

Chapter 5 X-ray imaging

5.1 The physics of diagnostic X-rays

Learning objectives:

- How are X-rays produced in an X-ray tube?
- How is the energy of the photons from an X-ray tube controlled?
- Why are photons in the energy range from 30 keV to 100 keV used for X-ray imaging?

The design of a diagnostic X-ray tube

X-rays are electromagnetic waves with wavelengths of the order of 10^{-10} m or less. They are used for medical imaging because they penetrate matter and darken photographic film. They ionise substances and damage living tissues so their use must be monitored very carefully.

Figure 1 shows the construction of a rotating X-ray tube used for medical imaging. Electrons are emitted by the filament wire when it is heated by an electric current. These electrons are attracted by the rotating metal anode when it is positive relative to the filament. The step-up transformer provides an alternating pd for the filament wire and a much larger alternating pd between the anode and the filament.

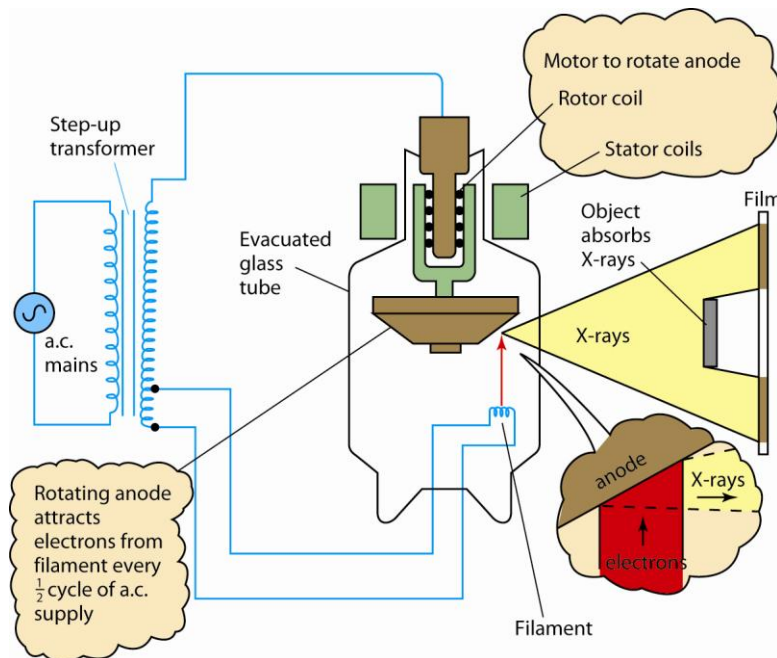


Figure 1 The rotating X-ray tube

The area of the anode from which X-rays are emitted is referred to as the **focal spot**. This area must be as small as possible otherwise features in the image would be blurred instead of being sharp.

The anode surface is at an angle of about 70° to the electron beam so that the X-rays effectively originate from a much smaller area than the impact area of the beam.

The anode is made to rotate at steady speed so the point of impact continually changes to prevent overheating.

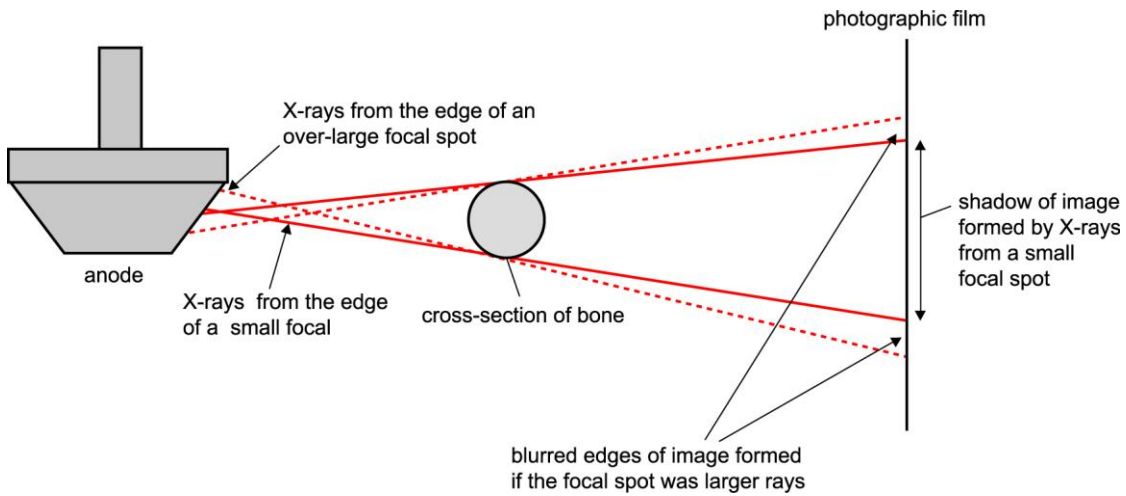


Figure 2 Image sharpness

Intensity

Figure 3 shows how the intensity of the X-rays from an X-ray tube varies with photon energy. The X-ray tube produces a continuous spectrum of photon energies up to a maximum value of eV , where V is the anode pd. This is the maximum because the kinetic energy of a beam electron at the anode is equal to the work done eV on it by the anode pd; an X-ray photon created by the stopping of an electron therefore cannot have more energy than eV .

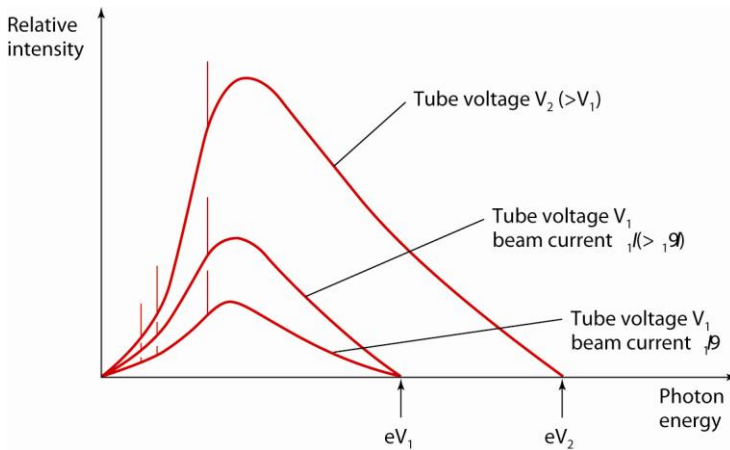


Figure 3 The energy spectrum of X-rays from an X-ray tube

Each curve for a certain anode pd V terminates on the x -axis at a photon energy equal to eV . The higher the tube pd, the further along the x -axis the curve terminates (because the maximum photon energy is greater) the higher the intensity peak (because more X-ray photons are emitted per second).

For a photon of maximum energy eV , its frequency f_{MAX} is given by

$$hf_{MAX} = eV$$

Thus an X-ray tube operating at pd V emits a continuous spectrum of frequencies up to a maximum frequency f_{MAX} .

Notes

- As the wavelength λ of a photon of frequency f is given by $\lambda = \frac{c}{f}$, an X-ray photon of frequency f_{MAX} has a wavelength $\lambda_{\text{MIN}} = \frac{c}{f_{\text{MAX}}} = \frac{hc}{eV}$ since $f_{\text{MAX}} = \frac{eV}{h}$. A graph showing the intensity distribution in terms of wavelength shows a continuous distribution from a minimum wavelength λ_{MIN} upwards, as shown in Figure 4.

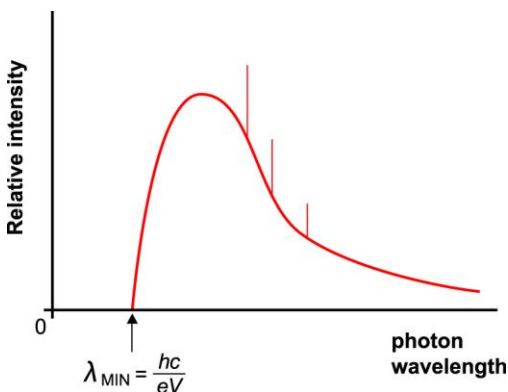


Figure 4 X-ray intensity v wavelength

- The spikes on the intensity curves are due to beam electrons knocking out electrons from the innermost shells of the atoms in the anode. These shells are at deep energy levels and when a vacancy in such a shell is filled by an outer shell electron or a conduction electron, an X-ray photon of specific energy is emitted.
- The energy of a X-ray photon is often expressed in kilo-electronvolts (keV) where $1 \text{ keV} = 1000 \text{ eV} = 1.6 \times 10^{-16} \text{ J}$.

Link

See AS Physics A Topic 1.3 for more about photons.

Controlling the power of the X-ray beam

The **anode pd** controls the maximum energy of the X-ray photons from an X-ray tube, as explained above.

The **electron beam current** controls the intensity of the X-ray beam. This is because increasing the electron beam current increases the number of electrons hitting the anode each second which therefore increases the number of X-ray photons emitted per second.

Note that increasing the electron beam current does not affect the maximum kinetic energy of the electrons and therefore does not increase the maximum energy of the emitted X-ray photons.

The electrical power supplied to an X-ray tube = beam current $I \times$ anode pd V . The anode pd for a diagnostic X-ray tube is generally between 20 kV and 100 kV. For a tube operating at 60 kV and a beam current of 0.05 A, the electrical power supplied to the tube would be 3000 W ($0.05 \text{ A} \times 60\,000 \text{ V}$).

The X-ray energy produced per second depends on the electrical power supplied to an X-ray tube and the efficiency of the tube which depends on factors such as the design of the anode and the filament.

$$\text{The efficiency of the tube} = \frac{\text{X-ray energy produced per second}}{\text{electrical power supplied}} \times 100\%$$

Typically, the efficiency of an X-ray tube is about 1%. If a tube that is 1% efficient is supplied with 3000 W of electrical power, the X-ray energy produced per second is 30 W (= 1% of 3000 W). The heat produced per second is therefore 2970 W which must be transferred to the surroundings. The target area in the anode, made of tungsten which has a high melting point, is set in copper which is a very effective heat conductor.

Summary questions

$$e = 1.6 \times 10^{-19} \text{ C}, h = 6.63 \times 10^{-34} \text{ J s}, c = 3.0 \times 10^8 \text{ m s}^{-1}$$

- 1 a** Describe the function of the following parts of an X-ray tube:
 - i** the filament wire
 - ii** the anode.
- b** What is the focal spot on the anode and why does it need to be as small as possible?
- 2 a** Sketch a graph to show how the intensity of the X-rays from an X-ray tube varies with frequency.
- b** The anode of an X-ray tube is at a constant potential and is made of tungsten set in copper. Explain why the X-ray photons emitted from this X-ray tube:
 - i** have a maximum energy
 - ii** are very intense at certain wavelengths.
- 3 a** Explain why the anode of an X-ray tube must be positive with respect to the filament in order for X-rays to be emitted.
- b** An X-ray tube in operation has a pd of 70 kV applied between the anode and the filament. Calculate:
 - i** the maximum kinetic energy of an electron at the anode
 - ii** the maximum frequency and minimum wavelength of the X-rays emitted.
- 4** The electron beam in a certain X-ray tube is accelerated through a pd of 50 kV and carries a current of 0.05 A.
 - a** Calculate the electrical power supplied to the X-ray tube.
 - b** The tube has an efficiency of 1%. Calculate:
 - i** the X-ray energy emitted per second
 - ii** the heat generated per second at the anode.

5.2 X-rays and matter

Learning objectives:

- What is the difference between the attenuation coefficient and the mass attenuation coefficient of a substance?
- How can the half-value thickness of a substance be calculated from its attenuation coefficient?
- Why are X-ray tubes operated between about 30 kV and 100 kV?

Attenuation

The **intensity** of a beam of X-radiation (and of any type of radiation or wave) is defined as the energy per second per unit area incident normally on a surface. The unit of intensity is $\text{J s}^{-1} \text{m}^{-2}$ or W m^{-2} .

When a beam of X-rays passes through matter, some X-ray photons are absorbed and some are scattered. The intensity of the transmitted beam is said to be **attenuated** because its intensity decreases the further it travels through the matter.

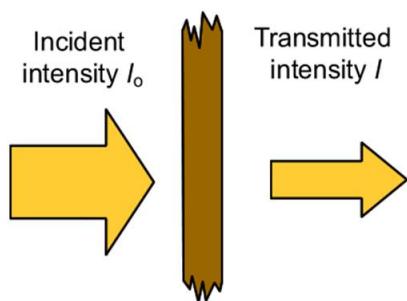


Figure 1 Attenuation

For a beam of X-rays that passes through an ‘absorber’ of thickness x , the transmitted intensity I is given by

$$I = I_0 e^{-\mu x}$$

where I_0 is the incident intensity of the beam and μ is the **attenuation coefficient** (sometimes called the absorption coefficient) of the absorber substance. The unit of μ is metre^{-1} .

The equation above shows that the intensity decreases exponentially with distance the further the beam travels through a substance. To understand how it arises, suppose an absorber of thickness t reduces the incident intensity to 80%:

- An absorber of thickness $2t$ will reduce the intensity to 80% of 80% of the incident intensity.
- An absorber of thickness $3t$ will reduce the intensity to 80% of 80% of 80% of the incident intensity etc.

Half-value thickness

The half-value thickness, $X_{\frac{1}{2}}$, of an absorber of X-rays is defined as the thickness required to reduce the intensity by 50%.

Since $I = 0.5I_0$ for $x = X_{\frac{1}{2}}$, substituting these values into the equation gives

$$0.5I_0 = I_0 e^{-\mu X_{\frac{1}{2}}}$$

Cancelling I_0 on both sides and taking natural logs therefore gives $\ln 0.5 = -\mu X_{\frac{1}{2}}$

As $\ln 0.5 = -\ln 2$, therefore $\mu X_{\frac{1}{2}} = \ln 2$

Notes

- 1 The ideas behind the above equations are the same as for radioactive decay so the mathematics is the same. This is shown in the table below.
- 2 Proof of these equations is not required in this specification.

Absorption of X-rays	Radioactive decay
Variation of intensity I variation with thickness x	Variation of activity A with time t
$I = I_0 e^{-\mu x}$	$A = A_0 e^{-\lambda t}$
absorption coefficient μ	decay constant λ
half value thickness $X_{\frac{1}{2}} = \frac{\ln 2}{\mu}$	half-life $T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Link

See A2 Physics A Topic 9.6 for radioactive decay.

Attenuation factors

The attenuation coefficient μ of an absorber is proportional to the density ρ of the absorber substance. This relationship is used to define the **mass attenuation coefficient** μ_m as $\frac{\mu}{\rho}$.

The unit of μ_m is $\text{m}^2 \text{kg}^{-1}$. This may be rearranged as $\frac{\text{W kg}^{-1}}{\text{W m}^{-2}}$ which indicates that μ_m is the energy per second absorbed per unit mass per unit intensity. For example, a value of 0.1 kg m^{-2} for μ_m would mean that 1 kg of absorber in a beam of intensity 1 W m^{-2} would absorb 0.1 W of power from the beam.

The mass attenuation coefficient of a substance depends on the atomic number Z of the absorber and the photon energy. For diagnosis, X-ray photons of energy less than 100 keV are used as higher energy photons are absorbed more equally in all types of tissue including bone so produce no distinction between different types of tissues.

- Photons of energies less than about 30 keV are absorbed by soft tissue as well as bone and are therefore removed from the X-ray beam by placing a suitable metal filter in the path of the X-ray beam as shown in Topic 5.3 Figure 4.

- Photons of energies between about 30 keV and 100 keV are absorbed more readily by bone than by soft tissue. This is because the elements in bone have higher atomic numbers than the elements in soft tissues and therefore have deeper atomic energy levels which can absorb photons in the 30–100 keV range. X-ray photons in this energy range are therefore used to distinguish between soft tissues and bone.

Summary questions

- 1 a** State what is meant by the half-value thickness of a substance.
- b** Bone has a half-value thickness for 50 keV X-ray photons of 15 mm. Calculate:
 - i** the attenuation coefficient of bone
 - ii** the percentage reduction in intensity when 50 keV X-rays pass through 10 mm of bone.
- 2** A beam of 80 keV X-rays of intensity $1.4 \times 10^{-2} \text{ W m}^{-2}$ is incident on a copper sheet of thickness 0.50 mm. The attenuation coefficient of copper for 80 keV X-rays is 690 m^{-1} .
 - a** Calculate:
 - i** the transmitted intensity of the beam after passing through the sheet
 - ii** the energy absorbed per second by a $0.10 \text{ m} \times 0.10 \text{ m}$ section of the sheet.
 - b** What thickness of copper would be needed to reduce the transmitted intensity to 25% of the incident intensity?
- 3** The mass attenuation coefficient of aluminium is $0.012 \text{ m}^2 \text{ kg}^{-1}$ for X-rays of a certain energy.
 - a** Explain what is meant by mass attenuation coefficient.
 - b** Calculate the half-value thickness of aluminium for X-rays of this energy. The density of aluminium is 2700 kg m^{-3} .
- 4 a i** Explain why a metal filter is placed in the path of an X-ray beam when an X-ray image of a patient is made.
 - ii** State a suitable metal for such a filter.
- b** Explain why X-ray photons of energies greater than about 100 keV are not used for X-ray imaging.

5.3 Image enhancement

Learning objectives:

- What measures are taken to produce sharp images with good contrast between different tissues?
- How does an image intensifier work?
- How does a CT scanner work?
- What are the comparative merits of CT scans, MRI scans and ultrasound scans?

Image quality

An X-ray image is a shadow on a photographic film caused by bones or organs that absorb X-rays more effectively than surrounding tissue does. Ideally, X-ray imaging should produce high-quality images with exposure to X-rays as low as possible.

Improving the quality of the image

- The **image sharpness** is improved by making the **focal spot**, the area on the anode from which the X-rays are emitted, as small as possible. As explained in Topic 5.1, the anode is designed so this area is as small as possible. This is to ensure image features are as sharp as possible. Topic 5.1 Figure 2 shows the idea.
- The **image contrast** is improved by using a **lead collimator grid** to prevent X-rays scattered by tissue and bone in the patient from reaching the film. Scattered X-rays would ‘fog’ the film in the shadow areas, thus reducing the contrast between clear and dark areas of the film. Only non-scattered X-rays pass through the channels in the grid to reach the film. Lead is much more effective at absorbing X-rays than any other commonly available metal.

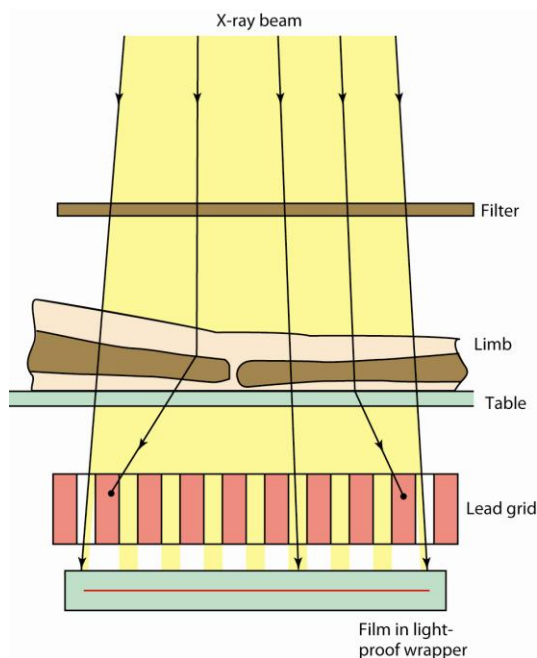


Figure 1 Using a lead collimator grid

- A **contrast medium** can be used to make the image of certain organs stand out. For example, a barium meal containing barium sulphate is given to a patient before an X-ray image of the stomach is taken. The image of the stomach stands out more because barium is a good absorber of X-rays. A solution containing iodine (in a compound) enables blood vessels to stand out.



Figure 2 Use of a contrast medium

Reducing exposure to X-rays

To reduce exposure to X-rays, the patient should not be exposed to the X-ray beam any longer than is necessary. In addition:

- **beam definers** consisting of two pairs of thick lead plates are used to ensure the beam is no wider than necessary to obtain the X-ray image

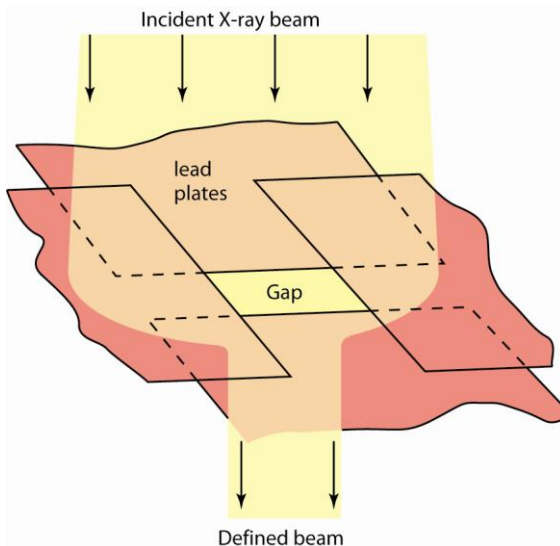


Figure 3 Beam definers

- a **metal filter** between the X-ray tube and the patient is used to filter out low energy X-rays which would otherwise be absorbed by tissue. The filters need to be made of a metal with a high atomic number such as copper, tin or lead. Figure 4 shows the effect on an X-ray beam of using a metal filter in the path of the beam.

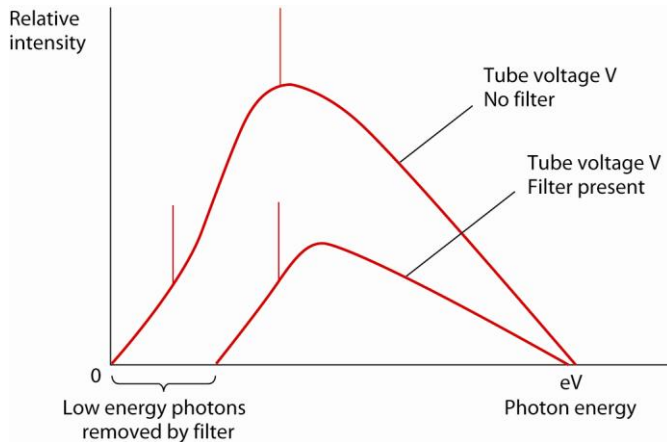


Figure 4 Using metal filters

Image intensification

An **intensifying screen** may be used to increase the rate of darkening of the film and thereby reducing the duration of exposure (i.e. the exposure time) to X-rays. A double-sided film is sandwiched between two sheets of fluorescent material such as calcium tungstate or zinc sulphide that absorbs X-rays and re-emits light, as shown in Figure 5. For each X-ray absorbed by the fluorescent material, many light photons are emitted. Since many of these light photons contribute to the darkening of the film, the image on the film forms about 30 times faster than it would without the use of the screen. Hence the duration of exposure to X-rays can be reduced significantly.

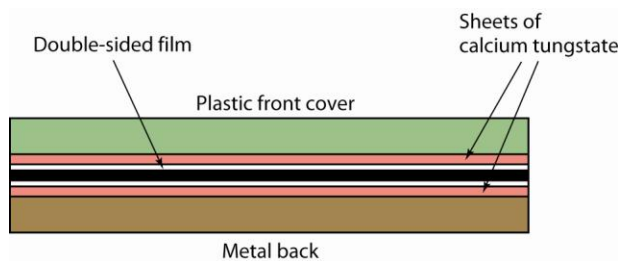


Figure 5 An intensifying screen

An **image intensifier** used with a CCTV camera enables the radiation dose to be reduced by a factor of 1000. This enables a ‘real time’ image to be observed and recorded where observation of an organ in action (e.g. swallowing) is necessary. The X-ray image is formed on a fluorescent screen in an evacuated tube as shown in Figure 6.

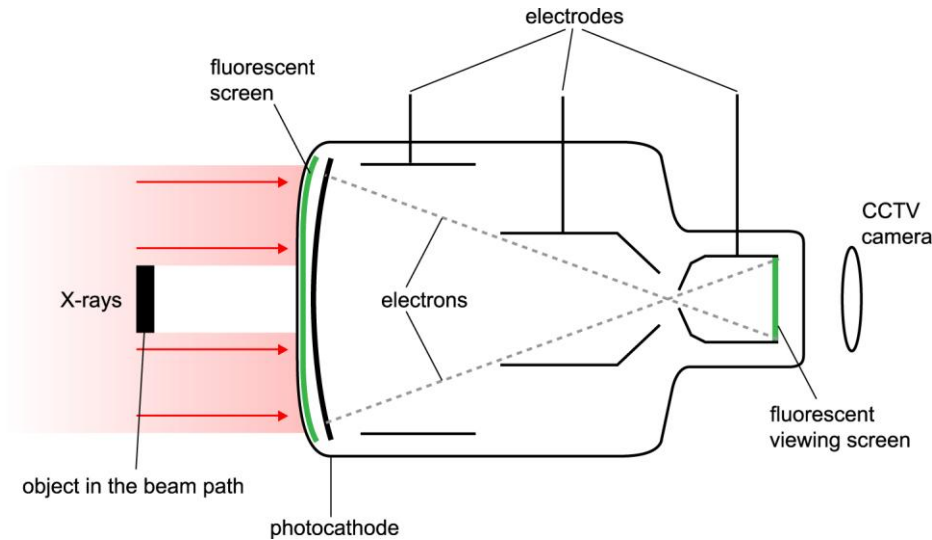


Figure 6 An image intensifier

Light from the screen strikes the photocathode causing electrons to be emitted. The electrodes attract the electrons through a pd of about 25 kV and focus them onto a small fluorescent viewing screen to form a visible image on the viewing screen. The image on the viewing screen is much more intense than the image on the first fluorescent screen because:

- each electron emitted from the photocathode gains kinetic energy as it is attracted through the electrodes. Therefore each electron hitting the viewing screen causes many visible photons to be emitted from the viewing screen.
- the electrons from the photocathode are focused by the electrodes onto a much smaller area than that of the photocathode.

As a result of the two factors above, the image on the viewing screen is about 5000 times brighter than the image on the first fluorescent screen. The image is viewed using a CCTV camera.

The CT scanner

An X-ray image taken using photographic film cannot be processed to make it sharper and clearer like a digital image can be, images of different organs in the path of the beam are superimposed on each other and it can't provide information about the depth of an organ in the body. The CT scanner provides a digital image which can be processed and it can give an image of any cross-section through the body. In addition, images of parallel planes in the body can be used to reconstruct a 3-D image of an organ.

Tomography covers any technique that gives an image of any cross-section of the body. Computed tomography or CT is any such technique that uses a computer to produce the image. CT scanning techniques are used with other forms of radiation such as positrons as well as with X-rays. The X-ray CT scanner was invented about 40 years ago and took several minutes to make a single scan. The latest scanners can take twenty or more images per second, giving much clearer and detailed images than the first scanners in less time.

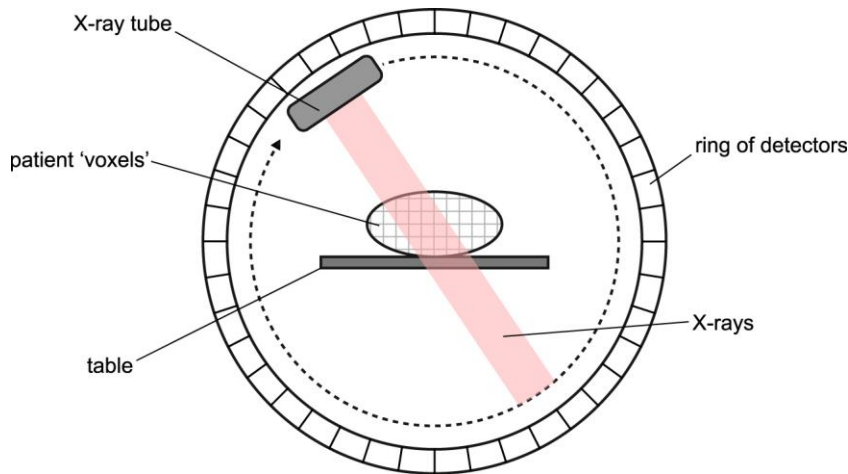


Figure 7 The CT scanner

How a CT scanner works

A CT scanner consists of an X-ray tube and a ring of thousands of small solid-state detectors, as represented in Figure 7, linked to a computer. The patient lies stationary on a bed which is suitably positioned along the axis of the ring according to the single scan image to be obtained. With the bed in position, the X-ray tube automatically moves round the inside of the ring, turning as it moves so the X-ray beam is always directed at the centre of the ring. The detector signals are simultaneously recorded by the computer each time the X-ray tube moves round the ring through a fraction of a degree until the tube has moved through 180° . In this way, the computer collects sufficient data to display a cross-sectional image of the patient.

For each position of the X-ray tube, the X-rays from the tube pass through the whole cross-section under investigation and reach the detector ring. Each detector in the path of the X-ray beam receives X-rays that have travelled through different tissue types in a straight path from the focal spot of the X-ray tube. The detector signal depends on the different types of tissue along the path and how far the X-ray beam passes through each type of tissue.

For the purpose of analysing and processing the signals, the body is considered as a collection of small 'volume elements' or **voxels**. Each detector signal is considered to have been reduced in intensity by each voxel along the path from the X-ray tube to the detector. The total attenuation along a given path is the sum of the attenuation by each voxel along the path. The computer is programmed to process all the detector signals to determine the attenuation due to each voxel, and then to display a digital image consisting of pixels, each corresponding to a voxel and of 'greytness' according to the attenuation caused by the voxel.

An X-ray tube in a CT scanner operates at potentials between 100 kV and 150 kV. The X-rays are filtered so that the X-rays that pass through the patient are monochromatic (i.e. monoenergetic). This is because the photons that pass through the patient need to be of a narrow range of energies. If this was not the case, absorption by a voxel would depend on the location of the voxel rather than its tissue type. For example, voxels near the incident body surface would absorb low energy photons more than voxels deeper inside the body.

How science works

Faster CT scans

The X-ray beam in the early scanners was confined to a narrow parallel beam to reduce the number of signals to a level that could be processed by the computer. The much greater processing capabilities of modern computers have led to the development of fan-beam CT scanners. For each position of the X-ray tube, the X-ray beam ‘fans out’ to cover the entire cross-section of the patient so it reaches more detectors than a narrow beam would. This enables an A-scan to be taken faster and thereby reduces exposure of the patient to X-rays.

Comparisons of CT, MR and ultrasound scanners

All scanners are non-invasive which means they enable the condition of body organs to be studied without cutting into the body. Each type of scanner has its own advantages and disadvantages in comparison with other types of scanners. These comparative advantages and disadvantages are summarised in the table below.

	CT scanner	MR scanner	Ultrasound scanner
Safety			
Ionising effect of radiation	yes (and the dose is greater than that for an ordinary X-ray image)	no	no
Heating effect of radiation on the body	low	mild	negligible
Other	High voltage equipment	magnetic materials must be removed; noisy; unsuitable for chemotherapy patients	-----
Image detail			
Resolution	≈ 0.5 mm	≈ 0.5 mm	≈ 1 mm
Good contrast	between hard and soft tissue and between different types of soft tissue	between hard and soft tissue and between different types of soft tissue	between different types of soft tissue
Scan time (depends on organ and scan extent*)	≈ 1 s	≈ 1000 s	≈ 10 seconds
Imaging organ movements	good	poor – takes too long	poor
Other			
Capital cost	high	significantly higher than CT	low
Availability	improving	limited due to high capital cost	commonplace
Examples of use	Tumours of the chest and abdomen	Imaging arteries in the brain, neck, chest	Imaging babies in the womb

Summary questions

- 1 a i** What is meant by a contrast medium as used in X-ray imaging?
 - ii** Give an example of a contrast medium, stating an organ it is used for?
 - iii** Describe how a lead collimator grid is used to improve the contrast of an X-ray image of a patient.
- 2 a** State the type of metal used in X-ray imaging for:
 - i** beam definer plates
 - ii** a filter used to remove low energy X-rays.
- b** Explain why the metal used for a filter would not be suitable for the beam definers?
- 3 a** State the function of an intensifying screen used in X-ray imaging and describe how it works.
 - b i** With the aid of a diagram, describe the essential features of an image intensifier.
 - ii** State two advantages of an image intensifier in comparison with an intensifying screen.
- 4 a** Describe the main differences between an X-ray CT scanner and an ordinary X-ray imaging system.
 - b** State two advantages of the use of a CT scanner in comparison with an ordinary X-ray imaging system.

Answers

1.1

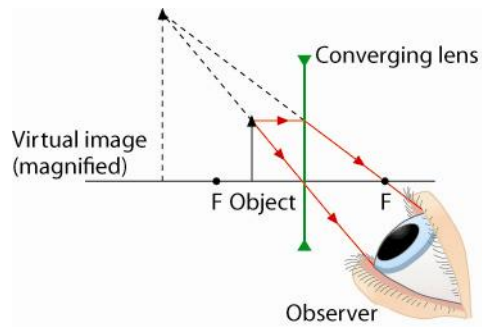
4 a $4 \mu\text{m}$

1.2

1 a i The top ray should refract at the lens and pass through F; the bottom ray should refract at the lens and then become parallel to the principal axis; the image should be formed at $1.4(3)f$ on the right-hand side of the lens.

ii real, inverted, diminished

b i



ii virtual, magnified, upright

2 a and c $v = +0.240 \text{ m}$

b i real ii inverted

3 a and c $v = -0.300 \text{ m}$

b i virtual ii upright

4 a $v = -0.086 \text{ m}$, image height = 5.7 mm

b $v = -0.111 \text{ m}$, image height = 4.4 mm

1.3

1 b ii -0.125 D

2 b ii $+2.0 \text{ D}$

3 a i -0.20 D ii $0.26(3) \text{ m}$

b i $+2.75 \text{ D}$ ii $0.36(3) \text{ m}$

2.1

3 a $1.0 \times 10^{-9} \text{ N}$ b $5.0 \times 10^4 \text{ Pa}$

2.2

1 a $2.2 \times 10^{-2} \text{ W m}^{-2}$ b 103 dB

2 2.0 W m^{-2}

3 a 70 dB **b** $3.2 \times 10^{-6} \text{ W m}^{-2}$

4 a $1.6 \times 10^{-2} \text{ W m}^{-2}$ **b** 105 dB

2.3

2 b i 20 dBA, 40 dB **ii** 60 dBA, 0 dB **iii** 15 dBA, approx 45 dB

3.1

2 $4.0 \times 10^{-15} \text{ A}$

3.2

1 b 173 kJ

3 b i 0.90 mV **ii** 0.89 mV

4.1

1 a 0.14 mm **ii** 0.62 mm

2 b ii about 0.15 m

3 a i 0.999 **ii** 1.66×10^{-3}

c i $1.64 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1}$ **ii** 2.3×10^{-5}

4.2

1 b ii 75.4°

4 b i 23.3° **ii** 33.8°

5.1

3 b i $1.1 \times 10^{-14} \text{ J}$ **ii** $1.7 \times 10^{-19} \text{ Hz}$, $1.8 \times 10^{-11} \text{ m}$

4 a 2500 W **b i** 25 W **ii** 2475 W

5.2

1 b i 46 m^{-1} **ii** 37%

2 a i $1.0 \times 10^{-2} \text{ W m}^{-2}$ **ii** $4.0 \times 10^{-5} \text{ J s}^{-1}$ **b** 2.0 mm

3 b 21 mm