AQA Physics A-level

Topic 6.1: Further Mechanics

Key Points

Radian Measures

When dealing with **circular motion**, it is often easier to make use of **angular** quantities. These make use of an angle unit, known as the **radian**. The conversion between degrees and radians is:

$$2\pi \text{ radians} = 360°$$

The equation to find the angle in radians you have turned through is:

$$\theta = \frac{s}{r}$$

Where '*s*' is arc length and '*r*' is the radius of the circle.

Angular speed is given by:

$$\omega = \frac{v}{r} \qquad \omega = 2\pi f$$

Circular Motion

From **Newton's first law**, we know that for an object to change velocity, a **resultant force** must act. In circular motion, since the **direction** of the object is continually changing, the **velocity** must also be changing. Therefore a resultant **centripetal force** is required. This force points towards the **centre** of the object's orbit and is given by the equation:

$$F = \frac{mv^2}{r}$$

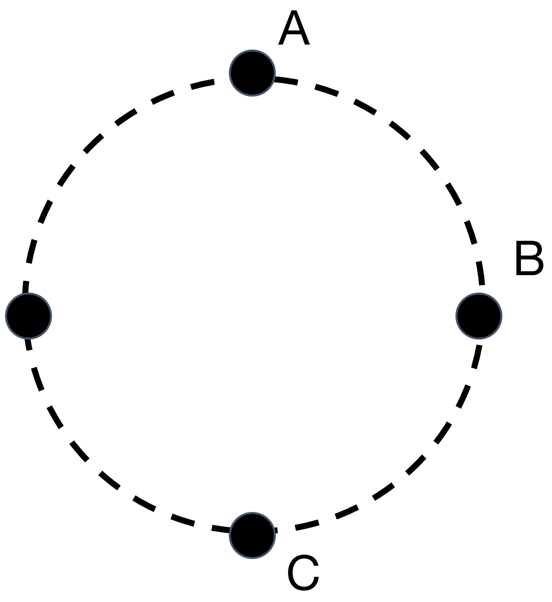
An alternative form of this equation can be given, which makes use of the **radian** and **angular speeds**.

$$F = m\omega^2 r$$

Where ω is the angular speed and is given by: $\omega = 2\pi f$

Circular Motion

It is important to consider what is contributing to the centripetal force at each point in the cycle. For a ball being spun on a string:



- At **position A**, the weight of the ball is directly contributing to the centripetal force since it is acting directly towards the centre of the circle - this means that the inwards force provided by the string is at a **minimum**
- At **position B**, the weight of the ball is acting **perpendicular** to the direction of the centripetal force, meaning it makes no contribution and the string provides the full force
- At **position C**, the weight of the ball is acting opposite to the direction of the centripetal force, meaning the inwards force of the string must overcome the weight and provide the required centripetal force- this means it is at a **maximum**

Simple Harmonic Motion

Simple harmonic motion is a mechanical process that is characterised by the following conditions:

- The object **oscillates** either side of an **equilibrium** position
- A **restoring** force always acts **towards** this equilibrium position
- The **force** is **proportional** to the object's **displacement**
- Consequently the object has an **acceleration** proportional to its displacement

The **defining equation** for SHM is:

$$F = -k\,x$$

Where 'x' is displacement and 'k' is a constant.

Further Equations

You should understand the following terms in the context of SHM:

- The **frequency** is the number of full cycles that occur each second
- A full **cycle** is the motion from maximum positive displacement, to maximum negative displacement and then back to the maximum positive displacement again
- The **time period** is the length of time it takes to complete a cycle

Like in circular motion, SHM make use of ω, the **angular frequency**. You need to be able to apply the following equations when analysing SHM scenarios:

$a = -\omega^2 x$ $\quad$ $x = A \cos\omega t$ $\quad$ $v = -A\omega \sin\omega t$ $\quad$ $a = -A\omega^2 \cos\omega t$

Make sure you are careful to substitute ‘a’ with **acceleration** and ‘A’ with **maximum displacement**.

Spring Oscillators

A **spring oscillator** consists of a **mass** on a spring which oscillates with **simple harmonic motion**.

The **time period** of oscillation for a spring oscillator is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Note that the period isn't affected by the **gravitational field strength**. This means that the period will be the same on all planets.

It is also unaffected by the **magnitude of displacement**, meaning the period will be the same for all initial displacements.

Pendulum Oscillators

A **pendulum oscillator** consists of a **mass** on a string which oscillates with **simple harmonic motion**.

The **time period** of oscillation for a pendulum oscillator is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Note that the period isn't affected by the **object's mass**. This means that the period will be the same regardless of how heavy the mass is.

It is also unaffected by the **magnitude of displacement**, meaning the period will be the same for all initial displacements.

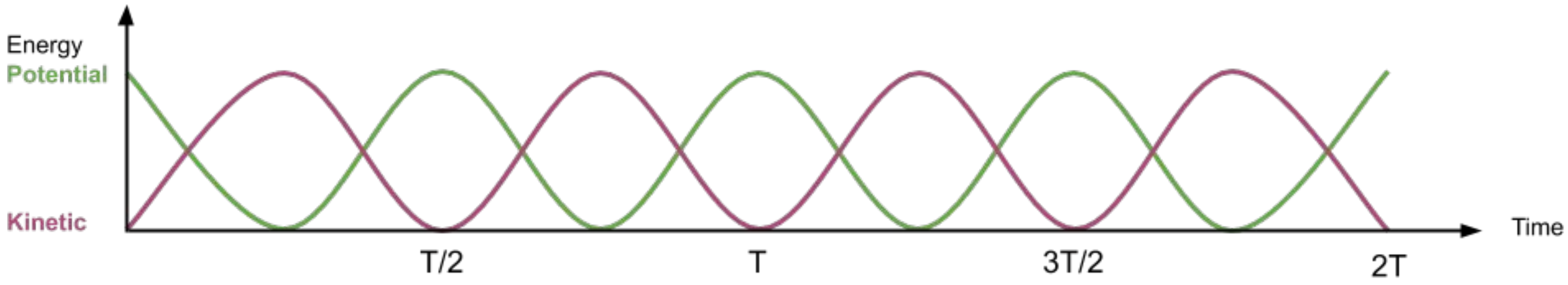
Energy of SHM

When an object oscillates with SHM, energy is **transferred** between kinetic and potential energies. These transfers are as follows:

- At maximum positive displacement, the potential energy is at a **maximum**
- As the object travels towards the equilibrium position, **potential energy** is transferred to **kinetic energy** as the object accelerates
- At the **equilibrium** position, the kinetic energy is at a **maximum** and the potential energy is at a minimum
- At maximum negative displacement, the **potential energy** is again at a maximum

SHM and Energy

The energy transfers that occur in SHM can be graphed:



Assuming **no damping** occurs:

$E_k + E_p$ = **total** energy and it is **constant** unless damping occurs.
There are **two** maximum E_k/E_p per **cycle**.

Damping

Damping of a SHM system occurs when **energy** is transferred **out** of the system. This results in the total energy no longer being **constant**. In reality **all** systems will experience some form of damping force, such as:

- Friction between components
- Air resistance

Systems are often also **deliberately** damped. Examples of this are:

- Car suspension systems
- Springed doors to prevent slamming
- Swings
- Speedometer dials

Types of Damping

There are **four** main types of damping that you need to be aware of:

1. **Light damping** is where the oscillations are damped slowly, and is normally the type of damping caused by forces such as air resistance and friction
2. **Heavy damping** is where the oscillations still continue but are brought to a stop more quickly
3. **Critical damping** involves stopping the oscillations in the quickest time possible
4. **Overdamping** is caused when the force is too great and stops the oscillations, but takes longer to return to the equilibrium position

Vibrations

The vibrations that occur in SHM can be one of **two** types:

1. **Free vibrations:** The frequency a system tends to vibrate at in a free vibration is called the natural frequency.
2. **Forced vibrations:** A driving force causes the system to vibrate at a different frequency. For higher driving frequencies, the phase difference between the driver and the oscillations rises to π radians. For lower frequencies, the oscillations are in phase with the driving force. When resonance occurs, which is when it most efficiently transfers energy to the system, the phase difference will be $\pi/2$ radians.

Resonance

Resonance takes place when the driving frequency is **equal** to the natural frequency of the system. You should know that at resonance:

- The rate of **energy transfer** is at a **maximum**
- The driving force is **π/2** out of phase and **ahead** of the oscillations
- The **amplitude** of oscillation is at a **maximum**

In some situations resonance is **wanted** such as for **musical instruments**, however in other situations resonance is **unwanted**. Bridges and towers often have to be damped so that footfall or earthquakes don't produce oscillations at the structure's natural frequency. This is because the large amplitude oscillations caused by resonance could **damage** the structure.