Chapter 3 Special relativity

3.1 About motion

Learning objectives:

- What do we mean by absolute motion and relative motion?
- What experimental evidence is there that all motion is relative?
- Why do we think the speed of light does not depend on the speed of the observer?

Absolute motion?

Galileo and Newton based their theories and laws of motion on the assumption that motion can always be detected. You can usually tell if an object is moving towards or away from you but can you tell if you are in motion or if the object is in motion? Galileo and Newton thought of space and time as absolute quantities that do not depend on the motion of any observer and are therefore the same throughout the universe. For example, a racing car that travels a distance of 10 000 metres in a straight line in 100 seconds according to timing devices at the trackside should record the same timing on an on-board timer.

Now imagine a 22nd century rocket race over a distance of 30 million kilometres. Light takes 100 seconds to travel this distance so you would expect a rocket travelling at half the speed of light to take 200 seconds. The rocket crew and the race officials would also expect a time of 200 s if, like Newton and Galileo, they thought in terms of absolute motion. However, according to Einstein's theory of special relativity, the rocket crew's on-board 'clock' would show they arrive at the finishing line 173 seconds after passing the starting line. The race officials at the starting or finishing line would still record 200 seconds on their clocks. Which timing is correct? In this chapter we will look at:

- why scientists began to question the concept of absolute motion
- how Einstein redefined our concept of motion
- experimental evidence that supports Einstein's theory of special relativity.

After Maxwell published his theory of electromagnetic waves (see Topic 2.2), many physicists thought that the waves are vibrations in an invisible substance which they called ether (or 'aether') and which they supposed exists throughout the universe. According to this hypothesis, the Earth must be moving through the ether and electromagnetic waves are vibrations in the ether. Light was thought to travel at a fixed speed relative to the ether. Detection of the ether was thought to be possible as a result of comparing the time taken by light to travel the same distance in different directions. The same idea applies to sound travelling through the air.

Imagine a very long ship with a hooter in the middle that sends out a short blast of sound on a wind-free day. If the ship is moving forwards, a person in the stern (the back end) will hear the sound before a person in the bow (the front end) because the ship's forward motion moves the person in the stern towards the sound whereas it moves the person in the bow away from the sound. By comparing when the sound is heard at each end, it is possible to tell if the ship is moving forwards or backwards or not at all.

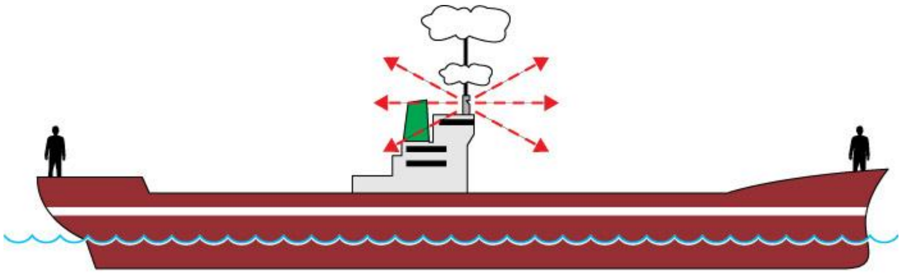

Figure 1 *At sea*

Now consider light instead of sound and the Earth as a 'ship' moving through space. The Earth moves on its orbit through space at a speed of about 30 km s^{-1} which is 0.01% of the speed of light so the differences in the travel time of light over a given distance in different directions would be very difficult to detect. This would not be so if the Earth travelled much faster so let us imagine the Earth's speed is 10% of the speed of light.

Light travels 300 m in 1 microsecond. Suppose a light pulse travels from its source to a mirror 300 m away and is then reflected back to the source, as shown in Figure 2.

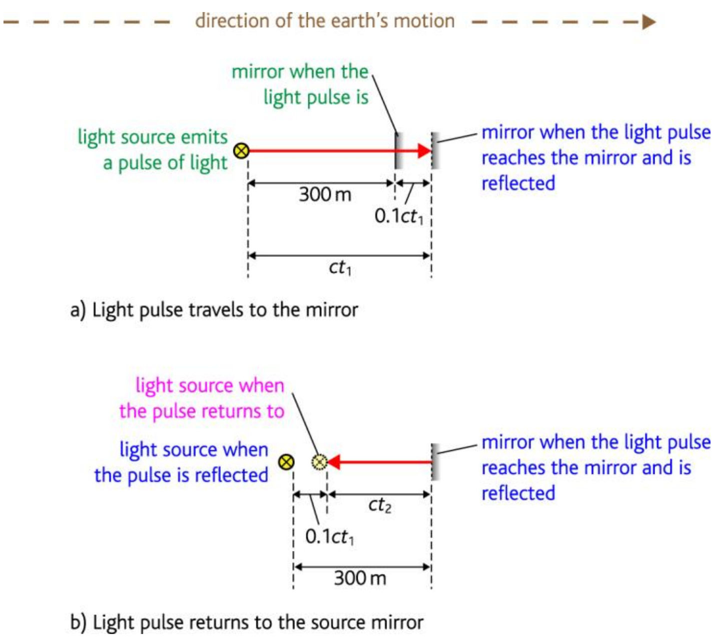

Figure 2 *Using the ether hypothesis*

According to pre-Einstein dynamics:

- If the line between the source and the mirror is parallel to the Earth's motion, in the time taken t_1 by the light pulse to reach the mirror, the pulse travels a distance ct_1. This distance is equal to 300 metres plus the distance travelled by the mirror in that time which is $0.1ct_1$. Hence $ct_1 = 300 + 0.1ct_1$ which gives $0.9ct_1 = 300$. This means that t_1 = 300 metres ÷ $0.9c$ = 1.111 microseconds.

  A similar argument for the return journey time, t_2, gives $ct_2 = 300 - 0.1ct_2$ since the light pulse is moving towards the light source. Hence t_2 = 300 metres ÷ $1.1c$ = 0.909 microseconds. So the total journey time of the light pulse would be 2.020 microseconds.

- If the line between the source and the mirror is perpendicular to the Earth's motion, light from the source would need to travel more than 300 m on each part of its journey. Its total journey time would be 2.010 microseconds, as explained in Note 1 below.

So the light pulse would take 0.01 microseconds longer to travel along the parallel path than along the perpendicular path. For light of frequency 6.0×10^{14} Hz, a difference in travel time of 0.01 microseconds corresponds to 6 million cycles (= 6.0×10^{14} Hz × 0.01 microseconds). If the same analysis is applied to the Earth moving through space at its correct speed of 0.01% of the speed of light, the difference in time travel would correspond to 6 cycles for the same distance of 300 m. A distance of 1.0 m would therefore correspond to 0.02 cycles. With this type of analysis in mind, physicists before Einstein realised that the phenomenon of light interference could be used to detect such very small differences. They confidently expected that the Earth's absolute motion through the ether could be detected.

Notes

1 Consider a pulse of light emitted from a source in a direction perpendicular to the Earth's motion towards a mirror at distance *d* then reflected back to the source. Because the source is moved by the Earth's motion while the light pulse is in transit, the pulse must travel as shown in Figure 3 from the position of the source when the light is emitted, S_1, to the mirror at position M_2 then back to the source at position S_3. $M_2S_2 = d$

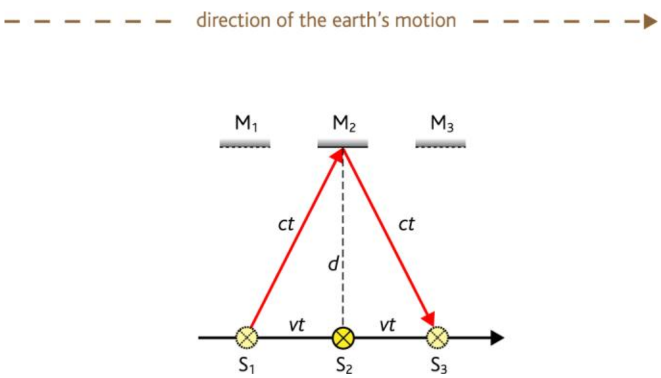

Figure 3 *Light travelling sideways to the Earth's motion*

Let *t* represent the time taken by the light source to travel from the source to the mirror (so light takes a time 2*t* to travel from the source to the mirror and back). In the time taken by the light to travel to the mirror:

- the light source moves a distance *vt* in the direction of the Earth's motion from S1 to S2, and
- the light travels a distance *ct* along the path S_1M_2.

Applying Pythagoras' theorem to triangle $S_1S_2M_2$ therefore gives $(ct)^2 = (vt)^2 + d^2$

Rearranging this equation gives $t = \dfrac{d}{\sqrt{(c^2 - v^2)}}$

Using this formula for $d = 300$ m and $v = 0.1c$ (10% of the speed of light) gives $t = 1.005$ microseconds. The total time taken $2t = 2.010$ microseconds.

2 The specification does not require the detailed analysis above. The key point to bear in mind is that the 'ether theory' predicted that absolute motion can be detected by a difference in the time taken for light to travel the same distance parallel and perpendicular to the Earth's motion.

Maxwell's hypothesis

An experiment to detect the ether was put forward by Maxwell in 1878. He suggested that a beam of light could be split into two perpendicular beams which could then be brought together again by reflection to produce an interference pattern. Interference would occur where the waves and crests of one beam overlapped with those of the other beam. According to Maxwell, rotating the whole apparatus horizontally through 90° would swap the directions of the beams and shift the interference fringes.

Maxwell's idea can be demonstrated using 3 cm microwaves as shown in Figure 4. The whole apparatus would need to be on a table that can be turned.

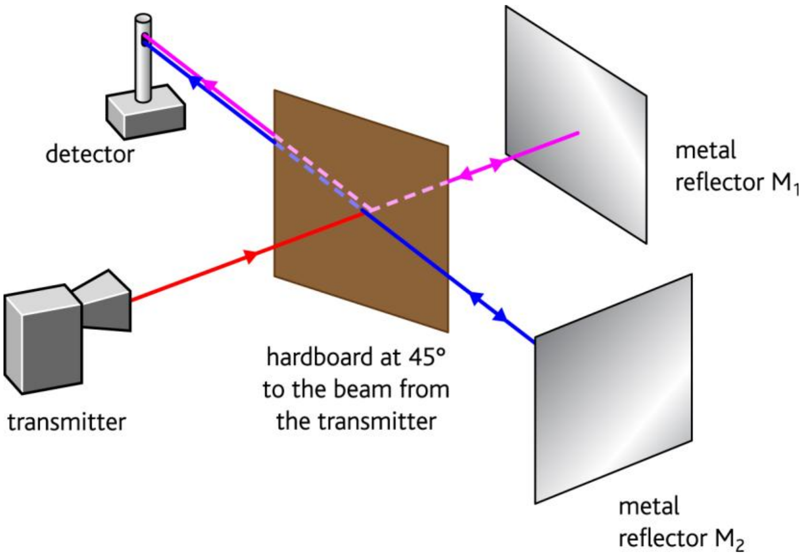

Figure 4 *Using microwaves*

The beam from the transmitter is split by the hardboard into a beam that passes through the hardboard to metal plate M_1 and a beam travelling towards metal plate M_2 due to partial reflection from the hardboard. Both beams are reflected back towards the hardboard where the beam from M_1 is partially reflected to the detector and the beam from M_2 passes straight through to the detector. The two beams reach the detector with a phase difference that depends on the difference in their path lengths. If either M_1 or M_2 are moved gradually away from the hardboard, the detector signal rises and falls repeatedly.

- When the detector signal is a maximum, the two beams arrive at the detector in phase and therefore reinforce.
- When the detector signal is a minimum, the two beams arrive at the detector out of phase by 180° and therefore cancel each other out.

To test Maxwell's idea, the whole apparatus would need to be turned through 90°. This would cause the detector signal to change if the 'ether hypothesis' is correct. Unfortunately for Maxwell, microwaves (and radio waves) were future discoveries so he could not test his idea. If the above test is carried out, no such change is detected.

Note

Movement by half a wavelength of either mirror from a position where the detector signal is a minimum takes the mirror to the next 'minimum signal' position. This is because the change of the path length of the beam that reflects from that mirror is 1 full wavelength. Hence the wavelength can be measured accurately by moving one of the mirrors though a known number of 'minimum signal' position.

The Michelson–Morley experiment

Could the ether hypothesis be tested with light using Maxwell's idea? Michelson and Morley were two American physicists who designed an experimental 'interferometer' apparatus for the purpose of testing the ether hypothesis. Figure 5 shows how their interferometer works.

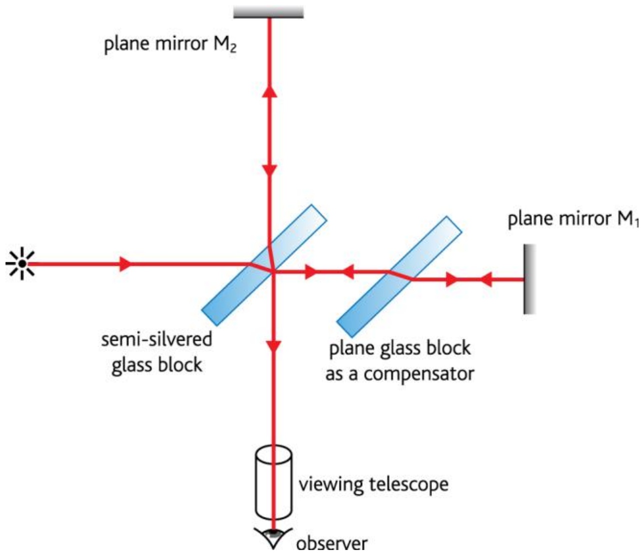

Figure 5 *The Michelson–Morley interferometer*

The light beam from the light source is split into two beams at the back surface of the semi-silvered glass block. One of the two beams continues towards M_1, reflects back to the glass block where it partially reflects into the viewing telescope. The other beam is due to partial reflection at the glass block so it travels towards mirror M_2 where it reflects back to the glass block and travels through it to the telescope. The compensator is present to ensure both beams travel through the same thickness of glass otherwise the two wave trains would not overlap.

An observer looking through the telescope sees a pattern of interference fringes because of the difference in the path lengths of the two beams.

- A bright fringe is where the two beams arrive in phase with each other.
- A dark fringe is where they arrive out of phase by 180° with each other.

Suppose the apparatus is initially aligned so the M_1 beam travels parallel to the direction of the Earth's motion and the M_2 beam travels perpendicular to the direction of the Earth's motion. Turning the apparatus through 90° in a horizontal plane would swap the beam directions relative to the Earth's motion. If the ether hypothesis is correct, this would cause the difference in the travel times of the two beams to reverse, resulting in a noticeable shift in the interference fringes.

Michelson and Morley had predicted using the ether theory that the fringes would shift by about 0.4 of a fringe width and they knew their apparatus was sensitive enough to detect a 0.05 fringe shift. However, they were unable to detect the predicted fringe shift. This 'null' result effectively finished the 'ether' theory off although some physicists tried to maintain the theory by supposing the Earth dragged the ether along with it but there was no astronomical evidence for this supposition. Physicists were presented with a major problem. Galileo's laws of dynamics and Newton's laws of motion didn't seem to work for light!

AQA Examiner's tip

Make sure you can explain why an interference pattern is observed and remember the pattern was expected to shift when the apparatus was turned through 90° – but it didn't!

Summary questions

1 **a** What is meant by:

- **i** absolute rest?
- **ii** absolute time?

b What was the purpose of the Michelson–Morley experiment?

2 In the Michelson–Morley experiment, state and explain the function of:

a the two plane mirrors

b the compensator glass block.

3 **a** Explain the formation of a dark fringe in the fringe pattern observed when looking through the telescope in the Michelson–Morley experiment.

b Describe what observation was expected when the apparatus was rotated through 90°.

4 The Michelson–Morley experiment produced a null result.

a State what the result was.

b **i** Explain why it was referred to as a null result.

- **ii** What was the significance of the null result?

3.2 Einstein's theory of special relativity

Learning objectives:

- What assumptions did Einstein make in his theory of special relativity?
- What do we mean by length contraction, time dilation and relativistic mass?
- Why do we believe that nothing can travel faster than light?

Relative motion

If we can't detect absolute motion, then perhaps absolute rest doesn't exist and all motion is relative. It seems absurd to suggest to a fellow passenger on a train or plane journey that your destination is in motion and moving towards you. Travellers can usually tell they are moving even if they can't see the ground outside because they can sense changes in their velocity. But sometimes your senses can deceive you – for example, in a train waiting at a station, you may have momentarily thought you were starting to move when an adjacent train starts to move.

In general, to detect the relative motion of an object, you need to observe the object's position against your **frame of reference** (i.e. markers that are fixed relative to your own position). Anyone in the adjacent train may think you are moving – until their own train starts to creak and rattle as it moves. Your position would be changing in the other person's frame of reference. If two trains glide perfectly smoothly past each other in darkness, could the passengers tell if their own train is moving? Yes, according to most physicists before Einstein. But no evidence was found for absolute motion from the Michelson and Morley experiment – a 'null' result which confounded the scientific community for many years.

Einstein's theory

Einstein put forward his theory of special relativity in 1905, the same year as he put forward his photon theory of light. He rejected the idea of absolute motion and started from two key ideas.

- The **laws of physics** should be the same in all **inertial** frames of reference which are frames of reference that move at constant velocity relative to each other. For example, an object released at rest in an inertial frame of reference (e.g. a passenger jet moving at constant velocity) will remain at rest in accordance with Newton's first law of motion. However, an object released at rest in an accelerating frame of reference (e.g. a passenger jet taking off or a rotating platform) will not remain at rest.
- The **speed of light** in free space, c, is **invariant** which means it is always the same and is independent of the motion of the light source and the motion of any observer.

In other words, in developing his theory of special relativity, Einstein started from two postulates or 'fundamental statements':

Physical laws have the same form in all inertial frames of reference.

The speed of light in free space is invariant.

Einstein realised that the equations representing physical laws be in the same form in any inertial frame of reference. As outlined in the notes below, he knew that this could not be achieved using the ideas of absolute space and time even for a simple equation such as $s = ct$ for the distance, s, travelled by a light pulse in time t. So he worked out mathematically how to do it by changing or 'transforming' the distance and time coordinates from any inertial frame of reference to any other. We will look at some of the consequences of these transformations such as time dilation and length contraction.

AQA Examiner's tip

Be sure you know what an *inertial frame of reference* is and what the word *invariant* means.

Time dilation

A moving clock runs more slowly that a stationary clock.

Einstein showed that if the time interval between two events measured by an observer at rest relative to the events is t_0, (called 'the proper time'), an observer moving at speed v relative to the events would measure a longer time interval, t, given by

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = t_0\left(1 - \frac{v^2}{c^2}\right)^{-0.5}$$

The time between the events is stretched out or 'dilated' according to the moving observer. Particle beam experiments provide direct evidence of time dilation.

For example, the half-life of muons at rest is 1.5 μs. A beam of muons travelling at 99.6% of the speed of light ($v = 0.996c$) would therefore decay to 50% of its initial intensity in a distance of 450 m (= $0.996c \times 1.5$ μs) in their own frame of reference.

However, experimental measurements in the Earth's frame of reference found that such a beam moving at this speed would take about 5000 m to decay to 50% of its initial intensity. Given $c = 3.0 \times 10^8\,\text{m s}^{-1}$, the time taken to travel 5000 m at a speed of $0.996c$ is 17 μs (= 5000 m ÷ $0.996c$). This agrees with Einstein's time dilation formula which gives a dilated time of 17 μs for a proper time of 1.5 μs (i.e. over 11 times longer than the half-life of stationary muons).

Note

The measurements were made using muons created in the upper atmosphere as a result of cosmic rays from space colliding with the nuclei of atoms in the atmosphere. A detector 2000 m below the top of the atmosphere recorded an intensity of about 80% of the initial intensity. A further 2000 m would cause a decrease to about 60% (≈ 80% of 80%.) To decrease to about 50%, a further 1000 m would be needed (about 5000 m in total).

Worked example

$c = 3.00 \times 10^8\,\text{m s}^{-1}$

The half-life of charged π mesons at rest is 18 ns. Calculate the half-life of charged mesons moving at a speed of $0.95c$.

Solution

$t_0 = 18$ ns

$$\frac{v}{c} = 0.95$$

Therefore

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \frac{18}{\sqrt{\left(1 - 0.95^2\right)}} = 3.2 \times 18 = 58\,\text{ns}$$

The twins paradox

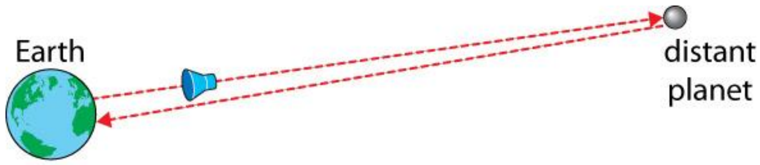

Figure 1 *A paradox*

An astronaut aged 25 says goodbye to his twin and travels at 0.95*c* in a spaceship to a distant planet, arriving there 5 years later. After spending a few weeks there, the astronaut returns to Earth at the same speed on a journey that takes another 5 years. The astronaut is 35 years old on his return. Use Einstein's time dilation formula to prove for yourself that 16 years on Earth elapse in the 5 years the astronaut, travelling at 0.95*c*, takes to travel each way. So his 'stay-at-home' twin is aged 57 when the astronaut returns!

Note that the proper time for each part of the journey was 5 years. This is the time between departure and arrival and only the astronaut and the other space crew were present at both events.

You can make up some strange scenarios round the above journey – if the astronaut was a parent of a child aged 4 on departure from Earth, the child would be a year older than the astronaut on return.

It could be argued that the Earth travels away from the spaceship at 0.95*c* and therefore the 'Earth' twin is the one who ages. However, this apparent paradox can be resolved on the grounds that the twin who travels to the distant planet has to accelerate and decelerate and therefore the twins' 'journeys' are not equivalent. Therefore the twin who has travelled to the distant planet was not in the inertial frame and so came back younger than his 'stay at home' brother.

Length contraction

A rod moving in the same direction as its length appears shorter than when it is stationary.

Einstein showed that the length *L of* a rod moving in the same direction as its length is given by

$$L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)} = L_0\left(1 - \frac{v^2}{c^2}\right)^{0.5}$$

where L_0, the **proper length** of the rod, is the length measured by an observer at rest relative to the rod.

Let's return to the charged π mesons travelling at $0.9995c$ in a particle beam. Remember their half-life of 18 ns when at rest is stretched to 570 ns in the laboratory by time dilation and they travel a laboratory distance of 171 m in this time. Imagine travelling alongside them at the same speed. The laboratory distance of 171 m would appear contracted to 5.4 m $(171 \times (1 - 0.995^2)^{1/2})$. This is the distance travelled by the charged π mesons in their frame of reference and is equal to how far they travel in one 'proper time' half-life of 18 ns.

Worked example

$c = 3.00 \times 10^8\,\text{m s}^{-1}$

A spaceship moving at a speed of $0.99c$ takes 2.3 seconds to fly past a planet and one of its moons. Calculate:

a the distance travelled by the spaceship in its own frame of reference in this time

b the distance from the planet to the moon in their frame of reference.

Solution

a Distance = speed × time = $0.99c \times 2.3 = 0.99 \times 3.0 \times 10^8 \times 2.3 = 6.8 \times 10^8$ m

b The distance from the planet to the Moon in their own frame of reference is the proper distance (L_0).

The distance travelled by the spaceship is the contracted distance (L).

Rearranging the length contraction formula to find L_0 therefore gives

$$L_0 = \frac{L}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \frac{6.8 \times 10^8}{\sqrt{\left(1 - 0.99^2\right)}} = 7.1 \times 6.8 \times 10^8 = 4.8 \times 10^9 \text{ m}$$

Relativistic mass

The mass of an object increases with speed.

By considering the law of conservation of momentum in different inertial frames of reference, Einstein showed that the mass m of an object depends on its speed v in accordance with the equation

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where m_0 is the rest mass or 'proper mass' of the object (i.e. its mass at zero speed).

The equation above shows that:

- the mass increases with increasing speed
- as the speed increases towards the speed of light, the equation shows that the mass becomes ever larger.

Experimental evidence for relativistic mass was obtained from electron beam experiments soon after Einstein published the theory of special relativity. The specific charge of the electron e/m was measured for electrons moving at different speeds. For example, at a speed of $0.69c$, e/m was measured as $1.28 \times 10^{11}\,\text{C kg}^{-1}$ which can be compared with the accepted value of

1.76×10^{11} C kg^{-1} for the 'rest' value, e/m_0. Prove for yourself using the relativistic mass formula that the value of e/m at $0.69c$ is indeed 1.28×10^{11} C kg^{-1}.

The cosmic speed limit

Figure 2 shows how the mass of an object varies with its speed in accordance with the relativistic mass equation. The graph shows that:

- at speed $v << c$, $m = m_0$
- at speed $v \rightarrow c$, m increases gradually to about $2m_0$ at $v \approx 0.9c$ and then increases sharply and tends to infinity as v approaches c.

Einstein's relativistic mass formula means that no material object can ever reach the speed of light as its mass would become infinite and therefore can never travel faster than light. Thus the speed of light in free space, c, is the ultimate cosmic speed limit.

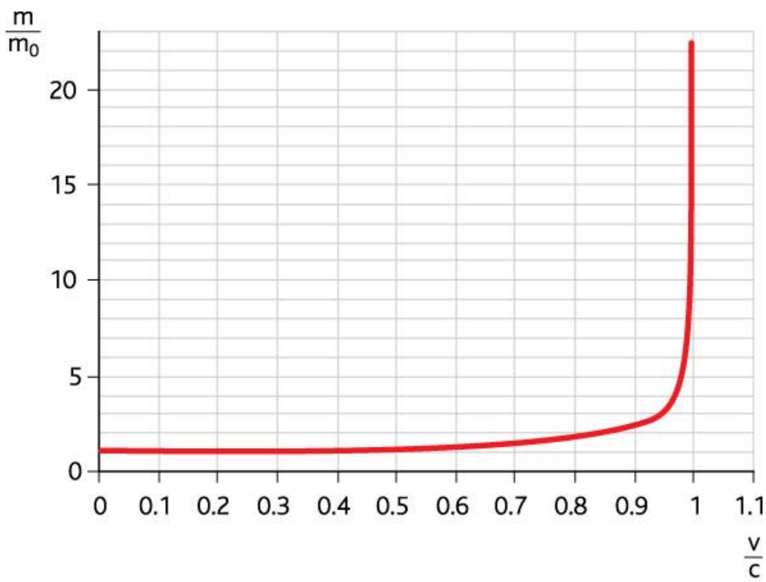

Figure 2 *Relativistic mass v speed*

Mass and energy

Increasing the speed of an object increases its kinetic energy. So Einstein's relativistic mass formula tells us that the mass of an object increases if it gains kinetic energy. In his theory of special relativity, Einstein went further and proved that transferring energy in any form:

- to an object increases its mass
- from an object decreases its mass.

He showed that energy E and mass m are equivalent (interchangeable) on a scale given by his now-famous equation

$$E = mc^2$$

Since the value of $c = 3.0 \times 10^8$ m s^{-1}, then 1 kg of mass is equivalent to 9.0×10^{16} J ($= 1 \times (3.0 \times 10^8)^2$).

In terms of the rest mass m_0 of an object, the above equation may be written as

$$E = \frac{m_0 c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

At zero speed, $v = 0$, therefore $\boldsymbol{E = m_0c^2}$ represents the **rest energy** of the object.

At speed v, the difference between its total energy E and its rest energy E_0 represents its energy due to its speed (i.e. its kinetic energy). Therefore,

its kinetic energy $\boldsymbol{E_k = mc^2 - m_0c^2}$

For example, if an object is travelling at a speed $v = 0.99c$, the relativistic mass formula gives

mass $m = \dfrac{m_0}{\sqrt{(1-0.99^2)}} = 7.1m_0$ so its kinetic energy $E_k = 7.1m_0c^2 - m_0c^2 = 6.1m_0c^2$

Notes

1 At speeds $v << c$, the above kinetic energy formula $E_k = mc^2 - m_0c^2 \rightarrow \frac{1}{2}mv^2$ as $\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \rightarrow \left(1 - \frac{v^2}{2c^2}\right)$ as $v \rightarrow c$. You don't need to know this for the option specification but it's helpful to know how the formula $E_k = \frac{1}{2}mv^2$ fits in.

2 If a charged particle of charge Q is accelerated from rest through a potential difference V to a certain speed, the work done on it is $W = QV$. Its kinetic energy after being accelerated is therefore equal to QV. Therefore its total energy $E = mc^2 = m_0c^2 + QV$.

3 The rest energy values in MeV of some particles will be supplied in an exam in the data booklet. These values are obtained by inserting mass values in kilograms and the value of the speed of light c into the rest energy formula $\boldsymbol{E = m_0c^2}$ to obtain the rest energy value in joules and then converting into MeV using the conversion factor 1 MeV = 1.6×10^{-13} J.

Worked example

$e = 1.6 \times 10^{-19}$ C, electron rest mass $m_0 = 9.1 \times 10^{-31}$ kg

An electron is accelerated from rest through a pd of 6.0 MV. Calculate:

a its gain of kinetic energy

b its mass in terms of its rest mass

c the ratio of its speed to the speed of light c.

Solution

a Its gain of kinetic energy = $QV = 1.6 \times 10^{-19} \times 6.0 \times 10^6 = 9.6 \times 10^{-15}$ J

b Its total energy = $mc^2 = m_0c^2 + QV$

Therefore $\dfrac{m}{m_0} = 1 + \dfrac{QV}{m_0c^2} = 1 + \dfrac{9.6\times10^{-13}}{9.1\times10^{-31}\times(3.0\times10^8)^2} = 1 + 11.7 = 12.7$

c Using the relativistic mass equation gives

$$12.7m_0 = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Cancelling m_0 from both sides gives

$$12.7^2 = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)}$$

Rearranging gives

$$\left(1 - \frac{v^2}{c^2}\right) = \frac{1}{12.7^2} = 6.2 \times 10^{-3}$$

Hence $\frac{v^2}{c^2} = 1 - 6.2 \times 10^{-3} = 0.9937$

Therefore $\frac{v}{c} = 0.997$

How science works

About Einstein

Einstein has become a byword for the ultimate in brain power. Although he displayed remarkable talent in maths and physics at school, Einstein was not rated very highly by his physics professors at university. He upset his university tutors by asking too many awkward questions and his unusual talents were dismissed. He secured a position as a patent officer in the Swiss Patent Office. The job was not too demanding and he had time to think about fundamental issues in physics. 'Thought' experiments have always been important in physics. Galileo and Newton developed their ideas about motion by developing thought experiments from their observations. In 1905, Einstein introduced the two key theories of modern physics, namely quantum theory and relativity theory. In his later theory of general relativity, he predicted that gravity can bend light. When this was confirmed by astronomical observations of the 1918 solar eclipse, Einstein became an overnight celebrity worldwide as 'the scientist who knew how to bend light'.

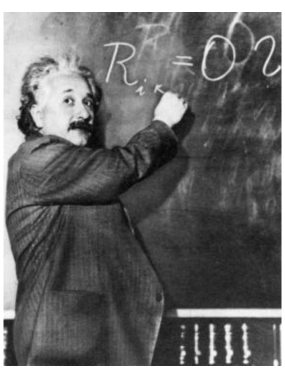

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Figure 3 *Einstein on tour*

How Einstein worked out the rules for relativity

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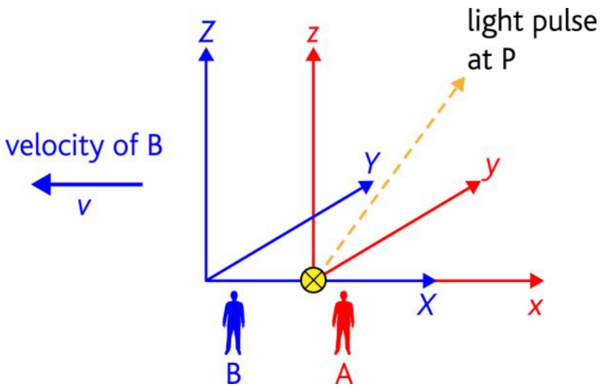

Figure 4 *Observers at work*

Consider the distance travelled by a light pulse in a certain time after being emitted from a light source, as shown in Figure 4, at the instant the axes of the frames of reference of two observers coincide with each other.

Observer A's frame of reference (in red in Figure 4) has the light source stationary at its origin. The equation $s = ct$ gives the distance, s, travelled by a light pulse in time t. The distance, s, travelled by the light pulse to a point P with coordinates (x, y, z) is given by

$$s^2 = x^2 + y^2 + z^2$$ (using Pythagoras theorem).

Therefore, according to observer A, the time taken by a light pulse to travel from the light source to P is given by

$$c^2t^2 = x^2 + y^2 + z^2$$

Observer B and the corresponding frame of reference (in blue in Figure 4) is travelling at velocity v relative to the light source in the negative x-direction.

In the observer B's frame of reference, the time taken T by a light pulse to travel from the light source to P is given by

$$c^2T^2 = X^2 + Y^2 + Z^2$$

where (X, Y, Z) are the coordinates of P in the observer B's frame of reference.

Not too difficult so far…

In pre-Einstein dynamics, space and time are absolute quantities so $T = t$, $X = x + vt$, $Y = y$ and $Z = z$. However, substituting these transformations into the equation above gives

$$c^2t^2 = (x + vt)^2 + y^2 + z^2$$

which is not the same as the corresponding equation for the frame of reference of observer A.

Still with it?

Einstein showed that there is a unique transformation solution given by

$$X = \beta(x + vt),\ T = \beta\left(t + \frac{vx}{c^2}\right),\ Y = y \text{ and } Z = z \text{ where } \beta = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Don't even try!

Now you know why Einstein was considered to be a genius! But *remember* you can *forget* this information for your examination – but not the consequences such as time dilation, length contraction and relativistic mass!

Summary questions

$e = 1.6 \times 10^{-19}$ C, electron rest mass $m_0 = 9.1 \times 10^{-31}$ kg, $c = 3.00 \times 10^8\ \mathrm{m\,s^{-1}}$

1 **a** State what is meant by an inertial frame of reference.

b Give an example of a frame of reference that is:

i inertial

ii non-inertial.

c Einstein put forward two postulates, one of which is that the speed of light in free space is invariant.

i What is meant by the word 'invariant' in this context?

ii State the other postulate put forward by Einstein.

2 A beam of particles travelling at a speed of 0.98*c* travels in a straight line between two stationary detectors which are 200 m apart.

Calculate:

a **i** the time taken by a particle in the beam to travel from one detector to the other in the frame of reference of the detectors

ii the proper time between a particle passing the detectors particles

b the distance between the detectors in the frame of reference of the particles.

3 An electron is accelerated from rest through a pd *V* to a speed 0.95*c*. Calculate:

a the mass of the electron at this speed

b its kinetic energy at this speed

c the pd, *V*, through which it was accelerated.

4 **a** In terms of the speed of light in free space, *c*, calculate the speed that a particle must have in order for its mass to be 100 times its rest mass.

b The kinetic energy of a particle can be increased by any amount but its speed cannot exceed the speed of light. Explain the apparent contradiction in this statement.

Answers

1.1

4 $3.75 \times 10^{7}\,\mathrm{m\,s^{-1}}$

1.2

4 $9.0 \times 10^{-4}\,\mathrm{V\,m^{-1}}$

1.3

1 **a** $3.39 \times 10^{7}\,\mathrm{m\,s^{-1}}$ **b** $1.75 \times 10^{11}\,\mathrm{C\,kg^{-1}}$

2 $1.80 \times 10^{11}\,\mathrm{C\,kg^{-1}}$

3 $1.79 \times 10^{11}\,\mathrm{C\,kg^{-1}}$

4 The value was many times larger than the largest known specific charge which was that of the hydrogen ion. The magnitude of the charge of the electron was not known at the time. However it was realised the electron either has much less mass than the hydrogen ion or it has much more charge.

1.4

1 **a** $4.78 \times 10^{-19}\,\mathrm{C}$ **b** 3

2 **b** **i** $6.40 \times 10^{-19}\,\mathrm{C}$, positive **ii** 4

3 **a** $3.96 \times 10^{-15}\,\mathrm{kg}$ **b** $3.18 \times 10^{-19}\,\mathrm{C}$

2.3

1 **a** $3.72 \times 10^{-19}\,\mathrm{J}$ **b** $7.60 \times 10^{-20}\,\mathrm{J}$

2 **a** $4.85 \times 10^{-19}\,\mathrm{J}$ **b** $1.61 \times 10^{-19}\,\mathrm{J}$ **c** $+\,1.31\,\mathrm{V}$

2.4

1 **a** $2.27 \times 10^{-10}\,\mathrm{m}$ **b** $1.24 \times 10^{-13}\,\mathrm{m}$

2 $3.14 \times 10^{6}\,\mathrm{m\,s^{-1}}$, $2.32 \times 10^{-11}\,\mathrm{m}$

3.2

2 **a** **i** 680 ns **ii** 135 ns, **b** 40 m

3 **a** $2.9 \times 10^{-30}\,\mathrm{kg}$ **b** $1.8 \times 10^{-13}\,\mathrm{J}$ **c** 1.1(3) MV

4 **a** $0.99995c$

