Chapter 3 Cosmology

3.1 The Doppler effect

Learning objectives:

- Why does the wavelength of waves from a moving source depend on the speed of the source?
- What is a Doppler shift?
- How can we measure the velocity of the two stars in a binary system?

Doppler shifts

The wavelengths of the light waves from a star moving towards the Earth are shorter than they would be if the star was stationary. If the star had been moving away from the Earth, the wavelengths of the light waves from it would be longer than if the star was stationary. This effect applies to all waves and is known as the Doppler effect. It is the reason why the pitch of a siren on an approaching emergency vehicle rises as the vehicle approaches then falls sharply as it passes by. The pitch is higher as the source approaches and lower as it retreats because the source moves a certain distance each time it emits each cycle of waves.

Consider a source of waves of frequency f moving at speed v. Figure 1 shows wave fronts representing successive wave peaks emitted by the source at time intervals $\Delta t = \frac{1}{f}$.

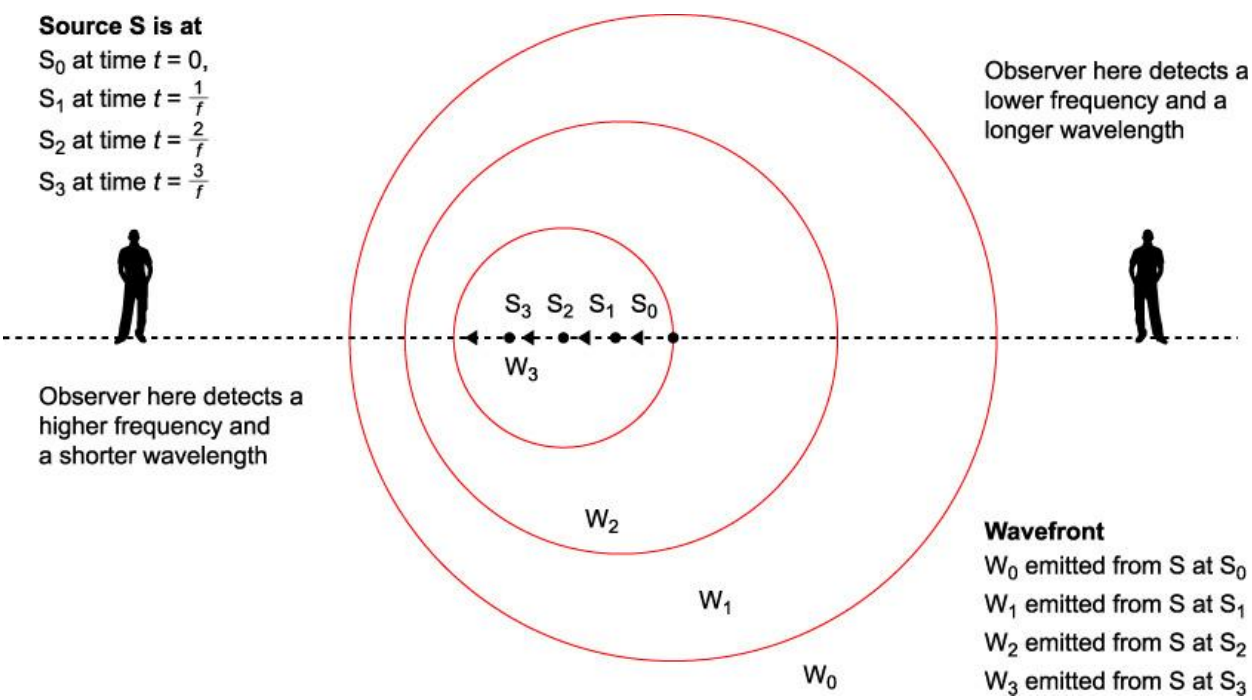

Figure 1 *The Doppler effect*

The distance between successive wave peaks is the wavelength of the waves. In time Δt, each wave peak shown travels a distance $c\Delta t$ ($= \frac{c}{f}$) before the next wave peak is emitted and the source travels a distance $v\Delta t$ ($= \frac{v}{f}$).

- Waves emitted in the opposite direction to the motion of the source ('behind' the source) are 'spaced out'. An observer in the path of these waves would therefore detect waves of longer wavelength and therefore lower frequency. For light, this shift to longer wavelengths is referred to as a **red shift** as it causes the lines of a line spectrum to shift towards the red end of the visible spectrum.
- Waves emitted in the same direction as the motion of the source are 'bunched together' ahead of the source. An observer in the path of these waves would therefore detect waves of shorter wavelength and therefore higher frequency. For light, this shift to longer wavelengths is referred to as a **blue shift** as it causes the lines of a line spectrum to shift towards the blue end of the visible spectrum.

It can be shown that for a source moving at speed v relative to an observer,

towards the observer:

- the change of frequency $\Delta f = \frac{v}{c} f$
- the change of wavelength $\Delta\lambda = -\frac{v}{c}\lambda$

away from the observer:

- the change of frequency $\Delta f = -\frac{v}{c} f$
- the change of wavelength $\Delta\lambda = \frac{v}{c}\lambda$

The **Doppler shift**, z, in frequency (or wavelength) is the fractional change $\frac{\Delta f}{f}$ (or $\frac{\Delta\lambda}{\lambda}$).

Mathematically, red shifts and blue shifts are **fractional** changes in frequency or wavelength.

Table 1 summarises the fractional change, z, in frequency and wavelength.

Doppler shift, *z*	Source moves towards observer	Source moves away from observer
in frequency $\frac{\Delta f}{f}$	$+\frac{v}{c}$	$-\frac{v}{c}$
in wavelength $\frac{\Delta\lambda}{\lambda}$	$-\frac{v}{c}$	$+\frac{v}{c}$

Table 1 *Summary of Doppler shifts*

AQA Examiner's tip

The frequency f is the frequency of the light **emitted** by the source which is the same as the frequency emitted by an identical source in the laboratory. The change of frequency Δf is the difference between this frequency and the **observed** frequency (the frequency of the light from the source as measured by an observer).

Notes

1 The formulas above and in Table 1 can only be applied to electromagnetic waves at source speeds much less than the speed of light *c*.

2 A star or galaxy may be moving through space with perpendicular velocity components parallel and at right angles to the line from the Earth to the star. The first component is the star's radial speed and the second component its tangential speed. Throughout this topic, speed *v* refers to its radial speed (i.e. the component of the star's velocity parallel to the line between the star and the Earth).

Astronomical velocities

The line spectrum of light from a star or galaxy is shifted to longer wavelengths if the star or galaxy is moving away from us and to shorter wavelengths if it is moving towards us. By measuring the shift in wavelength of a line of the star's line spectrum, the speed of the star or galaxy relative to Earth can be found. If the star is part of a binary system, its orbital speed can also be found as explained later.

In practice, the line spectrum of the star or galaxy is compared with the pattern of the prominent lines in the spectrum according to the star's spectral class. The change of wavelength of one or more prominent lines of known wavelength λ in the spectrum is then measured and the Doppler shift, z $(= \frac{\Delta\lambda}{\lambda})$ is then calculated.

For an individual star or galaxy, the speed *v* of the star or galaxy relative to a line between Earth and the star is then calculated from $z = \frac{v}{c}$.

The star or galaxy is moving:

- towards the Earth if the wavelength is shortened due to the star or galaxy's relative motion
- away from the Earth if the wavelength is lengthened due to the star or galaxy's relative motion.

For binary stars in orbit about each other in the same plane as the line from the Earth to the stars, the wavelength of each spectral line of each star changes periodically between:

- a minimum value of $\lambda - \Delta\lambda$ when the star is moving towards the Earth
- a maximum value of $\lambda + \Delta\lambda$ when the star is moving away from the Earth.

Worked example

A spectral line of a star is found to be displaced from its laboratory value of 434 nm by +0.087 nm. State whether the star is moving towards or away from the Earth and calculate its speed relative to the Earth. $c = 3.0 \times 10^8\,\text{m s}^{-1}$

Solution

The star is moving away from the Earth because the wavelength of its light is increased.

Rearranging $\Delta\lambda = \frac{v\lambda}{c}$ gives $v = \frac{c\Delta\lambda}{\lambda} = \frac{3.0\times10^8 \times 0.087\times10^{-9}}{434\times10^{-9}} = 6.0\times10^4\,\text{m s}^{-1}$

If the two stars cannot be resolved, they are referred to as a **spectroscopic binary**. Each spectral line splits into two after the stars cross the line of sight then merge into a single line as the two stars move towards the line of sight. Figure 2 shows the idea.

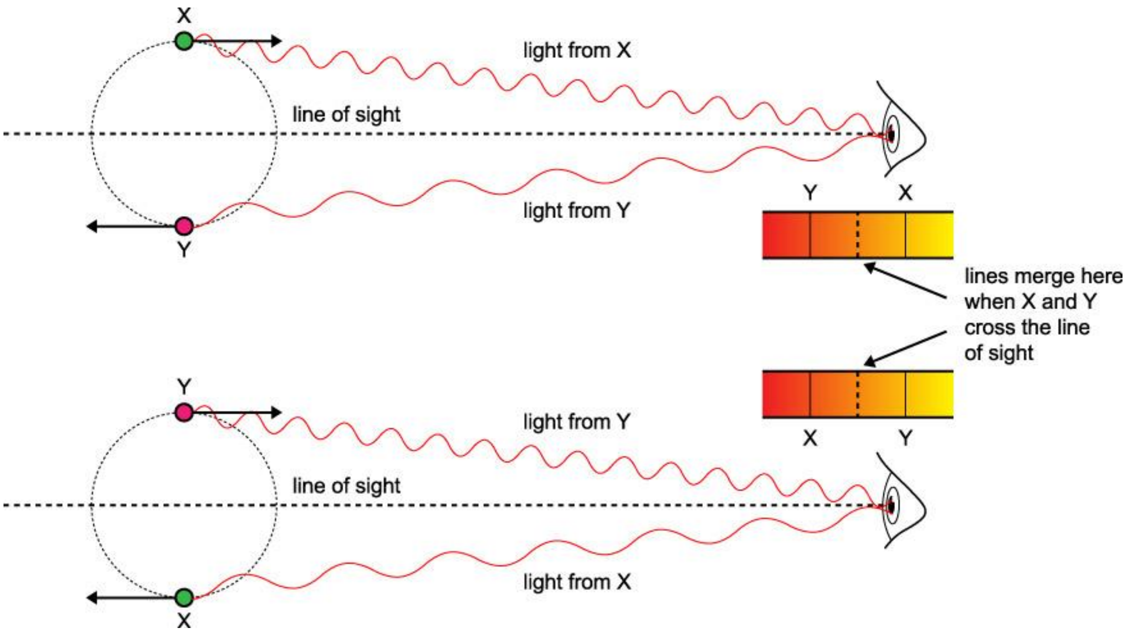

Figure 2 *A spectroscopic binary*

Note

If the stars are of different masses, they will move with the same period but at different speeds and orbital radii. The change of wavelength will be greater for the faster star (less massive star) than for the other star.

Worked example

$c = 3.0 \times 10^8\ \mathrm{m\,s^{-1}}$

A spectral line of a certain spectroscopic binary merges once every 1.5 years and splits to a maximum displacement of 0.042 nm and 0.024 nm from their laboratory wavelength of 486 nm. Calculate:

a the orbital speed of each star

b the radius of orbit of the larger orbit.

Solution

a For the slower star, $\Delta\lambda = 0.024$ nm

Rearranging $\Delta\lambda = \frac{v\lambda}{c}$ gives $v = \frac{c\Delta\lambda}{\lambda} = \frac{3.0\times10^8 \times 0.024\times10^{-9}}{486\times10^{-9}} = 1.5\times10^4\ \mathrm{m\,s^{-1}}$

For the faster star, $\Delta\lambda = 0.042$ nm

Rearranging $\Delta\lambda = \frac{v\lambda}{c}$ gives $v = \frac{c\Delta\lambda}{\lambda} = \frac{3.0\times10^8 \times 0.042\times10^{-9}}{486\times10^{-9}} = 2.6\times10^4\ \mathrm{m\,s^{-1}}$

b The orbital speed of the faster star, $v = 2\pi r/T$ where r is its radius of orbit and T is the time period.

Therefore, the radius of its orbit $r = \frac{vT}{2\pi} = \frac{(2.6\times10^4\ \mathrm{m\,s^{-1}})\times(1.5\times365.25\times24\times60\times60\,\mathrm{s})}{2\pi}$

Hence $r = 2.0 \times 10^{11}$ m

Summary questions

$c = 3.0 \times 10^8\ \mathrm{m\,s^{-1}}$

1 Explain why the wavelengths of the light waves from a star moving away from the Earth are longer than they would be if the star was stationary relative to the Earth.

2 A spectral line of a star is found to be displaced from its laboratory value of 656 nm by −0.035 nm. State whether the star is moving towards or away from the Earth and calculate its speed relative to the Earth.

3 The spectral lines of a star in a binary system vary in wavelength.

a Explain why this variation is:

i periodic

ii over a narrow well-defined range of wavelengths.

b **i** State what measurements can be made by observing the variation in wavelength of a spectral line from such a star.

ii Explain how the measurements can be used to find the radius of orbit of the star.

4 A spectral line of a certain star in a binary system changes from its laboratory wavelength of 618 nm by ±0.082 nm with a time period of 2.5 years.

Calculate:

a the orbital speed of the star

b its radius of orbit.

3.2 Hubble's law and beyond

Learning objectives:

- What do we mean by the term 'red shift'?
- Why do we think the Universe is expanding?
- What evidence led to the acceptance of the Big Bang theory
- What is dark energy?

Galaxies

The Andromeda galaxy is the nearest large galaxy to the Milky Way. Andromeda can just be seen by the unaided eye on a clear night. By taking photographs of Andromeda using a large telescope, Edwin Hubble was able to identify Cepheid variable stars in Andromeda. These stars vary in brightness with a period of the order of days and are named after the first one to be discovered, δ-Cephei, the fourth brightest star in the constellation Cepheus. Their significance is that the period depends on the absolute magnitude. Hubble measured the periods of the Cepheid variables in Andromeda that he had identified. He then used data obtained on Cepheid variables of known absolute magnitudes to find the absolute magnitude and hence the distance to each Cepheid variable in Andromeda. He found that Andromeda is about 900 kiloparsec away, far beyond the Milky Way galaxy which was known to be about 50 kiloparsec in diameter. His result settled the issue of whether or not Andromeda is inside or outside the Milky Way galaxy.

Astronomers realised that many spiral nebula they had observed like Andromeda must also be galaxies. The Universe consists of galaxies, each containing millions of millions of stars, separated by vast empty spaces. Hubble and other astronomers studied the light spectra of many galaxies and were able to identify prominent spectral lines as in the spectra of individual stars but 'red-shifted' to longer wavelengths. Hubble studied galaxies which were close enough to be resolved into individual stars. For each galaxy, he measured:

- its red shift and then calculated its speed of recession (the speed at which it was moving away)
- its distance from Earth by observing the period of individual Cepheid variables in the galaxy.

His results showed that galaxies are receding from us, each moving at speed v which is directly proportional to the distance, d. This discovery, referred to as **Hubble's law**, is usually expressed as the following equation:

$$v = Hd$$

where H, the constant of proportionality, is referred to as the Hubble constant.

For distances in megaparsec (Mpc) and velocities in $\mathrm{km\,s^{-1}}$, the accepted value of H is $65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

In other words, the speed of recession of a galaxy at a distance of:

- 1 Mpc is $65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$
- 10 Mpc is $650\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$
- 100 Mpc is $6500\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$

Figure 1 shows that the pattern of typical measurements of the speed of recession *v* and distance *d* plotted on a graph is a straight line through the origin. (This example is from actual data published in 2002. Notice the error bars on the distance estimates.) The slope of the graph is equal to the Hubble constant *H*.

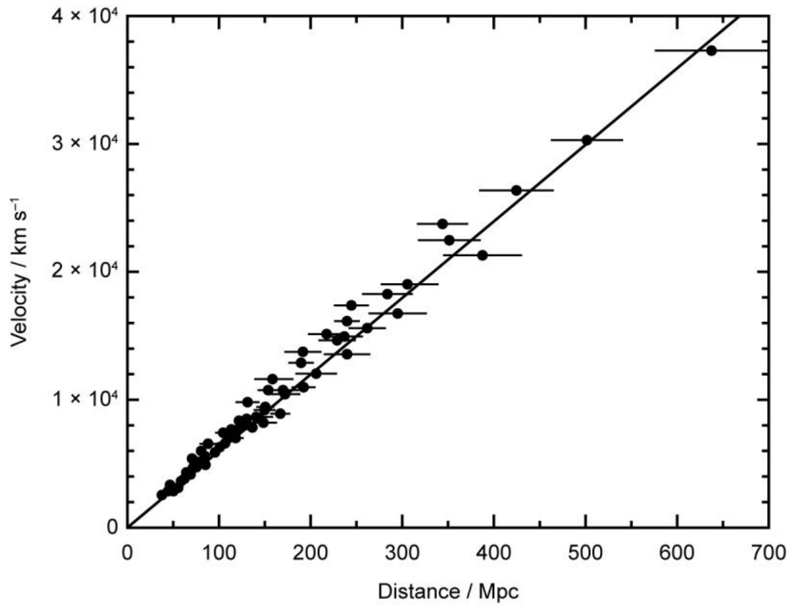

Figure 1 *Speed of recession against distance for galaxies*

Note

The galaxies local to the Milky Way galaxy such as Andromeda do not fit Hubble's law because their gravitational interactions have affected their direction of motion. Andromeda is known to be on course to collide with the Milky Way galaxy billions of years in the future.

Worked example

$c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$, $H = 65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$

The wavelength of a spectral line in the spectrum of light from a distant galaxy was measured at 398.6 nm. The same line measured in the laboratory has a wavelength of 393.3 nm. Calculate:

a the speed of recession of the galaxy

b the distance to the galaxy.

Solution

a $\Delta\lambda = 398.6 - 393.3 = 5.3\,\mathrm{nm}$

Rearranging $\Delta\lambda = \frac{v\lambda}{c}$ gives $v = \frac{c\Delta\lambda}{\lambda} = \frac{3.0\times10^{8}\times5.3\times10^{-9}}{393\times10^{-9}} = 4.0\times10^{6}\,\mathrm{m\,s^{-1}}$

b Converting *v* to $\mathrm{km\,s^{-1}}$ gives $v = 4.0 \times 10^3\,\mathrm{km\,s^{-1}}$

Rearranging $v = Hd$ gives $d = \frac{v}{H} = \frac{4.0\times10^{3}\,\mathrm{km\,s^{-1}}}{65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}} = 62\,\mathrm{Mpc}$

The Big Bang theory

Hubble's law tells us that the distant galaxies are receding from us. The conclusion we must draw from this discovery is that the galaxies are all moving away from each other and the Universe must therefore be expanding. At first, some astronomers thought this expansion is because the Universe was created in a massive 'primordial' explosion and has been expanding ever since. This theory was referred to by its opponents as the **Big Bang theory**.

With no evidence for a primordial explosion other than an explanation of Hubble's law, many astronomers supported an alternative theory that the Universe is unchanging, the same now as it ever was. This theory, known as the Steady State theory, explained the expansion of the Universe by supposing matter entering the Universe at 'white holes' pushes the galaxies apart as it enters. The Big Bang theory was accepted in 1965 when radio astronomers discovered microwave radiation from all directions in space. Steady state theory could not explain the existence of this microwave radiation but the Big Bang theory could.

Estimating the age of the Universe

The speed of light in free space, c, is 300 000 km s^{-1}. No material object can travel as fast as light. Therefore, even though the speed, v, of a galaxy increases with its distance d, **no** galaxy can travel as fast as light.

The Hubble constant tells us that the speed of a galaxy increases by 65 km s^{-1} for every extra million parsecs of distance or 3.26 million light years. Therefore, a galaxy travelling almost at the speed of light would be almost at a distance of $\frac{300\,000}{65} \times 3.26$ million light years.

To reach this distance, light would need to have travelled for 15 000 million years. Thus the Universe cannot be older than 15 000 million years.

Note

In mathematical terms, the speed of a galaxy $v < c$

Therefore, using the equation for Hubble's law gives $Hd < c$ or $d < \frac{c}{H}$

The distance $\frac{c}{H}$ represents the maximum expansion of the Universe and light could not have travelled further than this distance since the Universe began.

The age of the Universe, T, is therefore given by equating the distance travelled by light in time T (= cT) to the expansion distance $\frac{c}{H}$.

Hence $cT = \frac{c}{H}$ gives

$$\boldsymbol{T = \frac{1}{H}}$$

Substituting $H = 65\ \text{km s}^{-1}\ \text{Mpc}^{-1} = 2.1 \times 10^{-18}\ \text{s}^{-1}$ (as 1 Mpc = 3.1×10^{22} m) therefore gives

$$T = \frac{1}{H} = \frac{1}{2.1\times10^{-18}\ \text{s}^{-1}} = 4.7 \times 10^{17}\ \text{s} = 15\,000 \text{ million years.}$$

Evidence for the Big Bang theory

The spectrum of microwave radiation

The spectrum of microwave radiation from space matched the theoretical spectrum of thermal radiation from an object at a temperature of 2.7 K. Because the radiation was detected from all directions in space with little variation in intensity, it was realised it must be universal or 'cosmic' in origin.

This background cosmic microwave radiation is explained readily by the Big Bang theory as radiation that was created in the Big Bang has been travelling through the Universe ever since the Universe became transparent. As the Universe expanded after the Big Bang, its mean temperature has decreased and is now about 2.7 K. The expansion of the Universe has gradually increased the background cosmic microwave radiation to its present range of wavelengths.

Relative abundance of hydrogen and helium

Stars and galaxies contain about three times as much hydrogen by mass as helium. In comparison, other elements are present in negligible proportion. This 3 : 1 ratio of hydrogen to helium by mass means that for every helium nucleus (of mass 4 u approximately) there are 12 hydrogen nuclei (of mass 12 u in total). Thus there are 14 protons for every 2 neutrons (proton : neutron ratio of 7 : 1) is ratio is because the rest energy of a neutron is slightly greater than that of the proton. As a result, when the Universe cooled sufficiently to allow quarks in threes to form baryons, protons formed from the quarks more readily than neutrons. Precise calculations using the exact difference in the rest energies of the neutron and the proton yield a 7 : 1 ratio of protons to neutrons.

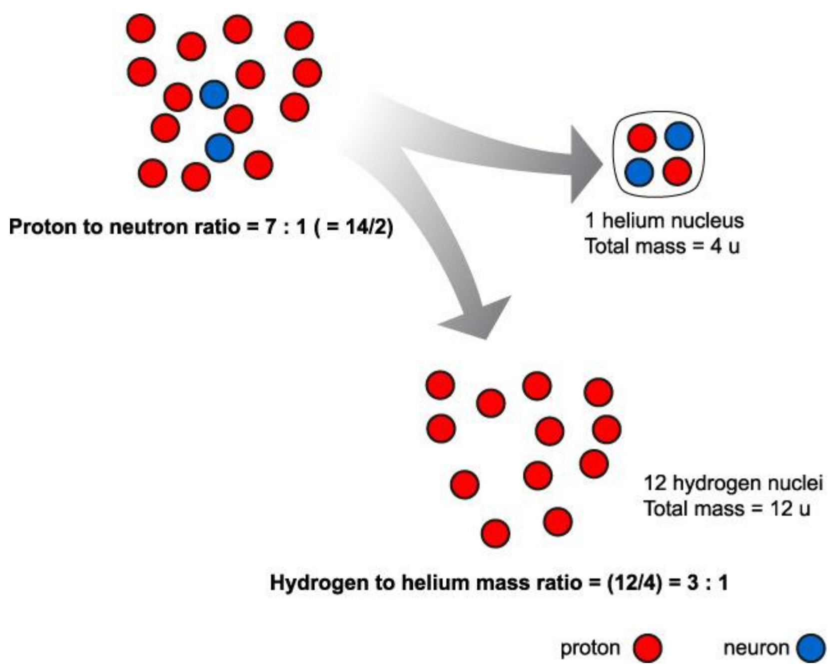

Figure 2 *Formation of hydrogen and helium nuclei*

Dark energy

Astronomers in 1998 studying type Ia supernova were astounded when they discovered very distant supernovae much further away than expected. To reach such distances, they must have been accelerating. The astronomers concluded that the expansion of the Universe is accelerating and has been for about the past 5000 million years. Before this discovery, most astronomers expected that the Universe was decelerating as very distant objects would be slowed down by the force of gravity from other galaxies. Many more observations since then have confirmed the Universe is accelerating. Scientists think that no known force could cause an acceleration of the expansion of the Universe and that a hitherto-unknown type of force must be releasing hidden energy referred to as **dark energy**.

Evidence for accelerated expansion of the Universe is based on differing distance measurements to type Ia supernova by two different methods:

1 The red shift method: measurement of the red shift of each of these distant type Ia supernova and use of Hubble's law gives the distance to each one.

2 The luminosity method: Type Ia supernova at peak intensity are known to be 10^9 times more luminous that the Sun, corresponding to an absolute magnitude of about −18. The distance to such a supernova can be calculated from its absolute magnitude M and its apparent magnitude m using the formula $m - M = 5\log\left(\frac{d}{10}\right)$.

The two methods give results that are different and indicate that the distant type Ia supernova are dimmer and therefore further away than their red shift indicates.

The nature of dark energy is unclear. It is thought to be a form of background energy present throughout space and time. It is more prominent than gravity at very large distances because gravity becomes weaker and weaker with increased distance whereas the force associated with dark energy is thought to be constant. Current theories suggest it makes up about 70% of the total energy of the Universe. The search for further evidence of dark energy will continue with observations using larger telescopes and more sensitive microwave detectors on satellites.

Summary questions

$c = 3.0 \times 10^8\,\text{m s}^{-1}$, $H = 65\,\text{km s}^{-1}\,\text{Mpc}^{-1}$

1 **a** State Hubble's law.

b Explain why Hubble's law leads to the conclusion that the Universe is expanding.

2 The wavelength of a spectral line in the spectrum of light from a distant galaxy was measured at 597.2 nm. The same line measured in the laboratory has a wavelength of 589.6 nm. Calculate:

a the speed of recession of the galaxy

b the distance to the galaxy.

3 State **two** pieces of experimental evidence other than Hubble's law that led to the acceptance of the Big Bang theory of the Universe.

4 A certain type Ia supernova has an apparent magnitude of +24.

a Calculate the distance to the supernova. (Assume the absolute magnitude of any type Ia supernova is −18.)

b Outline why measurements on type Ia supernovae have led to the conclusion that the expansion of the Universe is accelerating.

3.3 Quasars

Learning objectives:

- How were quasars discovered?
- What are the characteristic properties of a quasar?
- Why are there no nearby quasars?

The first quasar

The first quasar was announced two years after a previously discovered astronomical radio source, 3C 273, was identified as a dim star in 1962 using an optical telescope. The star presented a puzzle because its radio emissions were stronger than expected from an ordinary star and its visible spectrum contained strong lines that could not be explained. Astronomers in California realised the strong lines were due to a very large red shift of 0.15, corresponding to a light source with speed of recession of 0.15 *c* (i.e. 15% of the speed of light) at a distance of over 2000 million light years away.

Based on this distance, calculations showed that 3C 273 is 1000 times more luminous that the Milky Way galaxy yet variations in its brightness indicated it is much smaller than the Milky Way galaxy. Its variations on a time scale of the order of years or less tell us that its diameter cannot be much more than a few light years.

Astronomers concluded that 3C 273 is more like a star than a galaxy in terms of its size yet its light output is on a galactic scale or even greater. The object was referred to as a quasi-stellar object or **quasar**. Many more quasars have been discovered moving away at speeds up to 0.85 *c* or more at distances between 5000 and 10 000 light years away. The absence of quasars closer than about 5000 million light years indicates a 'quasar age' that commenced 2000 to 3000 million years after the Big Bang and lasted about 5000 million years.

Notes

1 To calculate the red shift of a quasar, the change of wavelength of one of its spectral lines of known wavelength λ is measured and then used to calculate the red shift $z \left(= \frac{\Delta\lambda}{\lambda}\right)$.

2 To calculate the speed of recession v, the equation $v = zc$ may be used only if $v << c$. Otherwise, a relativistic equation relating v and z must be used. Knowledge of this relativistic equation is not required in this option specification. Quasars generally have red shifts between 1 and 5 corresponding to speeds from 0.6 *c* to about 0.95 *c* which are not insignificant compared with the speed of light, *c*.

Quasar properties

Quasars are among the oldest and most distant objects in the Universe. A quasar is characterised by:

- its very powerful light output, much greater than the light output of a star
- its relatively small size, not much larger than a star
- a large red shift indicating its distance is between 5000 and 10 000 light years away.

Many quasars are not like 3C 273 in that they do not produce strong radio emissions.

What are quasars? Detailed optical and radio images of quasars indicate fast-moving clouds of gases and jets of matter being ejected. Quasars are found in or near galaxies which are often distorted, sometimes with lobes either side. Such 'active' galaxies are thought to have a supermassive black hole at their centres. As discussed in Topic 2.4, such a black hole could have a mass of more than 1000 million solar masses. With many stars near it, matter would be pulled in and would become very hot due to compression as it nears the event horizon. Overheating would result in clouds of hot glowing gas being thrown back into space. A spinning supermassive black hole would emit jets of hot matter in opposite directions along its axis of rotation.

Many astronomers think that a quasar is a supermassive black hole at the centre of a galaxy. When we observe a quasar, we are looking back in time at a supermassive black hole in action. The action ceases when there are no nearby stars for the black hole to 'consume'. Fortunately, the Milky Way galaxy and Andromeda and other galaxies close to us are relatively inactive because each galaxy no longer has many stars left near the supermassive black hole at its centre.

Summary questions

$c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$, $H = 65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$

1 State three characteristics of a quasar.

2 Light from a certain quasar was found to contain a spectral line of wavelength 540 nm that had been red-shifted from a normal wavelength of 486 nm.

- **a** Show that the red shift of this quasar is 0.11.
- **b** Calculate the speed of recession of this quasar, assuming its speed is much less than the speed of light. Ignore relativistic effects.

3
- **a** What features of the light from a quasar indicates a quasar is much more luminous than a star?
- **b** What feature of the light from a quasar indicates a quasar is much smaller in size than a galaxy?

4 Outline why astronomers think certain galaxies have a supermassive black hole at their centres.

Answers

1.1

1 a i The top ray should refract at the lens and pass through F; the bottom ray should refract at the lens and then become parallel to the principal axis; the image should be formed at 1.4(3)f on the right-hand side of the lens.

ii real, inverted, diminished

b ii virtual, magnified, upright

2 a and **c** $v = +\ 0.240$ m

b i real **ii** inverted

3 a and **c** $v = -0.300$ m

b i virtual **ii** upright

4 a i $v = -0.600$ m, image height = 40 mm **ii** $v = +1.000$ m, image height = 40 mm

b i 0.450 mm, virtual and upright **ii** 1.250 m, real and inverted

1.2

2 b i 7.5 **ii** 1.13° (1.125° to 4 s.f.)

4 a i 640 mm **ii** 680 mm **b i** 0.3° **ii** $1.4(3) \times 10^4$ m

1.3

4 a 100 **b** 40

1.4

3 a 0.06(3) m

4 b 80 m

1.5

2 a i 2×10^{-5} degree **ii** 3×10^{-6} degree

4 a 4, 1, 2/3, 5

2.1

2 b ii +11.63

3 b +5.9

4 b −23.2

2.2

2 b 4700 K

4 b 2.0×10^9 m

2.3

4 **a** Z, Y, X **b** X giant; Y main sequence; Z white dwarf

c X/Z = 6300, Y/Z = 100

2.4

4 **b** **i** 3.0 km **ii** 1.8×10^{19} kg m^{-3} **c** 1.8×10^{5}

3.1

2 1.6×10^{4} m s^{-1}

4 **a** 4.0×10^{4} m s^{-1} **b** 5.0×10^{11} m

3.2

2 **a** 3.9×10^{6} m s^{-1} **b** 59 Mpc

4 **a** 2500 Mpc

Image acknowledgements

Section 1.4 figure 4: NASA, ESA, and the Hubble Heritage Team (STScI/AURA)

Section 2.4 figure 1: NASA, ESA, J. Hester and A. Loll (Arizona State University)

Section 2.5 figure 1: ESO; figure 3: CSIRO; figure 4: ESA

Section 3.2 figure 1: PNAS