AQA Physics A-Level

Section 1: Measurements and their Errors

Notes

3.1 Measurements and their errors

3.1.1 - Uses of SI units and their prefixes

SI units are the fundamental units, they are made up of:

- Mass (m): **kg** (kilograms)
- Length (l): **m** (metres)
- Time (t): **s** (seconds)
- Amount of substance (n): **mol** (moles)
- Temperature (t): **K** (kelvin)
- Electric current (I): **A** (amperes)

The SI units of quantities can be **derived** by their equation, e.g. F=ma
For example, to find the SI units of force (F) multiply the units of mass and acceleration kg x m s^{-2} gives kgm s^{-2} (Also known as N)

The SI units of voltage can be found by a series of steps:

- V= $\frac{E}{Q}$ where E is energy and Q is charge , E=½ m v^2 so the SI units for energy is kg m^2s^{-2} (the units for speed (v) are ms^{-1} so squaring these gives m^2s^{-2})
- Q=It (where I is current) so the units for Q are As (ampere seconds)
- So V= $\frac{kgm^2s^{-2}}{As}$ V= $kgm^2s^{-3}A^{-1}$

Below are the **prefixes** which could be added before any of the above SI units:

Name	Symbol	Multiplier
Tera	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
Kilo	k	10^{3}
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}

Some examples:

6pF (picofarads) is **6 x 10^{-12} F**

9GΩ (gigaohms) is **9 x 10^{9} Ω**

10μm (micrometres) is **10 x 10^{-6} m**

Converting mega electron volts to joules:
1eV=1.6x 10^{-19} J

e.g. convert 76 MeV to joules:

First, convert from MeV to eV by multiplying by 10^6	76 x 10^6 eV
Then convert to joules by multiplying by 1.6x 10^{-19}	1.216 x 10^{-11} J

Converting **kWh** (kilowatt hours) to Joules:
1 kW = 1000 J/s   1 hour= 3600s

1kWh = 1000 x 3600
= 3.6x 10^6 J
= 3.6 MJ

3.1.2 - Limitation of Physical Measurements

Random errors affect precision, meaning they cause differences in measurements which causes a spread about the mean. You **cannot** get rid of all random errors.
An example of random error is electronic noise in the circuit of an electrical instrument
To reduce random errors:

- Take **at least 3 repeats** and calculate a **mean**, this method also allows anomalies to be identified.
- Use **computers/data loggers/cameras** to reduce human error and enable smaller intervals.
- Use **appropriate equipment**, e.g a micrometer has higher resolution (0.1 mm) than a ruler (1 mm).

Systematic errors affect accuracy and occur due to the apparatus or faults in the experimental method. Systematic errors cause all results to be too high or too low by the same amount each time.
An example is a balance that isn't zeroed correctly (zero error) or reading a scale at a different angle (this is a parallax error).
To reduce systematic error:

- **Calibrate** apparatus by measuring a known value (e.g. weigh 1 kg on a mass balance), if the reading is inaccurate then the systematic error is easily identified.
- In radiation experiments correct for background radiation by measuring it beforehand and excluding it from final results.
- Read the meniscus (the central curve on the surface of a liquid) at eye level (to reduce parallax error) and use controls in experiments.

Precision	Precise measurements are consistent, they fluctuate slightly about a mean value - this doesn't indicate the value is accurate
Repeatability	If the original experimenter can redo the experiment with the same equipment and method then get the same results it is repeatable

Reproducibility	If the experiment is redone by a different person or with different techniques and equipment and the same results are found, it is reproducible
Resolution	The smallest change in the quantity being measured that gives a recognisable change in reading
Accuracy	A measurement close to the true value is accurate

The **uncertainty** of a measurement is the bounds in which the accurate value can be expected to lie e.g. 20°C ± 2°C , the true value could be within 18-22°C

Absolute Uncertainty: uncertainty given as a fixed quantity e.g. 7 ± 0.6 V

Fractional Uncertainty: uncertainty as a fraction of the measurement e.g. $7 \pm \frac{3}{35}$ V

Percentage Uncertainty: uncertainty as a percentage of the measurement e.g. $7 \pm 8.6\%$ V

To reduce percentage and fractional uncertainty, you can measure larger quantities.

Resolution and Uncertainty

Readings are when one value is found e.g. reading a thermometer, measurements are when the difference between 2 readings is found, e.g. a ruler (as both the starting point and end point are judged).

The **uncertainty in a reading** is ± half the smallest division,

e.g. for a thermometer the smallest division is 1°C so the uncertainty is ±0.5°C.

The **uncertainty in a measurement** is at least ±1 smallest division,

e.g. a ruler, must include **both** the uncertainty for the start and end value, as each end has ±0.5mm, they are added so the uncertainty in the measurement is ±1mm.

Digital readings and given values will either have the uncertainty quoted or assumed to be ± the last significant digit e.g. 3.2 ± 0.1 V, the **resolution** of an instrument affects its uncertainty.

For **repeated data** the uncertainty is half the range (largest - smallest value), show as mean $\pm \frac{range}{2}$.

You can reduce uncertainty by fixing one end of a ruler as only the uncertainty in one reading is included. You can also reduce uncertainty by measuring multiple instances,
e.g. to find the time for 1 swing of a pendulum by measuring the time for 10 giving e.g. 6.2 ± 0.1 s, the time for 1 swing is 0.62 ± 0.01s (the uncertainty is also divided by 10).

Uncertainties should be given to the **same number of significant figures** as the data.

Combining uncertainties

- **Adding / subtracting data - ADD ABSOLUTE UNCERTAINTIES**

E.g. A thermometer with an uncertainty of ± 0.5 K shows the temperature of water falling from 298 ± 0.5 K to 273 ± 0.5K, what is the difference in temperature?

298-273 = 25K   0.5 + 0.5 = 1K (add absolute uncertainties)   difference = 25 ± 1 K

- **Multiplying / dividing data - ADD PERCENTAGE UNCERTAINTIES**

E.g. a force of 91 ± 3 N is applied to a mass of 7 ± 0.2 kg, what is the acceleration of the mass?

a=F/m =91/7 = 13ms^{-2} percentage uncertainty= $\frac{uncertainty}{value} \times 100$

Work out % uncertainties $\frac{3}{91} \times 100 + \frac{0.2}{7} \times 100$ = 3.3% + 2.9% add % uncertainties
= 6.2%

So a= $13 \pm 6.2\%$ ms^{-2} 6.2% of 13 is 0.8

a=13 ± 0.8 ms^{-2}

- **Raising to a power - MULTIPLY PERCENTAGE UNCERTAINTY BY POWER**

The radius of a circle is 5 ± 0.3 cm, what is the percentage uncertainty in the area of the circle?
Area = π x 25 = 78.5 cm^2
Area = πr^2
% uncertainty in radius = $\frac{0.3}{5} \times 100$ = 6% % uncertainty in area = 6 x 2 (2 is the power from r^2)
= 12%

78.5 $\pm$12% cm^2

Uncertainties and graphs
Uncertainties are shown as **error bars** on graphs,
e.g. if the uncertainty is 5mm then have 5 squares of error bar on either side of the point
A line of best fit on a graph should go through all error bars (**excluding anomalous points**).

The **uncertainty in a gradient** can be found by lines of best and worst fit, this is especially useful when the gradient represents a value such as the acceleration due to gravity:
- Draw a steepest and shallowest line of worst fit, it **must** go through all the error bars.
- Calculate the gradient of the line of best and worst fit, the uncertainty is the difference between the best and worst gradients.

percentage uncertainty = $\frac{|best\ gradient - worst\ gradient|}{best\ gradient} \times 100\%$

(modulus lines show it's always positive)

When the best and worst lines have different y intercepts, you can find the **uncertainty in the y-intercept**, which is |best y intercept-worst y intercept|:

percentage uncertainty = $\frac{|best\ y\ intercept - worst\ y\ intercept|}{best\ y\ intercept} \times 100\%$

Alternatively, the average of the two maximum and minimum lines can be used to calculate the percentage uncertainty:

percentage uncertainty = $\frac{max\ gradient - min\ gradient}{2} \times 100\%$

3.1.3 - Estimation of physical quantities

Orders of magnitude - Powers of ten which describe the size of an object, and which can also be used to compare the sizes of objects.
E.g: The diameter of nuclei have an order of magnitude of around 10^{-14} m.
100 m is two orders of magnitude greater than 1m.

You may be asked to give a value to the **nearest order of magnitude**, here you must simply calculate the value the question is asking you for and give it only as a power of ten
E.g If the diameter of a hydrogen atom is 1.06 $\times 10^{-10}$ m, find the approximate area of the entire atom (assuming it is perfectly spherical), to the nearest order of magnitude.
Find the area using A= πr^2 $\pi \times (0.53 \times 10^{-10})^2$ = 8.82×10^{-21} m = 1×10^{-20} m (to 1 s.f)
Therefore the area to the nearest order of magnitude is 10^{-20} m.

Estimation is a skill physicists must use in order to approximate the values of physical quantities, in order to make comparisons, or to check if a value they've calculated is reasonable.