# AQA Physics A-level

## Section 4: Mechanics and materials

Notes

## 3.4.1 Force, energy and momentum

### 3.4.1.1 - Scalars and vectors

Scalars and vectors are physical quantities, **scalars** describe only a magnitude while **vectors** describe magnitude and direction. Below are some examples:

| Scalars | Vectors |
|---|---|
| Distance, speed, mass, temperature | Displacement, velocity, force/weight, acceleration |

There are two methods you can use to add vectors:

**Calculation -** This should be used when the two vectors are perpendicular.

For example, two forces are acting perpendicular to each other and have magnitudes of 5 N and 12 N. Find the resultant force, and its direction from the horizontal.

To find the resultant vector (R) you can use **pythagoras**:

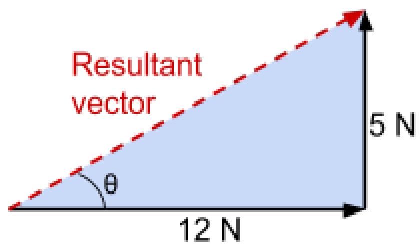
$$12^2 + 5^2 = 169 = R^2 \qquad R = 13\ N$$

In order to find the direction, you can use **trigonometry**:

$$\tan\theta = \frac{5}{12} \qquad \theta = 22.6°$$

Direction = 22.6° from the horizontal

It is very important to state how the angle you find signifies the direction.



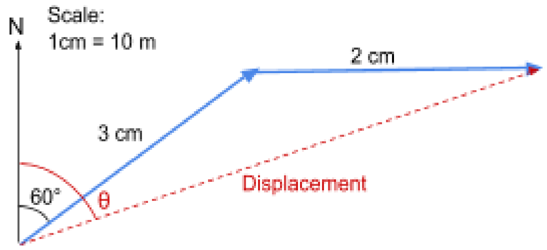

**Scale drawing -** This should be used when vectors are at angles other than 90°**.**

For example, a ship travels 30 m at a bearing of 060°, then 20 m east. Find the magnitude and direction of its displacement from its starting position.

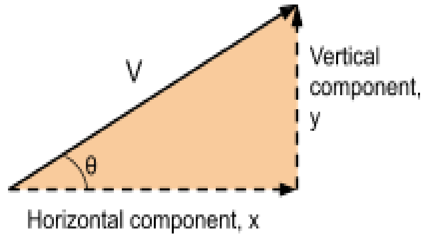
You will need to draw a **scale diagram**, using a ruler and a protractor as shown on the right. Make sure to show the scale you are using.

Finally, measure the missing side and convert it to the magnitude using your scale and measure the missing angle θ, to find the bearing of the displacement.

Magnitude = 4.9 cm = 49 m to scale          Direction = 072°




The opposite of adding two vectors is called **resolving vectors**, and is done using trigonometry. It is extremely helpful to do this in certain situations because vectors which are perpendicular don't affect each other, so can be evaluated separately.

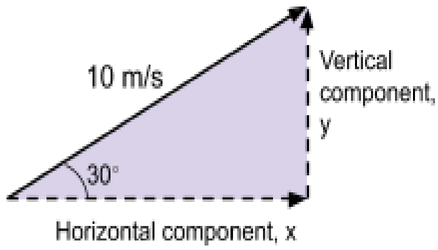



There are formulas to show how to resolve the vector V, into its components x and y.

**x = V cos θ**          **y= V sin θ**

However, if you struggle with remembering formulas a good hint to remember is:

If you are moving from the original vector through the angle θ to get to your component, use cos.
If you are moving away from the angle θ to get to your component, use sin.



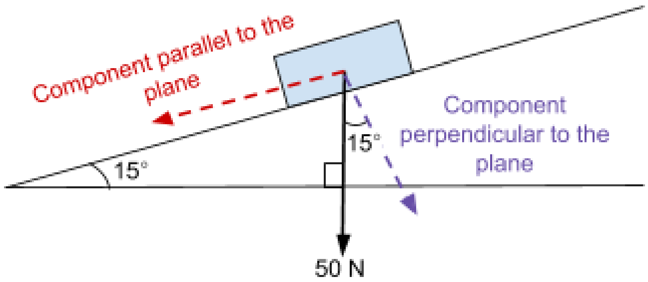

For example, a ball has been fired at a velocity of 10 m/s, at an angle of 30° from the horizontal, find the vertical and horizontal components of velocity.

x= 10 cos 30°
= 8.7 m/s

y= 10 sin 30°
= 5 m/s

This next example is much trickier as it involves a plane, however the method is exactly the same:

A block of weight 50N is resting on a plane inclined at 15°. Find the vertical and horizontal components of weight acting on the block.




Using the fact that the angles in a triangle add up to 180°, you can see that the angle between the component of weight perpendicular to the plane and weight is also 15°. Using this fact and the useful hint in red above, you can easily find the components of weight.

Parallel component = 50 sin 15°
(because you are moving away from the angle) = 12.9 N

Perpendicular component = 50 cos 15°
(because you are moving through the angle) = 48.3 N

For an object to be in **equilibrium**, the sum of all of the forces acting on it must be zero. If an object is in equilibrium it has no resultant force and therefore it is either at rest or moving at a constant velocity as according to Newton's first law.

You can show an object is in equilibrium by either:
- Adding the horizontal and vertical components of the forces acting on it, showing they equal zero.
- Or if there are 3 forces acting on the object you can draw a scale diagram, if the scale diagram forms a **closed triangle**, then the object is in equilibrium.

### 3.4.1.2 - Moments

The **moment** of a force about a point is the force multiplied by the perpendicular distance from the line of action of the force to the point.

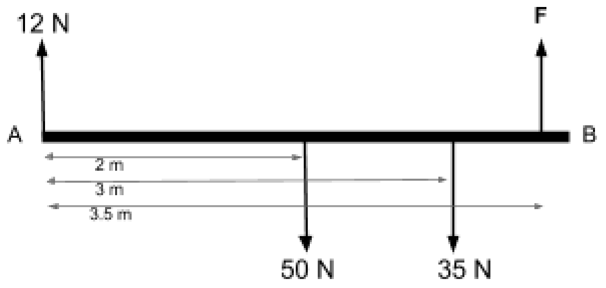
**Moment = Force X Perpendicular distance to line of action of force from the point**

A **couple** is a pair of coplanar forces (meaning they are forces within the same plane), where the two forces are equal in magnitude but act in opposite directions.

To find the **moment of a couple**, you multiply the one of the forces by the perpendicular distance between the lines of action of the forces.

**Moment of a couple = Force X Perpendicular distance between the lines of action of forces**

The **principle of moments** states that for an object in equilibrium, the sum of anticlockwise moments about a pivot is equal to the sum of clockwise moments.




You can use this fact to answer certain questions, for example:
Find the value of F from the diagram on the right.

**Σ clockwise moments = Σ anticlockwise moments**

Taking moments around A:

$$(2 \times 50) + (3 \times 35) = (3.5 \times F)$$

$$205 = 3.5F \qquad F = 58.6\ N$$

Note, in the example moments are taken about A, as the distance from A to A is 0, the moment caused by the 12 N force is also 0, therefore it can be ignored.

The **centre of mass** of an object is the point at which an object's mass acts.
If an object is described as **uniform**, its centre of mass will exactly at its centre.

### 3.4.1.3 - Motion along a straight line

**Speed** - This is a scalar quantity which describes how quickly an object is travelling.

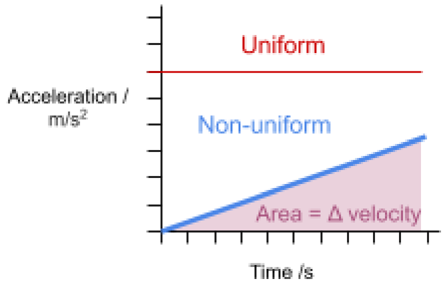
**Displacement (s)** - The overall distance travelled from the starting position (includes a direction as it is a vector quantity).

**Velocity (v)** - rate of change of displacement - $\frac{\Delta s}{\Delta t}$

**Acceleration (a)** - rate of change of velocity - $\frac{\Delta v}{\Delta t}$

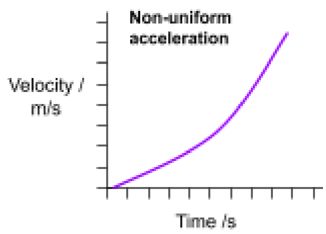
**Instantaneous velocity** is the velocity of an object at a specific point in time. It can be found from a displacement-time graph by drawing a tangent to the graph at the specific time and calculating the gradient.
**Average velocity** is the velocity of an object over a specified time frame. It can be found by dividing the final displacement by the time taken.

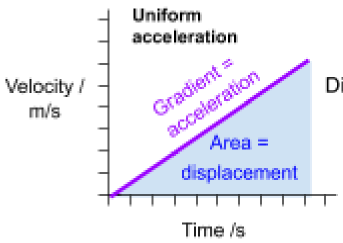
**Uniform acceleration** is where the acceleration of an object is constant.

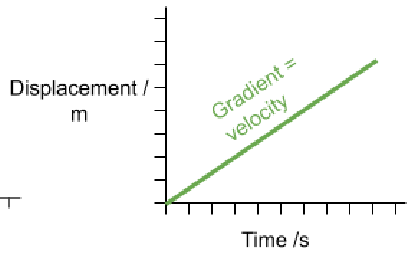



**Acceleration-time graphs** represent the change in acceleration over time. The area under the graph is change in velocity.

**Velocity-time graphs** represent the change in velocity over time. The the gradient of a velocity time graph is acceleration, and the area under the graph is displacement. **Displacement-time graphs** show change in displacement over time, and so their gradient represents velocity.




When an object is moving at **uniform acceleration**, you can use the following formulas:

$$v = u + at \qquad s = \left(\frac{u+v}{2}\right)t \qquad s = ut + \frac{at^2}{2} \qquad v^2 = u^2 + 2as$$

Where **s** = displacement, **u** = initial velocity, **v** = final velocity, **a** = acceleration, **t** = time

When approaching questions which require the use of these formulas, it is useful to write out the values you know, and the ones you want to find out in order to more easily choose the correct formula to use.

For example:
A stone is **dropped** from a bridge 50 m above the water below. What will be its final velocity (v) and for how long does it fall (t)?

Note, in this example the stone is dropped therefore we can assume, initial velocity is zero. Also because the stone is dropped we know its acceleration will be g (9.81 m/s$^2$), which is the acceleration due to gravity.

**s = 50 m** **u = 0 m/s** **v = ?** **a = 9.81 m/s²** **t = ?**

Using $v^2 = u^2 + 2as$, you can find v.

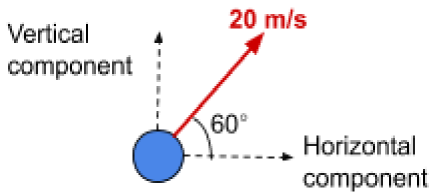
$v^2 = 0^2 + 2 \times 9.81 \times 50$ $v^2 = 981$ $v = 31.3$ m/s

Using $s = ut + \frac{at^2}{2}$, you can find t.

$50 = 4.905t^2$ $t^2 = 10.19$ $t = 3.19$ s

### 3.4.1.4 - Projectile motion

The vertical and horizontal components of a projectile's motion are **independent**, therefore the projectile's horizontal and vertical motion can be evaluated separately using the uniform acceleration formula, where acceleration is constant.




**For example:**
A ball is projected from the ground at 20 m/s, at an angle of 60° to the horizontal. Find the time taken for the ball to reach the ground and its maximum height. Ignore the effect of air resistance.

Firstly, you must resolve the initial speed into its components:

Vertical component = 20 sin 60° = 17.3 m/s
Horizontal component = 20 cos 60° = 10 m/s

(*To answer this particular question, you only need to consider the vertical component but this is not always the case*).

Maximum vertical height occurs when the vertical component of velocity first becomes 0, therefore:
**s = ?** **u = 17.3 m/s** **v = 0 m/s** **a = -g (-9.81 m/s² )** **t = ?**

Using $v^2 = u^2 + 2as$, you can find s.

$0 = 17.3^2 + 2 \times -9.81 \times s$ $19.62s = 300$ $s = 15.3$ m

**Maximum height = 15.3 m**

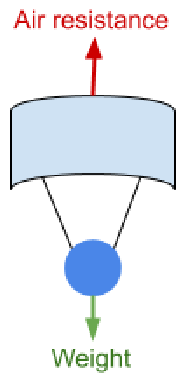
Using $v = u + at$, you can find t.

$0 = 17.3 - 9.81t \qquad 9.81t = 17.3 \qquad t = 1.76$ s

Using the fact that the time for the journey will be double the time to reach the maximum height:
**Time to reach ground = 3.5 s**

**Free fall** is where an object experiences an acceleration of g.

**Friction** is a force which opposes the motion of an object, and it is also known as drag or air resistance when considering friction experienced in a fluid. Frictional forces convert kinetic energy into other forms such as heat and sound.




The magnitude of **air resistance** increases as the speed of the object increases.

**Lift** is an upward force which acts on objects travelling in a fluid, it is caused by the object creating a change in direction of fluid flow, and it acts perpendicular to the direction of fluid flow.

**Terminal speed** occurs where the frictional forces acting on an object and driving forces are equal, therefore there is no resultant force and so no acceleration so the object travels at a constant speed. A good example of an object reaching terminal speed, or terminal velocity as it is also known, is a skydiver:

- As they leave the plane they accelerate because their weight is greater than the air resistance.
- As the skydiver's speed increases, the magnitude of air resistance also increases. This continues until the force of weight and air resistance become equal, at which point terminal velocity is reached.

**Air resistance** will affect both the vertical and horizontal components of a projectile's motion as shown in the diagram below:

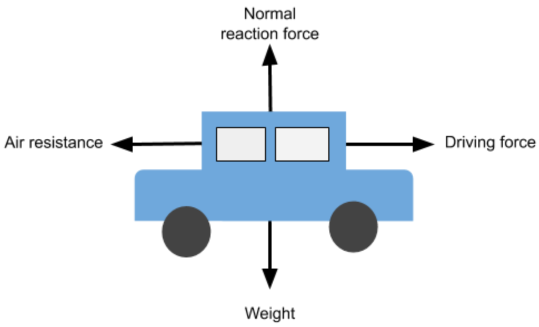

As you can see, with air resistance the maximum height is reached earlier, and the vertical and horizontal distance travelled decreases.

### 3.4.1.5 - Newton's laws of motion

➔ **Newton's 1st law -** An object will remain at rest or travelling at a constant velocity, until it experiences a resultant force.

- **Newton's 2nd law -** The acceleration of an object is proportional to the resultant force experienced by the object: $F = ma$ where **F** is the resultant force, **m** is the object's mass and **a** is its acceleration.
- **Newton's 3rd law -** For each force experienced by an object, the object exerts an equal and opposite force.

A **free-body diagram**, is a diagram which shows all the forces that act on an object, below is an example:




A free-body diagram will show you how each of the forces acting on the object compare with each other. In this example, all the arrows look equal therefore we know that the car is travelling at a constant velocity.

Here is an example where you would have to use Newton's 2nd law and a free-body diagram:

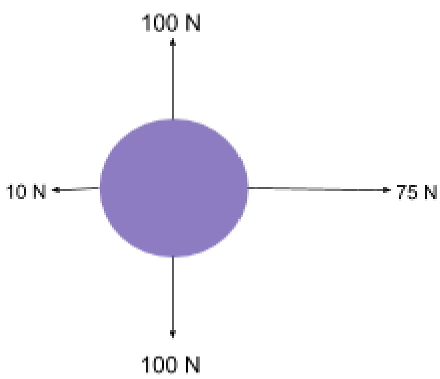
Find the acceleration of the ball in the diagram below:

Firstly, you must find the mass (m) of the ball as you are only given the weight.

As Weight = mass x g, the mass = $\frac{100}{9.81}$ = 10.2 kg

Next, you must find the resultant force (F).
$75 - 10 = 65$ N to the right

Finally, you can use F = ma, to find acceleration:
$65 = 10.2 \times a$ $\quad a = \frac{65}{10.2}$ $\quad a =$ **6.4 m/s²**



### 3.4.1.6 - Momentum

**Momentum** is the product of mass and velocity of an object.

$$Momentum = mass \times velocity$$

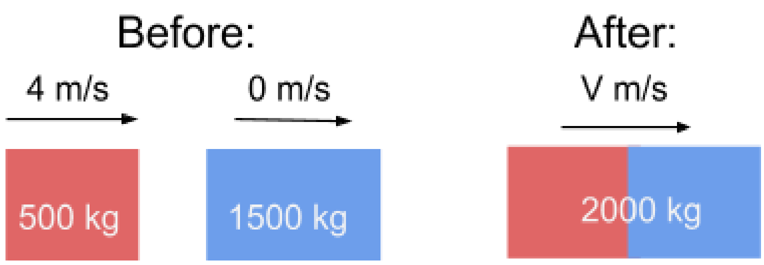
**Momentum** is always conserved in any interaction where no external forces act, which means the momentum before an event (e.g a collision) is equal to the momentum after. This fact is used to find the velocity of objects after collisions, for example:

A car with a mass of 500 kg, and a velocity of 4 m/s, collides with a stationary truck with a mass of 1500 kg. The two vehicles join together and move on with a velocity V. Find the value of V.

First find the momentum before the collision.
Total momentum before = $(500 \times 4) + (1500 \times 0)$ = 2000 kgm/s

**Total momentum before = Total momentum after**

Therefore, $2000 = 2000 \times V$ $V =$ **1 m/s**

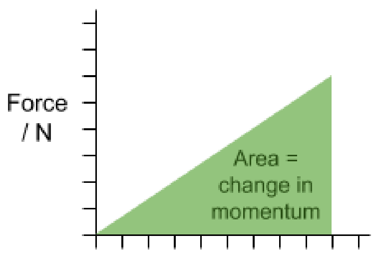



Newton's 2nd law states $F = ma$ , therefore, $F = \frac{\Delta(mv)}{\Delta t}$ as $a = \frac{\Delta v}{\Delta t}$ . From this you can see that force is the rate of change of momentum.

Rearranging the above equation leads to $F\Delta t = \Delta(mv)$ (where F is constant, and t is an impact time).

$F\Delta t$ is known as impulse, and **impulse** is the change in momentum as demonstrated in the equation above.

The **area of a force-time graph** is $F \times \Delta t$ , therefore it is also equal to change in momentum:



Here is an example question about impulse.

A ball is hit with a baseball bat with a force of 100 N, with an impact time of 0.5 s. What is the change in momentum of the ball?

To find the impulse we must use the equation $F\Delta t = \Delta(mv)$.
Change in momentum = $100 \times 0.5$ = 50 kgm/s

An important application of calculating impulse is during the design of car safety features. For example, cars have **crumple zones**, which crumple upon impact, **seat belts** which stretch upon an impact, and **air bags** all of which increases the impact time of the car or the passenger. This causes the force exerted on passengers to decrease, meaning they are less likely to be seriously injured.

There are two types of collisions:

- **Elastic** - where **both** momentum and kinetic energy are conserved
- **Inelastic** - where **only** momentum is conserved, while some of the kinetic energy is converted into other forms (e.g heat, sound, gravitational potential) and may be larger or smaller after a collision

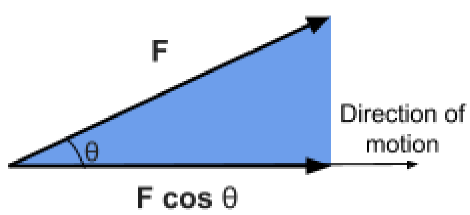
If the objects in a collision stick together after the collision, then it is an **inelastic collision**.

An **explosion** is another example of an inelastic collision as the kinetic energy after the collision is greater than before the collision.

### 3.4.1.6 - Work, energy and power

**Work done (W)** is defined as the force causing a motion multiplied by the distance travelled in the direction of the force.

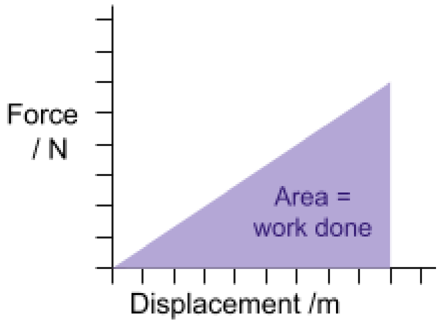
$W = Fs\cos\theta$ where **s** is the distance travelled and θ is the angle between the direction of the force and the direction of motion.




As work is a measure of energy transfer, the rate of doing work = the rate of energy transfer.

**Power (P)** is the rate of energy transfer therefore, $P = \frac{\Delta W}{\Delta t} = \frac{F \times \Delta s}{\Delta t} = Fv$ as $v = \frac{\Delta s}{\Delta t}$.

For a variable force you can’t use the formula to find work done, however the area under a **force-displacement graph** is equal to the work done.




**Efficiency** is a measure of how efficiently a system transfers energy. It is calculated by dividing the useful power output by total energy input.

$$Efficiency = \frac{useful\ output\ power}{input\ power} \qquad Efficiency\ (percentage) = \frac{useful\ output\ power}{input\ power} \times 100$$

If you multiply the value of efficiency by 100, you receive the value of efficiency as a percentage.

### 3.4.1.7 - Conservation of energy

The **principle of conservation of energy** states that energy cannot be created or destroyed, but can be transferred from one form to another. Therefore, the total energy is a closed system stays constant.

$$Total\ energy\ in = Total\ energy\ out$$

You may need to use this when answering questions involving gravitational potential energy and kinetic energy.

Change in gravitational potential energy =

$$\Delta E_p = mg\Delta h$$

Kinetic energy =

$$E_k = \frac{1}{2}mv^2$$

As an example, think about a ball being thrown up into the air. The thrower gives the ball kinetic energy therefore it moves upwards, however as it does, the ball slows down because kinetic energy is transferred to gravitational potential energy. Eventually, all of the kinetic energy is transferred to gravitational potential energy and the ball stops momentarily, after which the ball's gravitational potential energy is converted back into kinetic energy and the ball falls to the ground.

It is very important to note that work is being done by the ball to work against resistive forces, therefore the initial kinetic energy given to the ball is not equal to the maximum gravitational potential when the ball has stopped in mid-air. This is because the kinetic energy of the ball is being transferred to the environment in the form of heat due to air resistance.

Here is an example of a question where you could use the principle of conservation of energy, (noting that the effect of air resistance is ignored, therefore $\Delta E_p = \Delta E_k$ ):

As a simple pendulum of mass 500g swings, it rises up by a height of 10cm at its maximum amplitude from its equilibrium position. What is the maximum speed the pendulum can reach during its oscillation? (Ignore the effect of air resistance)

Firstly, find the maximum gravitational potential energy (which will be at the amplitude).
$\Delta E_P = 0.5 \times 9.81 \times 0.1$ = 0.4905 J

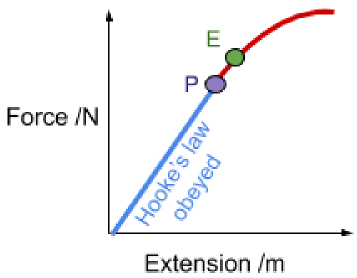
Then, equate this to the kinetic energy formula (subbing in known values), and rearrange to find v.
$\frac{1}{2} \times 0.5 \times v^2 = 0.4905$ $v^2 = 1.962$ $v = 1.4$ m/s

## 3.4.2 Materials

### 3.4.2.1 - Bulk properties of solids

The **density** of a material is its mass per unit volume, and it's a measure of how compact a substance is.

**Hooke's law** states that extension is **directly proportional** to the force applied, given that the environmental conditions (e.g temperature) are kept constant. This can be shown by the straight part of the force-extension graph shown to the right; a **straight line graph through the origin**, which shows the force and extension are **directly proportional**.




The **limit of proportionality (P)** is the point after which Hooke's law is no longer obeyed. The **elastic limit (E)** is just after the limit of proportionality and if you increase the force applied beyond this, the material will deform plastically (be permanently stretched).

Hooke's law can be described as the equation $F = k\Delta L$, where k is the spring constant, which is a measure of the stiffness of the spring, and $\Delta L$ is the extension.

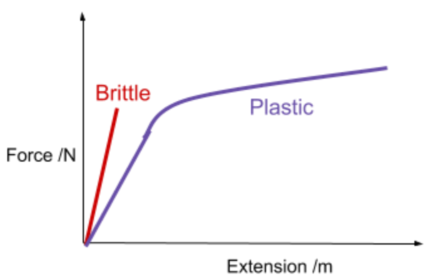
**Tensile stress** - Force applied per unit cross-sectional area.

**Tensile strain** - This is caused by tensile stress, and is defined as the extension over the original length.

$$Stress = \frac{F}{A}$$
$$Strain = \frac{\Delta L}{L}$$

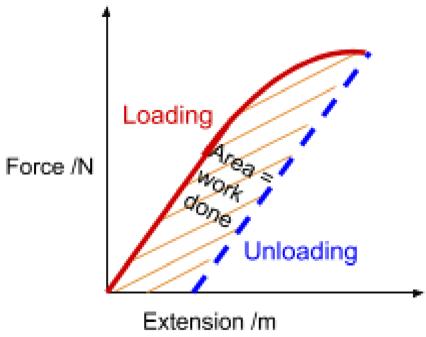
When work is done on a material to stretch or compress it, this energy is stored as **elastic strain energy**. This value cannot be calculated using the formula $W = Fs\cos\theta$ because the force is variable, however you can find it by calculating the area under a force-extension graph. Therefore, **elastic strain energy =** $\frac{1}{2}F\Delta L$.

**Breaking stress** is the value of stress at which the material will break apart, this value will depend on the conditions of the material e.g its temperature.



Force-extension graphs can show the properties of a specific object. There are two main behaviours that a material can exhibit on a force-extension graph:

- **Plastic** - This is where a material will experience a large amount of extension as the load is increased, especially beyond the elastic limit
- **Brittle** - This is where a material will extend very little, and therefore is likely to fracture (break apart) at a low extension.

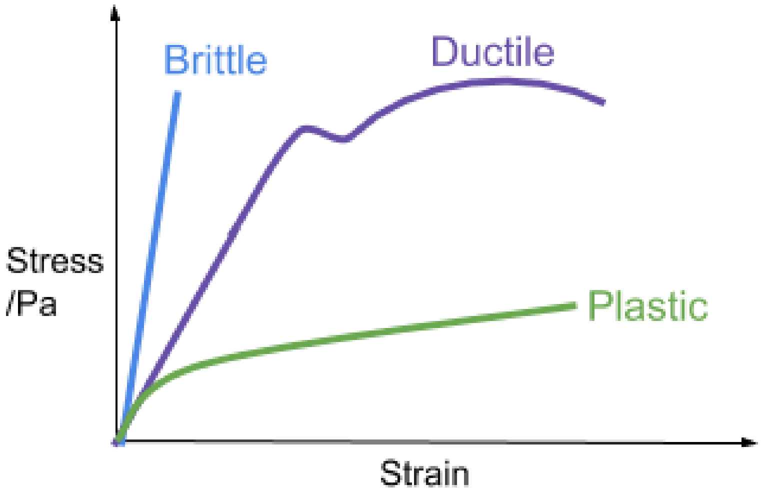



Once a material is stretched beyond its elastic limit, a force-extension graph showing loading and unloading, will not return to the origin, however the loading and unloading lines will be parallel because the material's stiffness is constant, as shown on the left. The area between the loading and unloading line is the work done to permanently deform the material.

When a stretch is **elastic** (material returns to original shape once force is removed), all the work done is stored as elastic strain energy, however when a stretch is **plastic** work is done to move atoms apart, so energy is not stored as elastic strain energy but is dissipated as heat.
This fact is used when designing safety features for cars, for example work is done to deform crumple zones plastically to decrease the car's kinetic energy, and seat belts stretch in order to convert some of the passenger's kinetic energy into elastic strain energy.

When a spring is hung vertically and stretched, kinetic energy is converted into elastic strain energy, and if it the force is removed, the elastic strain energy will be transferred back to kinetic energy (and in a similar fashion to the ball example in **3.4.1.7 - Conservation of energy**), this kinetic energy is then transferred to gravitational potential energy as it rises.

**Stress-strain graphs** are similar to force-extension graphs, however they describe the behaviour of a material rather than the behaviour of a specific object. They show a material's **ultimate tensile stress** (UTS), which is the highest point on the graph as it shows the maximum stress the material can withstand. Their shape can also show whether a material is ductile (can undergo a large amount of plastic deformation before fracturing), brittle, or plastic.




### 3.4.2.2 - Young modulus

The **Young modulus** is a value which describes the stiffness of a material.
It is known that up to the limit of proportionality, for a material which obeys Hooke's law, stress is proportional to strain, therefore the value of stress over strain is constant, this value is the Young modulus.

$$Young\ Modulus\ (E) = \frac{Tensile\ Stress}{Tensile\ strain}$$

Using the formulas from the previous section, this can be rewritten as:

$$E = \frac{FL}{\Delta LA}$$

You can find the Young modulus of a material from a stress-strain graph by finding the **gradient** of the straight part of the graph.