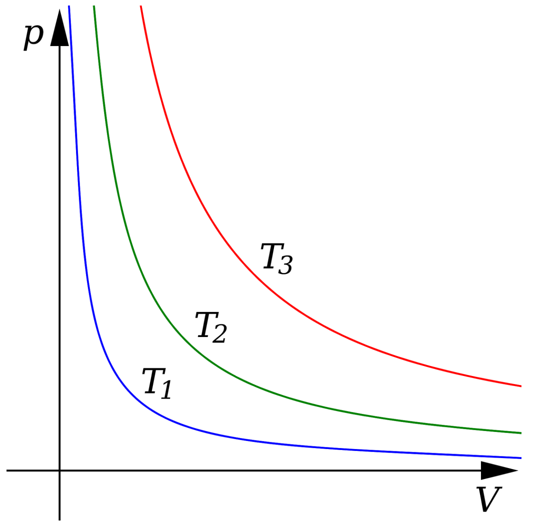
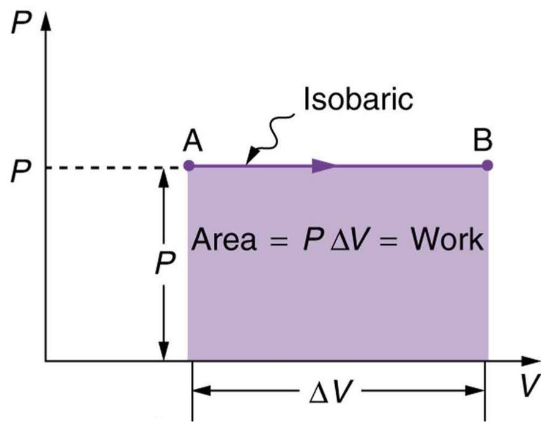
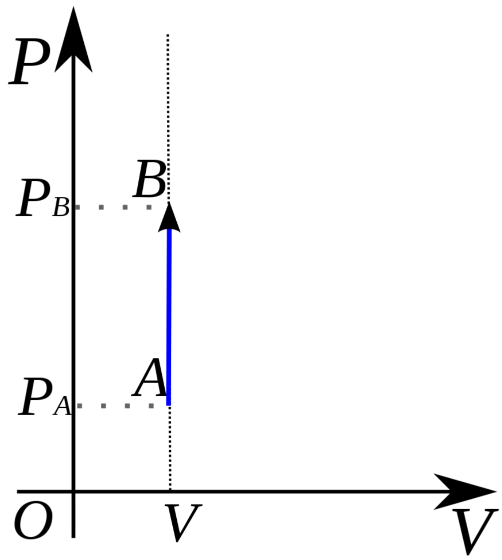
3.11.2.3 - The p-V diagram

Diagrams of **pressure against volume** can be used to represent non-flow processes, and an arrow is used on the graph to indicate the direction of the change. A useful feature of a p-V diagram is that the **area beneath the curve is equal to the work done** during the process. You can estimate the work done by estimating the area beneath the curve by using one of the following methods:

- **Counting squares** - count the squares beneath the curve which are more than half full, then multiply this by the area of one square
- **Trapezium method** - split the curve into a number of trapeziums, which fit the curve as well as possible and calculate the sum of their area. The more trapeziums used, the more accurate the answer.

It is important to note that in general, the increase in volume of a system implies that work is done **by** the system (**W is positive**) and a decrease in volume implies that work is done **on** the system (**W is negative**).

Process	Description	First law application	Diagrams
Adiabatic	No energy is transferred in or out of the system. As you can see from the p-V graph the curve for an adiabatic process is steeper than the curve for an isothermal change (labelled as isotherms on graph). Therefore, an adiabatic compression does more work than an isothermal one and so an adiabatic expansion does less work than an isothermal expansion.	Q = 0 ΔU = − W	 Image source: Stannered,CC BY-SA 3.0

Isothermal	The temperature of the system is kept constant. The p-V graphs for isothermal processes are known as isotherms. The higher the temperature, the further the curve is from the origin therefore **T_3>T_2>T_1** in the diagram to the right.	ΔU = 0 Q = W	 Image source: Andrew Jarvis, CC BY-SA 4.0
Constant pressure	The pressure of the system is kept constant. This is also known as an isobaric change. The work done by this process is calculated using the formula: **Work = P ΔV**		 Image source: OpenStax College, CC BY 4.0
Constant volume	The volume of the system is kept constant. As the curve is a straight vertical line, it has no area and so **no work is done** by this process. This is known as an isochoric change.	W = 0 Q = ΔU	 Image source: IkamusumeFan, CC BY-SA 3.0, Image is edited (some text is removed)

A **cyclic process** is where the system undergoes **two or more processes** one after another and **returns to its initial volume, temperature and pressure**. Therefore the p-V diagram for a cyclic process forms a loop.
To find the **net work done** during a cyclic process you must find the **difference** between the work done **by** the system and the work done **on** the system. This is equal to the **area of the loop** formed by the cyclic process.

$$Work\ done\ per\ cycle\ =\ area\ of\ loop$$

Cyclic processes can be **repeated continuously**, meaning they could potentially release a large amount of energy.

3.11.2.4 - Engine cycles

In order for a cyclic process to be useful, the amount of energy done **by** the system must be greater than the amount of energy done **on** the system.

An **internal combustion engine** contains cylinders of air which form **systems**. The air inside the cylinders is **compressed when the engine is at a low temperature** and and **expanded when it is at a high temperature**. As less energy is needed to compress the air at a low temperature than the amount of energy released when the gas is expanded at a high temperature, there is a net amount of energy output by the system.

A **four-stroke petrol engine** is a type of internal combustion engine, which burns fuel every four strokes of the piston (every two revolutions), similarly two-stroke engines burn fuel once every two strokes. A **stroke** is a single movement of the piston, either up or down. Each cylinder in this type of engine follows this sequence of operations (sometimes known as the otto cycle):

1. **Induction -** The **piston moves down**, causing the volume of the cylinder to increase, and so the volume of the gas (a mixture of air and petrol vapour) above the piston also **increases**. The gas mixture is drawn into the cylinder through an **open inlet valve**. The pressure of the gas remains constant, and is just below atmospheric pressure.
2. **Compression -** The **inlet valve is closed** and the **piston moves up**, doing work on the gas, causing its volume to **decrease** and its pressure to **increase**. Almost at the end of the piston's stroke, the spark plug produces a spark, which ignites the gas mixture. This causes the temperature and pressure of the gas to **increase** dramatically, at an almost constant volume.
3. **Expansion -** The gas mixture expands and so does work on the piston **causing it to move down** the cylinder. As the gas is now at a higher temperature, the **work done by the gas here is higher** than the work used to compress it. Almost at the end of the piston's stroke, the **exhaust valve opens** and the pressure **reduces** to almost atmospheric pressure.
4. **Exhaust -** The **piston moves up** the cylinder, forcing the burnt gas out of the cylinder through the **open exhaust valve**. The pressure stays at slightly above atmospheric pressure.

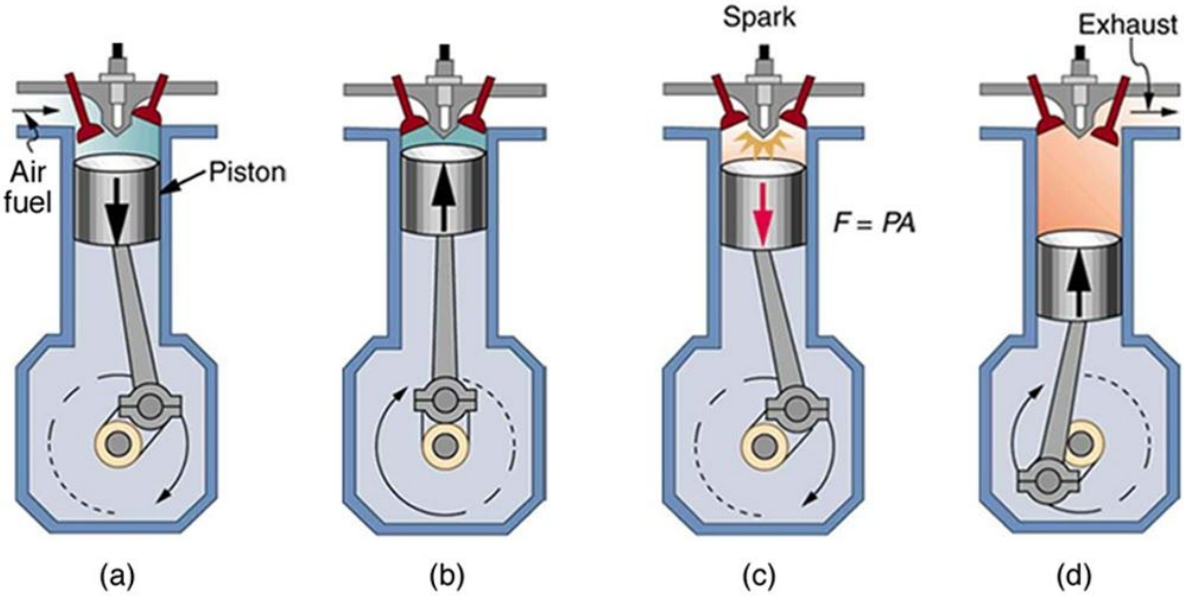


Image source: OpenStax College,CC BY 4.0,Image is cropped
A - Induction, B - Compression, C - Expansion, D - Exhaust

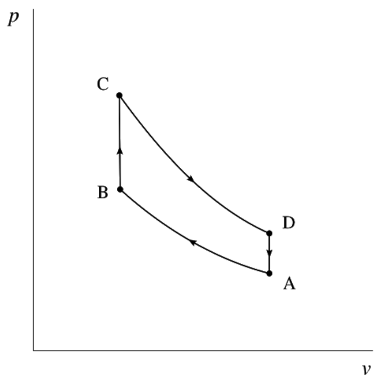
One of the main differences between the petrol engine described above and a diesel engine is the **ignition process**. In a petrol engine, the mixture of gas and petrol is **ignited by a spark** just before the piston reaches the end of its compression stroke. This is so that there is enough time for ignition to occur before the piston finishes its stroke, and so the **maximum amount of pressure is output** on the piston. In a diesel engine, **only air is drawn into the cylinder** during the induction stroke. During the compression stroke, the temperature of the air rises to over 550°C, which is high enough to ignite diesel, so **just before** the end of the compression stroke, **diesel is input directly into the cylinder as a fine spray** using a **fuel injector**, causing it to ignite. The expansion and exhaust strokes work in the same way as in a petrol engine, however the variations in pressure and volume are slightly different in a petrol engine.

Pressure-volume diagrams for a engines are known as **indicator diagrams**, and these can be used to calculate the **output power** and **efficiency** of an engine. There are two types of indicator diagrams:

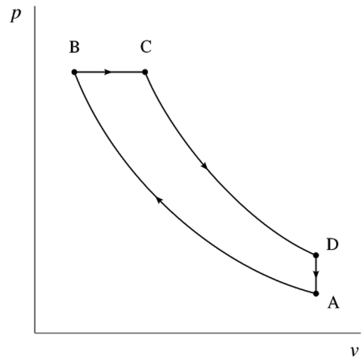
- **Theoretical** - These diagrams can be formed by modelling the engine using the following assumptions:
  - ➔ The same gas is constantly moving through the cycle. The gas is pure air, which has an adiabatic constant of 1.4.
  - ➔ Pressure and temperature can change instantaneously.
  - ➔ The engine experiences no friction whatsoever.
  - ➔ The heat source is external.
- **Actual** - formed using recorded data, found using a pressure sensor and transducer in the cylinder.

The two different diagrams vary greatly and **their comparison allows analysis of the engine's power and efficiency**.

You must be familiar with the theoretical and actual indicator diagrams for four-stroke petrol and diesel engines.

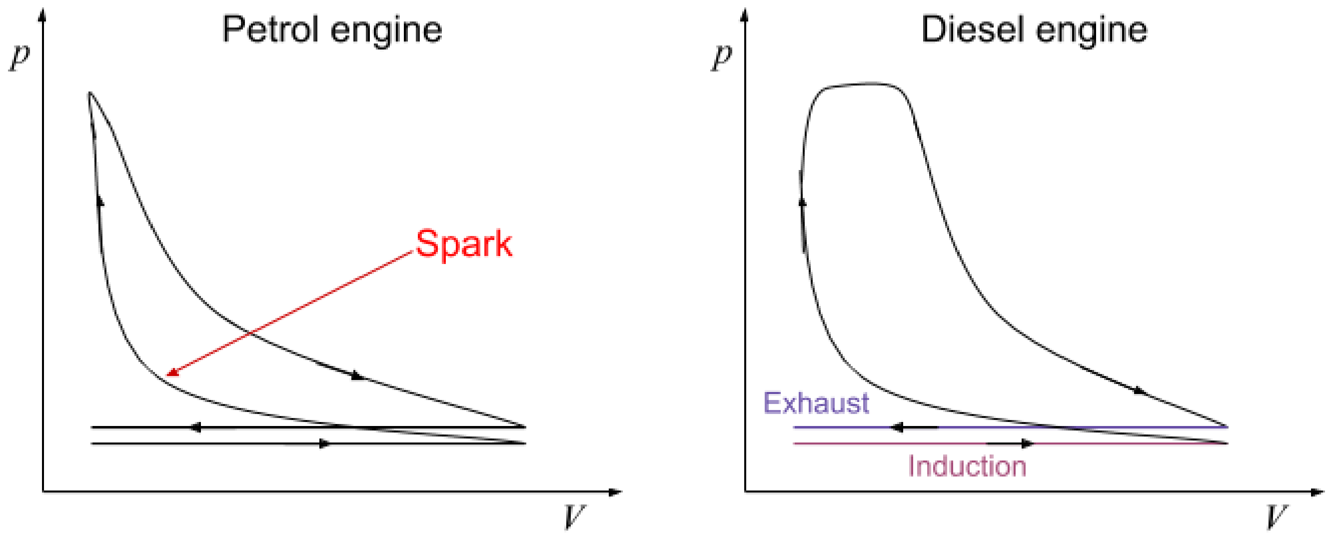


Four-stroke petrol engine
The theoretical diagram is shown on the right:
A-B - The gas is compressed **adiabatically.**
B-C - **Heat** is supplied, **volume is kept constant.**
C-D - The gas expands **adiabatically** (and therefore cools).
D-A - The system is **cooled at a constant volume.**



Four-stroke diesel engine
The theoretical diagram is shown on the right:
A-B - The gas is compressed **adiabatically.**
B-C - **Heat** is supplied, **pressure is kept constant.**
C-D - The gas expands **adiabatically** (and therefore cools).
D-A - The system is **cooled at a constant volume.**

The **actual** indicator diagrams are shown below. As you can see, the most distinct differences between the petrol and diesel diagrams is that **there is not a sharp peak at the start of the expansion stroke (C-D) for the diesel engine**.

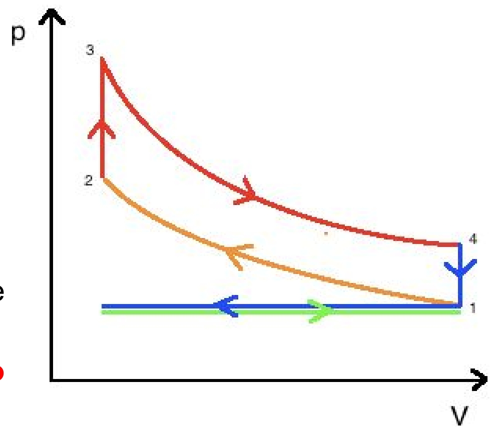


There are some **key differences** between the theoretical and actual indicator diagrams, and these differences also demonstrate why the **efficiency of a real engine is far lower than its theoretical efficiency**:

- The actual diagrams have **rounded edges** - this is because the valves in the engine take time to open and close, and so the same air is not used continuously as assumed when producing theoretical indicator diagrams.
- In a petrol engine, **heating does not take place at a constant volume** as this requires temperature and pressure to increase instantaneously (or the piston to pause at the top of its stroke).

- The actual diagrams show the amount of **negative work done** by the engine, shown by the **small loop between the induction and exhaust lines**, whereas the theoretical diagrams don't because it is assumed the same air is used continuously.
- The theoretical diagrams have **higher peaks** as they assume an external heat source, which could cause production of higher pressures. In reality, the heating occurs internally so the temperature rise is lower, as the fuel is not completely burnt.
- The **area of the loop is smaller** in the actual diagrams because the net work done is lower than predicted as work must be done to overcome **friction** between moving parts of the engine.

The area beneath the compression curve (shown in orange) is the work done **on** the gas during the compression stroke, while the area beneath the expansion curve (shown in red) is the work done **by** the gas during the expansion stroke.



The area of the small loop between the induction and exhaust lines (shown in green and blue respectively), is the **negative work done** by the engine and this value should be subtracted from the area of the main loop to find the **true value** of net work done. In an actual diagram, this area is **so small it is negligible**, so the **net work done by the gas** can be calculated by finding the **area of the main loop**.

The **indicated power** is the net work done by the engine **each second** or simply the **power developed by the engine**. In order to calculate this value, you must calculate the number of cycles occurring in the engine per second:

$$Number\ of\ cycles\ per\ second = \frac{1}{Time\ for\ one\ cycle}$$

The indicated power of a **single cylinder** is the **product of the area of the main p-V loop** (for this cylinder) and the **number of cycles it undergoes per second**. To find the **indicated power of the whole engine**, you must multiply the indicated power of a single cylinder by the number of cylinders in the engine:

$$Indicated\ power = (area\ of\ p - V\ loop) \times (no.\ of\ cycles\ per\ second) \times (no.\ of\ cylinders)$$

The **brake power** (also known as the **output power**) is the power output by the engine. **Part of the indicated power must be used to overcome the frictional forces** in the engine, this is known as the **friction power**. The **brake power is therefore lower than the indicated power**, because part of the indicated power is used in overcoming friction (friction power).

$$brake\ power = indicated\ power - friction\ power$$
$$friction\ power = indicated\ power - brake\ power$$

You can also calculate the brake power by finding the **product of the torque (T) produced by the engine and its angular velocity (ω)**:

$$P = T\omega$$

The **input power** of the engine can be calculated by finding the **product of the calorific value of the fuel and its flow rate**. The **calorific value** of a fuel is a measure of how much energy the fuel stores per unit volume, however this may sometimes be given to you per unit mass, in this case you will need to calculate the flow rate in terms of mass, rather than in terms of volume, and perform the calculation in the same way.

$$input\ power = calorific\ value \times fuel\ flow\ rate$$

There are 3 types of engine efficiencies you need to know about:

- **Overall efficiency** - simply the overall efficiency of the engine (the product of thermal and mechanical efficiency).

$$Overall\ efficiency = \frac{brake\ power}{input\ power}$$

- **Thermal efficiency** - a measure of how efficiently the chemical energy from the fuel is transformed into work.

$$Thermal\ efficiency = \frac{indicated\ power}{input\ power}$$

- **Mechanical efficiency** - this value depends on the amount of energy lost due to moving parts in the engine (e.g friction).

$$Mechanical\ efficiency = \frac{brake\ power}{indicated\ power}$$

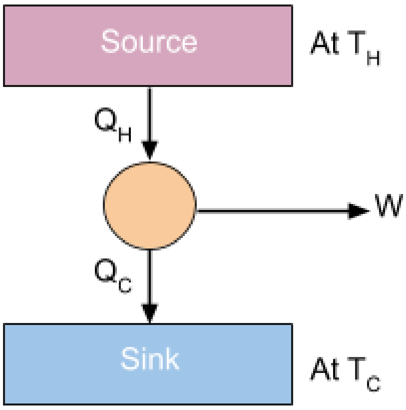
3.11.2.5 - Second law and engines

The **second law of thermodynamics** states that a heat engine must have **both** a **source** and a **sink** to operate. This means that the engine must be heated by the source and it **must** lose part of the energy it gains to the sink. The source must be at a higher temperature than the sink.

For example, if the engine reached the temperature of the source, **no more heat would flow** (as they have reached thermal equilibrium) and so **no work would be done**. This shows that **an engine cannot work by following only the first law of thermodynamics** (which is that the energy transferred to a system through heating is equal to the sum of the increase in internal energy and work done by the system) and so **cannot be 100% efficient**.

As shown in the diagram below:

1. Heat energy (**Q_H**) is transferred from a source at a temperature of **T_H** to the heat engine.
2. **Some** of the energy is transferred into work, **W**.
3. The **rest of the energy (Q_C)** is transferred to the sink (this is usually the surrounding of the engine), which is at a lower temperature, **T_C**.



The efficiency of an engine can be calculated using the values described above:

$$efficiency = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$

Where Q_H is the energy transferred from the source, Q_C is the energy transferred to the sink and W is work output.

If an **ideal gas** is used as the substance in the engine, the following equation for maximum theoretical efficiency can be formed:

$$maximum\ theoretical\ efficiency = \frac{T_H - T_C}{T_H}$$

Where T_H is the temperature of the source and T_C is the temperature of the sink.

From the equation above, you can see that to **make an engine as efficient as possible**, the source temperature should be made as high as possible while the sink temperature should be made as low as possible. Also, this equation shows that an engine can only be 100% efficient when the sink is the temperature of **absolute zero**.

The efficiency of real engines is much lower than their theoretical maximum, this is due to a number of reasons:

- ➔ **Work must be done** in order to overcome the **frictional forces** in the engine.
- ➔ The **fuel is not completely burned** in the process, so the **temperature rise is not as high** as expected.
- ➔ **Power is used to drive internal components** of the engine (e.g. pumps, motors).

In heat engines, the amount of energy transferred to the sink (Q_C) is usually higher than the useful power generated (W). Because of this, several ways have been developed in order to **maximise** the use of power generated (W) and the energy transferred to the source (Q_H):

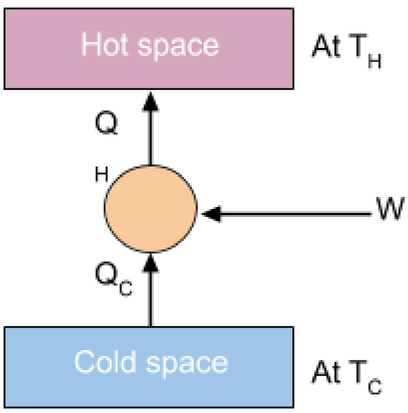
- **Combined heat and power (CHP) schemes -**
  These rely on the fact that power plants transfer huge amounts of heat to their surroundings (e.g through cooling towers), therefore this instead of removing this heat through cooling, it is **used to heat homes and businesses** which are close by. The main problem with CHP schemes is that most power plants in the UK are positioned far away from homes and

businesses and so the heat has too far to travel (as it would cool down before reaching its destination).

- **Regenerative braking -**
  This allows the power caused by braking in a vehicle to be stored (in a flywheel as described in 3.11.1.2) and be **used later on to accelerate when needed** instead of being entirely dissipated, mostly as heat to the surroundings.

3.11.2.6 - Reversed heat engines

Reversed heat engines have **work done on them** in order **to transfer energy from a colder region to a warmer one**. The reason work must be done is because heat will naturally flow from a warmer space to a colder one (as in heat engines).



Reversed heat engines can be used in two forms:

- **Refrigerators - extract as much energy from the cold region** as possible **for each joule of work done**.
  An example is the practical refrigerator, which is used in homes to store food. The inside of the refrigerator is the cold space, while the room the refrigerator is in is the hot space, and so the room is heated while the refrigerator is kept cool.
- **Heat pumps - transfer as much energy to the hot region** as possible **for each joule of work done**. An example is a heat pump used to heat a house. The inside of the house is the hot space, while the outdoors is the cold space.

The **efficiency** of a reversed heat engine depends on its aim, so whether it acts as a refrigerator or a heat pump. Because of this it is very difficult to form an equation for efficiency, therefore the coefficient of performance is used instead. The **coefficient of performance (COP)** is **not** a measure of efficiency as it can be greater than 1, whereas due to the law of conservation of energy, efficiency cannot be greater than one. The **COP** is a **measure of how effective a reversed heat engine is at transferring heat per unit of work done**, for example a COP of 5 means that 5 J of heat energy is transferred per 1 J of work done. The ways of finding the COP for a refrigerator and heat pump are different and are outlined below:

- **Refrigerator** - the ratio of heat transferred from the cold region to the work done.

$$COP_{ref} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$$

- **Heat pump** - the ratio of heat transferred into the hot region to the work done.

$$COP_{hp} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$

When using the temperature of the regions in order to calculate COP, you are assuming that the engine is running at maximum theoretical efficiency

A reversed heat pump can be used to perform both the function of a refrigerator and a heat pump at once. The main advantage of using a heat pump over a conventional electric or gas heater is that the **energy transferred by a heat pump exceeds the work done on the pump**. An electric or gas heater will **at most** transfer 1 J of energy per 1 J of work done, therefore heat pumps are cheaper to run.

It is important to note that you should be very careful when talking about energy transfer in reversed heat engines, **always specify the region that the heat is moving into** (either hot or cold). This is because general phrases like “input energy” can mean many things such as the work done on the engine, the energy input into the hot region or even the energy input into the engine from the cold region.