Chapter 2 Surveying the stars

2.1 Star magnitudes

Learning objectives:

- How is the distance to a nearby star measured?
- What do we mean by apparent and absolute magnitude?
- How can we calculate the absolute magnitude of a star?

Astronomical distances

One **light year** is the distance light travels through space in 1 year and equals 9.5×10^{15} m. Light from the Sun takes 500 s to reach the Earth, about 40 minutes to reach Jupiter, about 6 hours to reach Pluto and about 4 years to the nearest star, Proxima Centauri.

As there are 31.536 million seconds in one year, it follows that one light year = speed of light $\times$ time in seconds for one year = $3.00 \times 10^{8}\,\text{m}\,\text{s}^{-1} \times 3.15 \times 10^{7}\,\text{s} = 9.45 \times 10^{15}\,\text{m}$.

The **Sun and nearby stars** are in a spiral arm of the Milky Way galaxy. The galaxy contains almost a million million stars. Light takes about $100\,000$ years to travel across the Milky Way galaxy.

Galaxies are assemblies of stars prevented from moving away from each other by their gravitational attraction. Galaxies are millions of light years apart, separated from one another by empty space.

The **most distant galaxies** are about ten thousand million light years away and were formed shortly after the Big Bang. The Universe is thought to be about 13 thousand million (i.e. 13 billion) years old. The most distant galaxies are near the edge of the observable Universe.

Measurement of the distance to a nearby star

Astronomers can tell if a star is relatively near us because nearby stars shift in position against the background of more distant stars as the Earth moves round its orbit. This effect is referred to as **parallax** and it occurs because the line of sight to a nearby star changes direction over six months because we view the star from diametrically opposite positions of the Earth's orbit in this time. By measuring the angular shift of a star's position over six months, relative to the fixed pattern of distant stars, the distance to the nearby star can be calculated as explained below.

The Earth's orbit round the Sun is used as a baseline in the calculation, so accurate knowledge of the measurement of the mean distance from the centre of the Sun to the Earth is required. This distance is referred to as one **astronomical unit** (AU) and is equal to 1.496×10^{11} m.
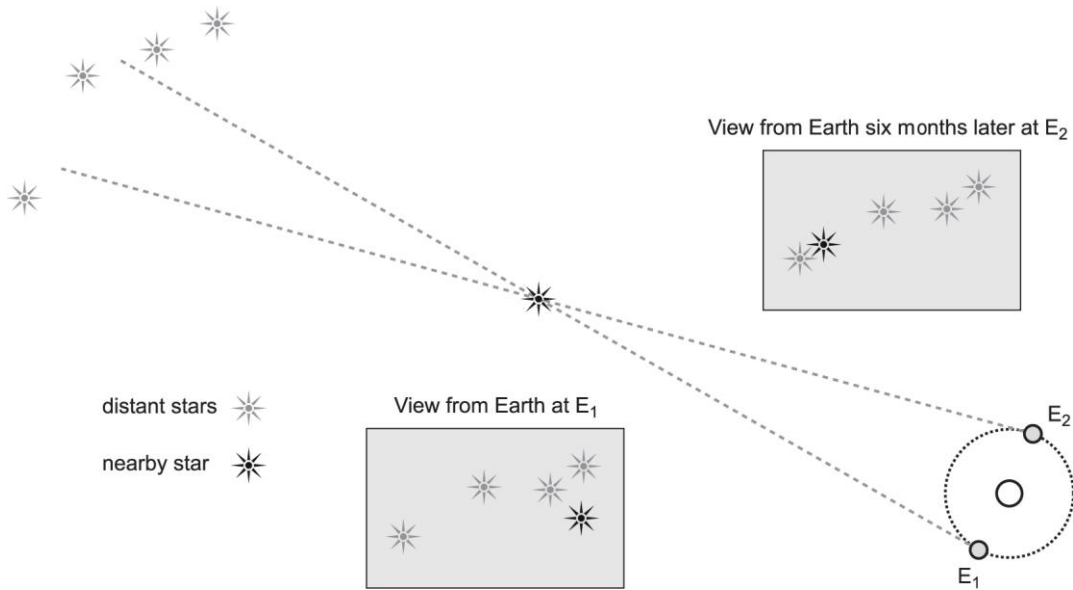
Figure 1 *Star parallax*

To calculate the distance to a nearby star, consider Figure 2 which shows the 'six month' angular shift of a nearby star's position relative to stars much further away.
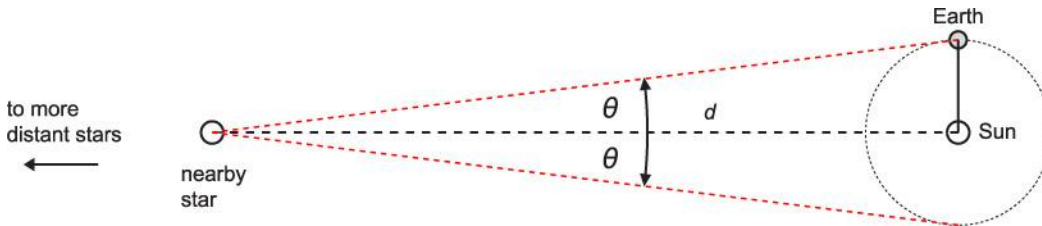

Figure 2 *Parallax angle*

The **parallax angle** θ is defined as the angle subtended by the star to the line between the Sun and the Earth, as shown in Figure 2. This angle is half the angular shift of the star's line of sight over six months.

■ From the triangle consisting of the three lines between the Sun, the star and the Earth as shown in Figure 2, $\tan\theta = \dfrac{R}{d}$.

■ Since θ is always less than 10°, using the small angle approximation gives $\theta = \dfrac{R}{d}$, where θ is in radians. So, $d = \dfrac{R}{\theta}$. Note that 360° = 2π radians.

Parallax angles are generally measured in arc seconds where 1 arc second = $\dfrac{1\,\text{degree}}{3600}$. For this reason, star distances are usually expressed for convenience in terms of a related non-SI unit called the **parsec** (abbreviated as pc).

> **1 parsec is defined as the distance to a star which subtends an angle of 1 arc second to the line from the centre of the Earth to the centre of the Sun.**

Since 1 arc second $= \dfrac{1\,\text{degree}}{3600} = 4.85 \times 10^{-6}$ radians and 1 AU $= 1.496 \times 10^{11}$ m, using the

equation $d = \dfrac{R}{\theta}$ gives:

- 1 parsec $= 3.08 \times 10^{16}$ m $\left(= \dfrac{1.496 \times 10^{11}\,\text{m}}{4.85 \times 10^{-6}\,\text{radians}} \right) = 3.26$ light years

- The distance, in parsecs, from a star to the Sun $= \dfrac{1}{\theta}$, where θ is the parallax angle of the star angle, in arc seconds

$$d(\text{in parsecs}) = \frac{1}{\theta(\text{in arc seconds})}$$

The smaller the parallax angle of a star, the further away the star is. For example:

- $\theta = 1.00$ arc second, $d = 1.00$ pc
- $\theta = 0.50$ arc seconds, $d = 2.00$ pc
- $\theta = 0.01$ arc seconds, $d = 100$ pc

Notes

1   For telescopes sited on the ground, the parallax method for measuring distances works up to about 100 pc. Beyond this distance, the parallax angles are too small to measure accurately because of atmospheric refraction. Telescopes on satellites are able to measure parallax angles much more accurately and thereby measure distances to stars beyond 100 pc.
2   1 parsec $= 3.09 \times 10^{16}$ m $= 3.26$ light years $= 206\,265$ AU

Star magnitudes

The brightness of a star in the night sky depends on the **intensity** of the star's light at the Earth which is the light energy per second per unit surface area received from the star at normal incidence on a surface. The intensity of sunlight at the Earth's surface is about $1400\,\text{W m}^{-2}$. In comparison, the intensity of light from the faintest star that can be seen with the unaided eye is more than a million million times less. With the Hubble Space Telescope the intensity is more than 10 000 million million times less.

Astronomers in ancient times first classified stars in six magnitudes of brightness, a first magnitude star being one of the brightest in the sky and a sixth magnitude star being just visible on a clear night. The scale was established on a scientific basis in the 19th century by defining a difference of five magnitudes as a hundredfold change in the intensity of light received from the star. In addition, the terms 'apparent magnitude' and 'absolute magnitude' are used to distinguish between light received from a star and light emitted by the star respectively. The term 'absolute magnitude' is important because it enables a comparison between stars in terms of how much light they emit.

On the scientific scale, stars such as Sirius which give received intensities greater than 100 times that of the faintest stars are brighter than first magnitude stars and therefore have zero or negative apparent magnitudes.

Apparent magnitude

The apparent magnitude, m, of a star in the night sky is a measure of its brightness which depends on the intensity of the light received from the star.

Consider two stars X and Y of apparent magnitudes m_X and m_Y which give received intensities I_X and I_Y. Every difference of 5 magnitudes corresponds to 100 times more light intensity from X than from Y. Generalising this rule gives $\dfrac{I_X}{I_Y} = 100^{\frac{\Delta m}{5}}$, where $\Delta m = m_Y - m_X$

Taking base 10 logs of this equation gives:

$$\log\left(\frac{I_X}{I_Y}\right) = \log\left(100^{\frac{\Delta m}{5}}\right) = \log\left(100^{0.2\Delta m}\right) = 0.2\Delta m \log 100 = 0.4\Delta m$$

Multiplying both sides of the equation by 2.5 gives $2.5\log\left(\dfrac{I_X}{I_Y}\right) = \Delta m$

Hence

$$m_Y - m_X = 2.5\log\left(\frac{I_X}{I_Y}\right)$$

The absolute magnitude, M, of a star is defined as the star's apparent magnitude, m, if it was at a distance of 10 parsecs from Earth.

It can be shown that for any star at distance d, in parsecs, from the Earth:

$$m - M = 5\log\left(\frac{d}{10}\right)$$

To prove this equation, recall that the intensity I of the light received from a star depends on its distance d from Earth in accordance with the **inverse square law** ($I \propto 1/d^2$). In using the inverse square law here, we assume the radiation from the star spreads out evenly in all directions and no radiation is absorbed in space.

Link

The inverse square law for gamma radiation was looked it in Topic 9.3 of *A2 Physics A*.

Comparing a star X at a distance of 10 pc from Earth with an identical star Y at distance d from Earth, the ratio of their received intensities $\dfrac{I_X}{I_Y}$ would be $\left(\dfrac{d}{10}\right)^2$.

Therefore, the difference between their apparent magnitudes,

$$m_Y - m_X = 2.5\log\frac{I_X}{I_Y}$$

$$= 2.5\log\left(\frac{d}{10}\right)^2$$

$$= 5 \log\left(\frac{d}{10}\right)$$

Since the stars are identical, the absolute magnitude of X, M_X = absolute magnitude of Y, M_Y.

Also, because X is at 10 pc, its apparent magnitude $m_X = M_X$

So, $m_Y - M_Y = 5 \log\left(\frac{d}{10}\right)$

More generally, for any star at distance d, in parsecs, from the Earth:

$$m - M = 5 \log\left(\frac{d}{10}\right)$$

Proof of this formula is not required in this specification.

AQA Examiner's tip

When you use this equation, make sure you use base 10 logs not base e.

Worked example

A star of apparent magnitude $m = 6.0$ is at a distance of 80 pc from the Earth.

Calculate its absolute magnitude M.

Solution

Rearranging $m - M = 5 \log\left(\frac{d}{10}\right)$ gives $M = m - 5 \log\left(\frac{d}{10}\right)$

Hence $M = 6.0 - 5 \log\frac{80}{10} = 6.0 - 5 \log 8 = 1.5$ (= 1.48 to 3 significant figures)

Summary questions

1 parsec = 206 000 AU

1 With the aid of a diagram, explain why a nearby star shifts its position over six months against the background of more distant stars.

2 a State what is meant by the absolute magnitude of a star.

   b A star has an apparent magnitude of +9.8 and its angular shift due to parallax over six months is 0.45 arc seconds.

     i Show that its distance from Earth is 4.4 pc.

     ii Calculate its absolute magnitude.

3 a Show that a star with an apparent magnitude

     i $m = 3.0$ at 100 pc has an absolute magnitude of −2.0

     ii $m = -1.4$ at 2.7 pc has an absolute magnitude of +1.4

   b Calculate the apparent magnitude of a star of absolute magnitude $M = +3.5$ which is 30 pc from Earth.

4 The apparent magnitude of the Sun is −26.8.

a Show that its absolute magnitude is +4.8.

b Calculate the apparent magnitude of the Sun as seen from the planet Jupiter at a distance of 5.2 AU from the Sun.

2.2 Classifying stars

Learning objectives:

- What does the colour of a star tell us about the star?
- How can we classify stars?
- What can we tell from the absorption spectrum of a star?

Starlight

Stars differ in colour as well as brightness. Viewed through a telescope, stars that appear to be white to the unaided eye appear in their true colours. This is because a telescope collects much more light than the unaided eye thus activating the colour-sensitive cells in the retina. CCDs with filters and colour-sensitive photographic film show that stars vary in colour from red to orange and yellow to white to bluish-white.

Like any glowing object, a star emits thermal radiation which includes visible light and infrared radiation. For example, if the current through a torch bulb is increased from zero to its working value, the filament glows dull red then red then orange-yellow as the current increases and the filament becomes hotter. The spectrum of the light emitted shows that there is a continuous spread of colours which change their relative intensities as the temperature is increased. This example shows that:

- the thermal radiation from a hot object at constant temperature consists of a continuous range of wavelengths
- the distribution of intensity with wavelength changes as the temperature of the hot object is increased.

Figure 1 shows how the intensity distribution of such radiation varies with wavelength for different temperatures.
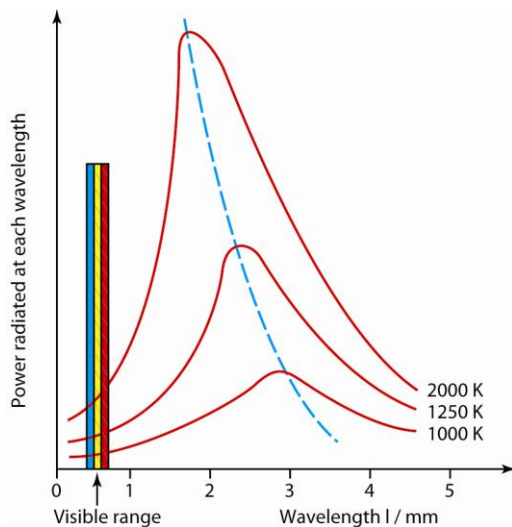

Figure 1 *Black body radiation curves*

The curves are referred to as **black body radiation** curves, a black body being defined as a body that is a perfect absorber of radiation (absorbs 100% of radiation incident on it at all wavelengths) and therefore emits a continuous spectrum of wavelengths. Remember from GCSE that a matt black surface is the best absorber and emitter of infrared radiation. In addition to a star as an example of a black body, a small hole in the door of a furnace is a further example: any thermal radiation that enters the hole from outside would be completely absorbed by the inside walls. We can assume a star is a black body because any radiation incident on it would be absorbed and none would be reflected or transmitted by the star. In addition, the spectrum of thermal radiation from a star is a continuous spectrum with an intensity distribution that matches the shape of a black body radiation curve.

The laws of thermal radiation

Black body radiation curves are obtained by measuring the intensity of the thermal radiation from a black body at different constant temperatures. Each curve has a peak which is higher and at shorter wavelength than the curves at lower temperatures. The following two laws of thermal radiation were obtained by analysing the black body radiation curves.

Wien's law

The wavelength at peak intensity, λ_P, is inversely proportional to the absolute temperature T of the object, in accordance with the following equation known as **Wien's law**:

$$\lambda_{max}T = 0.0029 \, m \, K$$

Therefore, if λ_{max} for a given star is measured from its spectrum, the above equation can be used to calculate the absolute temperature T of the light-emitting outer layer, the **photosphere**, of the star. The photosphere is sometimes referred to as the surface of a star.

Notice that the unit symbol 'm K' stands for 'metre kelvin' not milli kelvin!

Worked example

The peak intensity of thermal radiation from the Sun is at a wavelength of 500 nm.

Calculate the surface temperature of the Sun.

Solution

Rearranging $\lambda_{max} T = 0.0029 \, m \, K$ gives $T = \dfrac{0.0029 \, m \, K}{500 \times 10^{-9} \, m} = 5800 \, K$

Stefan's law

The total energy per second, P, emitted by a black body at absolute temperature T is proportional to its surface area A and to T^4, in accordance with the following equation known as **Stefan's law**

$$P = \sigma A T^4$$

where σ is the Stefan constant which has a value of $5.67 \times 10^{-8} \, W \, m^{-2} \, K^{-4}$. In effect, P is the power output of the star and is sometimes referred to as the **luminosity** of the star.

Therefore, if the absolute temperature T of a star and its power output P are known, the surface area A and the radius R of the star can be calculated.

Worked example

$\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$

A star has a power output of $6.0 \times 10^{28}\,\mathrm{W}$ and a surface temperature of 3400 K.

a Show that it surface area is $7.9 \times 10^{21}\,\mathrm{m^2}$

b Calculate:

   i its radius

   ii the ratio of its radius to the radius of the Sun.

   radius of Sun $= 7.0 \times 10^{8}\,\mathrm{m}$

Solution

a Rearranging $P = \sigma A T^4$ gives $A = \dfrac{P}{\sigma T^4}$

   Hence $A = \dfrac{6.0 \times 10^{28}}{5.67 \times 10^{-8} \times (3400)^4} = 7.9 \times 10^{21}\,\mathrm{m^2}$

b i For a sphere of radius R, its surface area $A = 4\pi R^2$

   Rearranging this equation gives $R^2 = \dfrac{A}{4\pi} = \dfrac{7.9 \times 10^{21}}{4\pi} = 6.3 \times 10^{20}\,\mathrm{m^2}$

   Hence $R = 2.5 \times 10^{10}\,\mathrm{m}$

   ii Ratio of radius to Sun's radius $= \dfrac{2.5 \times 10^{10}\,\mathrm{m}}{7.0 \times 10^{8}\,\mathrm{m}} = 36$

Note

Two stars that have the same absolute magnitude have the same power output. For two such stars X and Y:

- power output of X $= \sigma A_X T_X^4$, where A_X = surface area of X and T_X = surface temperature of X
- power output of Y $= \sigma A_Y T_Y^4$, where A_Y = surface area of Y and T_Y = surface temperature of Y

For equal power output, $\sigma A_X T_X^4 = \sigma A_Y T_Y^4$

Hence $\dfrac{A_X}{A_Y} = \dfrac{T_Y^4}{T_X^4}$

Therefore, if their surface temperatures are equal, they must have the same radius. If their surface temperatures are unequal, the cooler star must have a bigger radius than the hotter star.

Stellar spectral classes

The spectrum of light from a star is used to classify it as shown in Table 1. When the scheme was first introduced, stars were classified on an alphabetical scale A, B, C etc according to colour. The scale was re-ordered later according to surface temperature when the surface temperatures were first measured. As shown in Table 1, the main spectral classes in order of decreasing temperature are O, B, A, F, G, K and M.

Spectral Class	Intrinsic Colour	Temperature (K)	Prominent Absorption Lines
O	blue	25 000–50 000	He+, He, H
B	blue	11 000–25 000	He, H
A	blue-white	7500–11 000	H (strongest), ionised metals
F	white	6000–7500	ionised metals
G	yellow-white	5000–6000	ionised & neutral metals
K	orange	3500–5000	neutral metals
M	red	≈ 2500–3500	neutral atoms, TiO

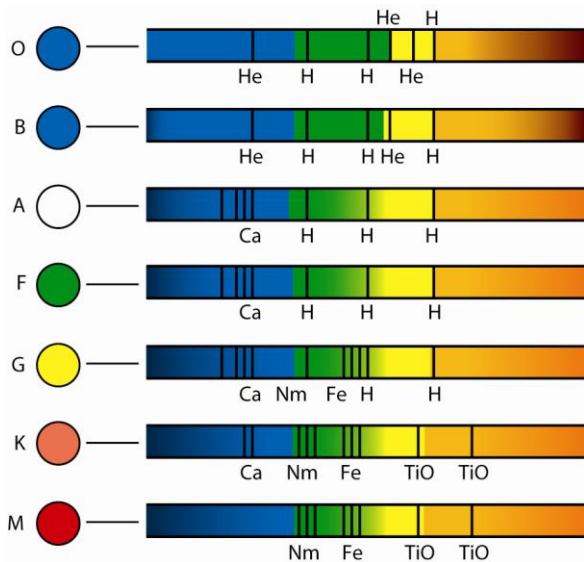**Table 1** *Characteristics of the main spectral classes*
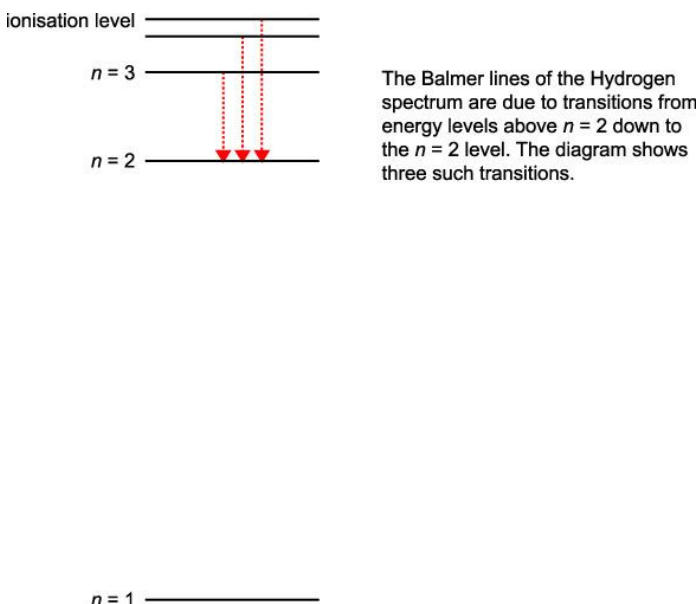

Figure 2 *Star classification*

The spectrum of light from a star contains absorption lines due to a 'corona' or 'atmosphere' of hot gases surrounding the star above its photosphere. The photosphere emits a continuous spectrum of light as explained earlier. Atoms, ions and molecules in these hot gases absorb light photons of certain wavelengths. The light that passes through these hot gases is therefore deficient in these wavelengths and its spectrum therefore contains absorption lines.

The wavelengths of the absorption lines are characteristic of the elements in the corona of hot gases surrounding a star. By comparing the wavelengths of these absorption lines with the known absorption spectra for different elements, the elements present in the star can be identified. The last column in Table 1 shows how the elements present in a star differ according to the spectral class of the star.
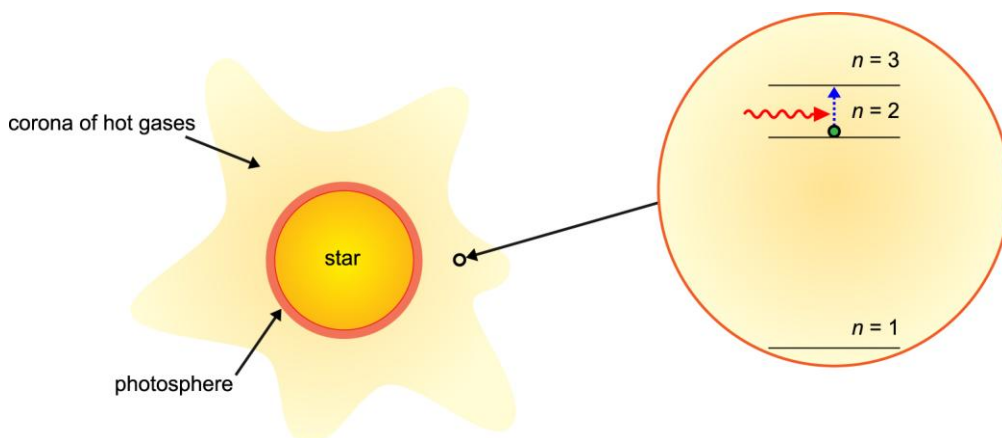
Figure 3 *Excitation in the hydrogen atom*

Since the absorption lines vary according to temperature, they can therefore be used in addition to temperature to determine the spectral class of the star. Note that the hydrogen absorption lines correspond to excitation of hydrogen atoms from the $n = 2$ state to higher energy levels. These lines, referred to as the **Balmer lines**, are only visible in the spectra of O, B and A class stars as other stars are not hot enough for excitation of hydrogen atoms due to collisions to the $n = 2$ state. In other words, hydrogen atoms in the $n = 2$ state exist in hot stars (i.e. O, B and A class stars); such atoms can absorb visible photons at certain wavelengths hence producing absorption lines in the continuous spectrum of light from the photosphere.

The Balmer lines of the Hydrogen spectrum are due to transitions from energy levels above $n = 2$ down to the $n = 2$ level. The diagram shows three such transitions.

Figure 4 *The origin of the Balmer lines*

Note that hydrogen atoms in the $n = 1$ state (the ground state) do not absorb visible photons because visible photons do not have sufficient energy to cause excitation from $n = 1$.

AQA Examiner's tip

Don't forget temperature in Wien's law and Stefan's law is always in kelvin.

Summary questions

Wien's law constant = $0.0029\,\text{m K}$, $\sigma = 5.67 \times 10^{-8}\,\text{W m}^{-2}\,\text{K}^{-4}$

1 With the aid of a diagram, explain what is meant by a black body spectrum and describe how such a spectrum from a star is used to determine the temperature of the star's light-emitting surface.

2 a State the main spectral classes of a star and the approximate temperature range of each class.

   b The spectrum of light from a star has its peak intensity at a wavelength of 620 nm. Calculate the temperature of the star's light-emitting surface.

3 A star has a surface temperature which is twice that of the Sun and a diameter that is four times as large as the Sun's diameter. Show that it emits approximately 250 times as much energy per second as the Sun.

4 Two stars X and Y are in the same spectral class. Star X emits 100 times more power that star Y.

   a State and explain which star, X or Y, has the bigger diameter.

   b X has a power output of $6.0 \times 10^{26}\,\text{W}$ and a surface temperature of 5400 K. Show that its surface area is $1.2 \times 10^{19}\,\text{m}^2$ and calculate its diameter.

2.3 The Hertzsprung–Russell diagram

Learning objectives:

■ What does the colour of a star tell us about the star?
■ How do stars form?
■ Why do we think the Sun will eventually become a white dwarf star?

The power of the Sun

The intensity of solar radiation at the Earth is about $1400\,W\,m^{-2}$. This means that a solar panel (area = $1\,m^2$) facing the Sun directly will receive $1400\,J$ of solar energy per second. In practice, absorption due to the atmosphere occurs and there is also some reflection. So how much radiation energy does the Sun emit each second? The mean distance from the Earth to the Sun is 1 AU which is $1.5 \times 10^{11}\,m$.
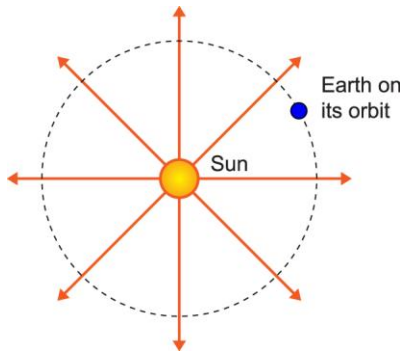

Earth on its orbit

Sun

Figure 1 *Solar radiation*

Imagine the Sun at the centre of a sphere of radius $1.5 \times 10^{11}\,m$. Each square metre of surface of this sphere will receive $1400\,J$ of solar energy per second.

The total amount of solar energy per second received by the sphere surface must be $1400\,J\,s^{-1}$ per square metre × the surface area of the sphere. This must be equal to the amount of solar energy per second emitted by the Sun (its luminosity or power output) as no solar radiation is absorbed between the Sun and the sphere's surface.

Since the surface area of a sphere of radius r is equal to $4\pi r^2$, the power output of the Sun is therefore $4.0 \times 10^{26}\,J\,s^{-1}$ (= $1400\,J\,s^{-1}\,m^{-2} \times 4\pi \times (1.5 \times 10^{11}\,m)^2$).

Dwarfs and giants

Topic 2.2 looked at how the spectrum of a star can be used to find the surface temperature of the star and its spectral class. It also looked at how the output power of a star can be calculated if the surface temperature and diameter are known. However, star diameters except for the Sun cannot be measured directly and are determined by comparing the absolute magnitude of the star with that of the Sun which is 4.8.

For example, a G class star which has an absolute magnitude of −0.2 is five magnitudes more powerful than the Sun and is therefore 100 times more powerful. Therefore, its power output is

$4.0 \times 10^{28} \, \text{J s}^{-1}$ (= $100 \times$ the power output of the Sun). Substituting this value of power output and the star's surface temperature into Stefan's law therefore enables its surface area and diameter to be calculated.

- A **dwarf star** is a star that is much smaller in diameter than the Sun.
- A **giant star** is a star that is much larger in diameter than the Sun.

Stefan's law gives the power output across the entire spectrum, not just across the visible spectrum. Absolute and apparent magnitudes relate to the visible spectrum. For a star that emits a significant fraction of its radiation in the non-visible spectrum, magnitude values that take account of non-visible radiation would need to be used. Such modifications are not part of the specification.

Worked example

$\sigma = 5.67 \times 10^{-8} \, \text{W m}^{-2} \text{K}^{-4}$

A K-class star has a power output of $4.0 \times 10^{28} \, \text{J s}^{-1}$ and a surface temperature of $4000 \, \text{K}$.

a Calculate:

   i   its surface area

   ii   its diameter.

b The diameter of the Sun is $1.4 \times 10^{9} \, \text{m}$. State whether it is a giant star or a dwarf star or neither.

Solution

a   i   Rearranging $P = \sigma A T^4$ gives $A = \dfrac{P}{\sigma T^4}$

        Hence $A = \dfrac{4.0 \times 10^{28}}{5.67 \times 10^{-8} \times \left(4000\right)^4} = 2.8 \times 10^{21} \, \text{m}^2$

   ii   For a sphere of radius R, its surface area $A = 4\pi R^2$

        Rearranging this equation gives $R^2 = \dfrac{A}{4\pi} = \dfrac{2.8 \times 10^{21}}{4\pi} = 2.2 \times 10^{20} \, \text{m}^2$

        Hence $R = 1.5 \times 10^{10} \, \text{m}$ so its diameter $= 2R = 3.0 \times 10^{10} \, \text{m}$

b   The star is 21 times the diameter of the Sun and so it is a giant star.

A ready-reckoner

To compare a star X with the Sun,

- power output of X, $P_X = \sigma A_X T_X^4$, where A_X = surface area of X and T_X = surface temperature of X
- power output of the Sun, $P_S = \sigma A_S T_S^4$, where A_S = surface area of the Sun and T_S = its surface temperature.

Therefore, $\dfrac{\text{power output of X, } P_X}{\text{power output of the Sun, } P_S} = \dfrac{\sigma A_X T_X^{\,4}}{\sigma A_S T_S^{\,4}} = \dfrac{A_X}{A_S} \times \left(\dfrac{T_X}{T_S}\right)^4$

Rearranging this gives $\dfrac{A_X}{A_S} = \dfrac{P_X}{P_S} \div \left(\dfrac{T_X}{T_S}\right)^4 = $ (power output ratio) $\div$ (temperature ratio)^4

For example, if the power ratio is 100 and the temperature ratio is 0.7, using the above expression gives 420 (to 2 significant figures) for the area ratio. So the diameter ratio is 420$^{\frac{1}{2}}$ as the area ratio is equal to the diameter ratio squared. So, the diameter of X is 20 times the diameter of the Sun.

Note

X is 100 times more powerful than the Sun so its absolute magnitude $= M_S - 5$ where M_S is the absolute magnitude of the Sun. Each magnitude difference of 1 corresponds to a power ratio of $100^{1/5}$ which is equal to 2.5.

In general, for two stars:

- with the same surface temperature and unequal absolute magnitudes, the one with the greater power output has the larger surface area and hence diameter
- with the same absolute magnitude and unequal surface temperatures, the hotter star has a smaller surface area and hence a smaller diameter.

The Hertzsprung–Russell diagram

Stars of known absolute magnitude and known surface temperature can be plotted on a chart in which the absolute magnitude is plotted on the *y*-axis and temperature on the *x*-axis as shown in Figure 2. This was first undertaken independently by Enjar Hertzsprung in Denmark and Henry Russell in America. The chart is known as a Hertzsprung–Russell (or HR) diagram.
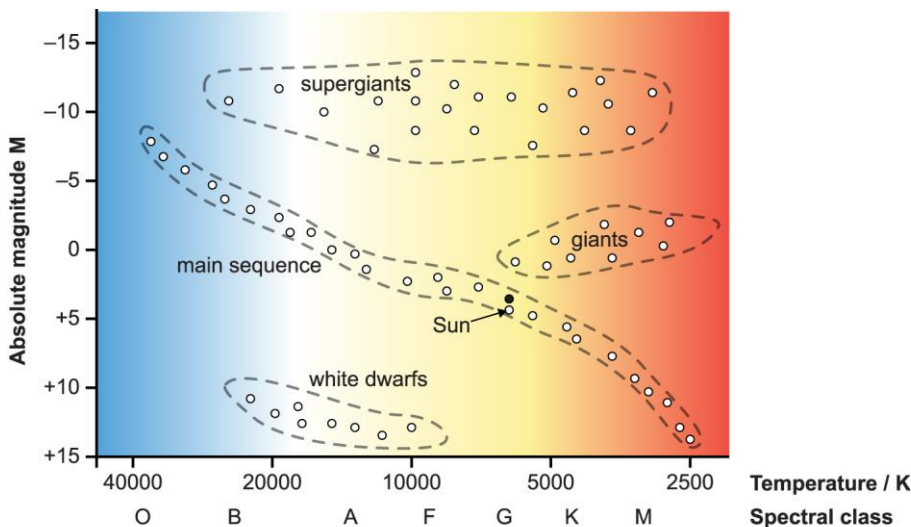
The Hertzsprung-Russell diagram
Notice that the temperature scale is non-linear and decreases from left to right and the absolute magnitude scale becomes more negative (which corresponds to increased brightness) as you go up !

Figure 2 *The Hertzsprung–Russell diagram*

The main features of the HR diagram are as follows:

- The **main sequence**, a heavily-populated diagonal belt of stars ranging from cool low-power stars of absolute magnitude +15 to very hot high-power stars of absolute magnitude about −5. The greater the mass of a star, the higher up the main sequence it lies. Star masses on the main sequence vary from about 0.1 to 30 or more times the mass of the Sun.

- **Giant stars** have absolute magnitudes in the range of about +2 to −2 so they emit more power than the Sun and are 10 to 100 times larger. Red giants are cooler than the Sun.
- **Supergiant stars** have absolute magnitudes in the range from about −5 to −10 and are much brighter and larger than giant stars. They have diameters up to 1000 times that of the Sun. They are relatively rare compared with giant stars.
- **White dwarf stars** have absolute magnitudes between +15 and +10 and are hotter than the Sun but they emit much less power. They are much smaller in diameter than the Sun.

Worked example

A red giant and a main sequence star have the same absolute magnitude of 0. Their surface temperatures are 3000 K and 15 000 K respectively. Show that the radius of the red giant is 25 times larger than that of the main sequence star.

Solution

Power output of a star, $P = \sigma A T^4$, where A = its surface area and T = its surface temperature.

The two stars have the same power output as they have the same absolute magnitude.

Therefore, $\sigma A T^4$ for the red giant = $\sigma A T^4$ for the main sequence star.

Cancelling σ on both sides of this equation and rearranging gives

$$\frac{A_{RG}}{A_{MS}} = \frac{T_{MS}^4}{T_{RG}^4} = \left(\frac{15000}{3000}\right)^4 = 5^4 = 625$$

Since the surface area $A = 4\pi(\text{radius})^2$, the radius of the red giant is therefore 25 times ($= 625^{\frac{1}{2}}$) the radius of the main sequence star.

Stellar evolution

The Sun is a middle-aged star about 4600 million years old. It produces energy as a result of nuclear fusion in its core converting hydrogen into helium. The core temperature must be of the order of millions of kelvin to maintain fusion. The fusion reactions release energy which maintains the core temperature. Radiation from the core heats the outer layers of the Sun causing light to be emitted from its surface (photosphere).

Main sequence stars like the Sun are in a state of internal equilibrium in the sense that gravitational attraction acting inwards is balanced by radiation pressure due to the outflow of gases which expand and cool. The star will move from its position on the main sequence and become a red giant star.

All stars evolve through a sequence of stages from their formation to the main sequence stage and beyond.

Formation

A star is formed as dust and gas clouds in space contract under their own gravitational attraction becoming denser and denser to form a protostar (a star in the making).

- In the collapse, gravitational potential energy is transformed into thermal energy as the atoms and molecules in the clouds gain kinetic energy so the interior of the collapsing matter becomes hotter and hotter.

■ If sufficient matter accretes to form the protostar, the temperature at the core of the protostar becomes high enough for nuclear fusion to occur. If there is insufficient matter, the star does not become hot enough for nuclear fusion to occur and it gradually cools once it has stopped contracting.

■ Energy released as a result of nuclear fusion of hydrogen to form helium increases the core temperature so fusion reactions continue to occur as long as there are sufficient light nuclei. As a result of continuing fusion reactions, the outer layers of the protostar become hot and a light-emitting layer (the photosphere) is formed and the protostar becomes a star.

Main sequence

The newly-formed star reaches internal equilibrium as the inward gravitational attraction is balanced by the outward radiation pressure. The star therefore becomes stable with constant luminosity.

■ Its absolute magnitude depends on its mass; the more mass it has, the greater its luminosity so it joins the main sequence at a position according to its mass.

■ The star remains at this position for most of its lifetime, emitting light as a result of 'hydrogen burning' in its core.

Application

Cepheid variables

Most stars have constant luminosity. In other words, their power output is constant and their brightness does not vary. Some stars do vary in their luminosity because they pulsate. Cepheid variables pulsate with a period of the order of days that depends on their average luminosity. By measuring the period of all 'nearby' Cepheid variables at known distances (and therefore known absolute magnitudes), the absolute magnitude of and hence distance to any other Cepheid variable (e.g. in a distant galaxy) can be determined by measuring its period. Prove for yourself that the Cepheid variable represented in Figure 3 is about 280 parsecs from the Sun. The graph in (b) shows the relationship for metal-rich Cepheids (group 1) and metal-poor Cepheids (group 2). The absolute magnitude for the period of the Cepheid variable represented in (a) is shown by the red dashed line.
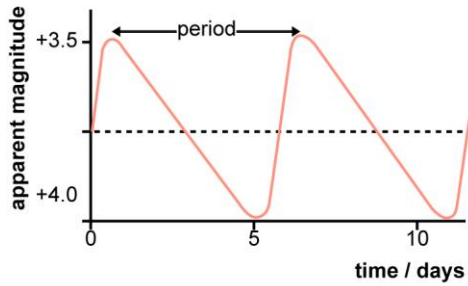
(a) Variation of apparent magnitude with time for a typical Cepheid variable
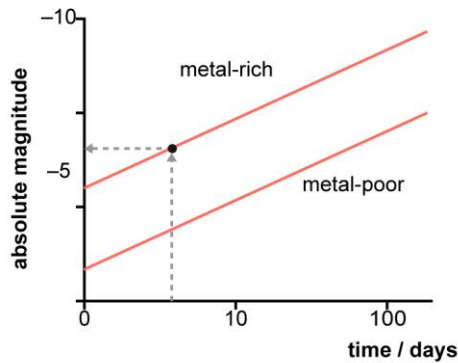
(b) Absolute magnitude v period relationship

Figure 3 *Using Cepheid variables*

Red giants

Once most of the hydrogen in the core of the star has been converted to helium, the core collapses on itself and the outer layers of the star expand and cool as a result. The star swells out and moves away from its position on the main sequence to become a giant or a supergiant star.

- The temperature of the helium core increases as it collapses and causes surrounding hydrogen to form a 'hydrogen-burning' shell which heats the core further.
- When the core temperature reaches about 108 K, helium nuclei in the core undergo fusion reactions in which heavier nuclei are formed, principally beryllium, carbon and oxygen. The luminosity of the star increases and the wavelength at peak intensity increases because it becomes cooler.
- The red giant stage lasts about a fifth of the duration of the main sequence stage. The evolution of a star after the red giant stage follows one of two paths according to its mass. Below a mass of about 8 solar masses, a red giant star sooner or later becomes a white dwarf. A star of higher mass swells out even further to become a supergiant which explodes catastrophically as a supernova.

White dwarfs

When nuclear fusion in the core of a giant star ceases, the star cools and its core contracts, causing the outer layers of the star to be thrown off.

- The outer layers are thrown off as shells of hot gas and dust which form so-called planetary nebulae around the star. This happens through several mechanisms including ionisation in the star's outer layers as the layers cool causing the layers to trap radiation energy which suddenly breaks out.

■ If the mass of the red giant star is between 4 and 8 solar masses, the core becomes hot enough to cause energy release, through further nuclear fusion, to form nuclei as heavy as iron in successive shells. The process stops when the fuel (i.e. the light nuclei) has all been used up.

After throwing off its outer layers, the star is now little more than its core which at this stage is white hot due to release of gravitational energy. If its mass is less than 1.4 solar masses, the contraction of the core stops as the electrons in the core can no longer be forced any closer. The star is now stable and has become a white dwarf which will gradually cool as it radiates its thermal energy into space and eventually becomes invisible. In the next topic, we will see that if its mass at this stage is greater than 1.4 solar masses, it does not form a white dwarf. Instead, it explodes catastrophically as a supernova.

Application

The future of the Sun

From what is known about the stars, astronomers have predicted the evolutionary path of the Sun. In about 5000 million years time, the Sun will become a red giant and swell out as far as the Earth. Its increased luminosity will evaporate Mars and blaze away the gaseous atmospheres of the planets beyond Mars. After the red giant stage which will last about 1 to 2 billion years, the Sun will throw off most of its mass into space and evolve into a white dwarf not much wider than the Earth and about ten times dimmer than at present. Over the next few billion years, it will become fainter and fainter and gradually fade away.
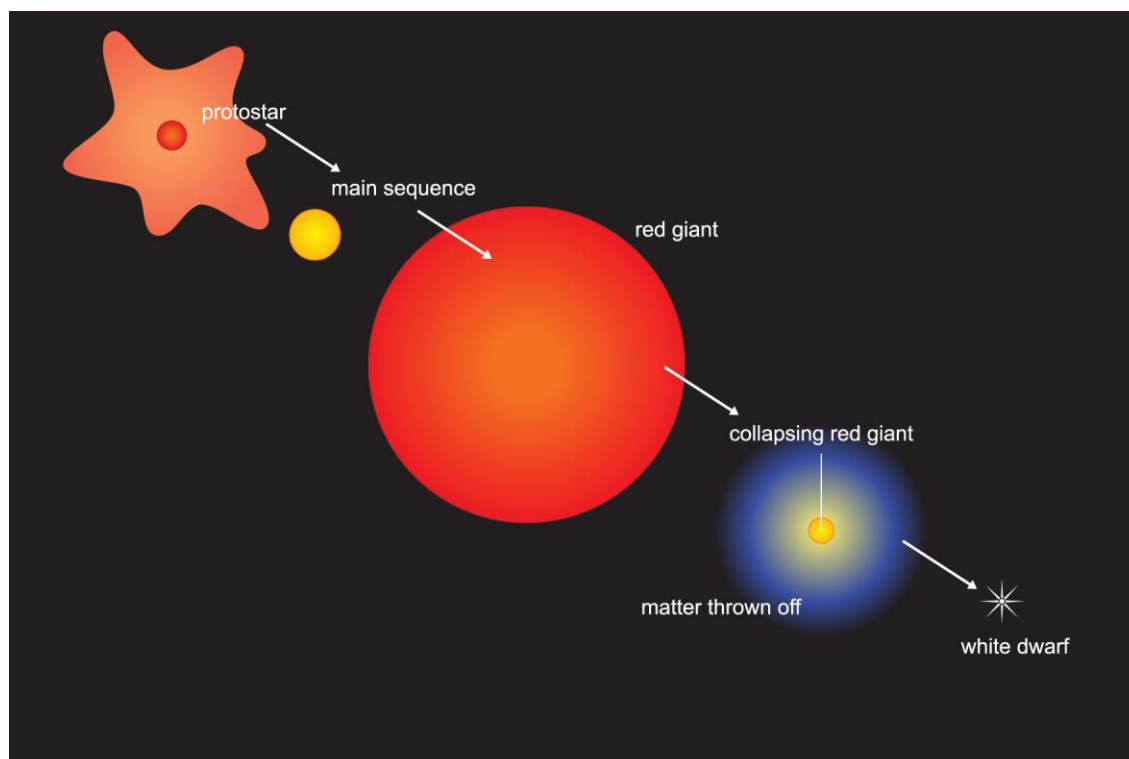

Figure 4 *Evolution of the Sun*

Summary questions

1 a Sketch a Hertzsprung–Russell diagram to show the full range of main sequence, giant and supergiant stars and white dwarfs. Show the relevant scales on each axis.

  b Show on your diagram the present position of the Sun and its evolutionary path after it leaves the

main sequence.

2 a Describe the formation of a star from gas and dust clouds.

b A protostar first becomes visible as a very dim cool star then moves onto a fixed position on the main sequence.

    i Indicate on your HR diagram the position where the protostar first becomes visible.

    ii What physical property of a newly formed star determines its position on the main sequence?

3 When a certain red giant star evolves into a white dwarf, it becomes very hot without loss of brightness, then it becomes fainter and it cools before stabilising as a white dwarf.

a Indicate on your HR diagram the evolutionary path of this star after the red giant stage.

b State the defining characteristics of a white dwarf star and list two other properties it possesses.

4 Three stars X, Y and Z have surface temperatures of 4000 K, 8000 K and 20 000 K respectively and absolute magnitude −2, +4 and +10 respectively.

a List the stars in order of increasing power output.

b State the evolutionary stage of each star, giving your reason for each statement.

c Calculate the ratio of the diameter of X and of Y relative to Z.

# 2.4 Supernovae, neutron stars and black holes

Learning objectives:

- Why is a supernova called a supernova?
- What is a neutron star?
- How is a black hole formed?

The death of a high-mass star

As discussed in the previous topic, when nuclear fusion ceases in the core of a red giant star, the outer layers of the star are thrown off and, if mass of the core and remaining matter is less than 1.4 solar masses, the star stabilises as a white dwarf. The repulsive force between the electrons in the core pushing outwards counterbalances the gravitational force pulling the core inwards.

Nuclear fusion ceases when there are no longer any nuclei in the core that release energy when fused. This happens when iron nuclei are formed by fusion as they are more stable than any other nuclei so cannot fuse to become even more stable.

If the core mass exceeds 1.4 solar masses, the electrons in the iron core can no longer prevent further collapse as they are forced to react with protons to form neutrons. The equation for this reaction is:

$$p + e^- \rightarrow n + \nu_e$$
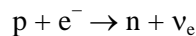
The sudden collapse of the core makes the core more and more dense until the neutrons can no longer be forced any closer. The core density is then about the same as the density of atomic nuclei, about $10^{17}\,\mathrm{kg\,m^{-3}}$. The core suddenly becomes rigid and the collapsing matter surrounding the core hits it and rebounds as a shock wave propelling the surrounding matter outwards into space in a cataclysmic explosion. The exploding star releases so much energy that it can outshine its host galaxy. The event is referred to as a **supernova** as it is much brighter than a **nova** or 'new' star in the same galaxy.
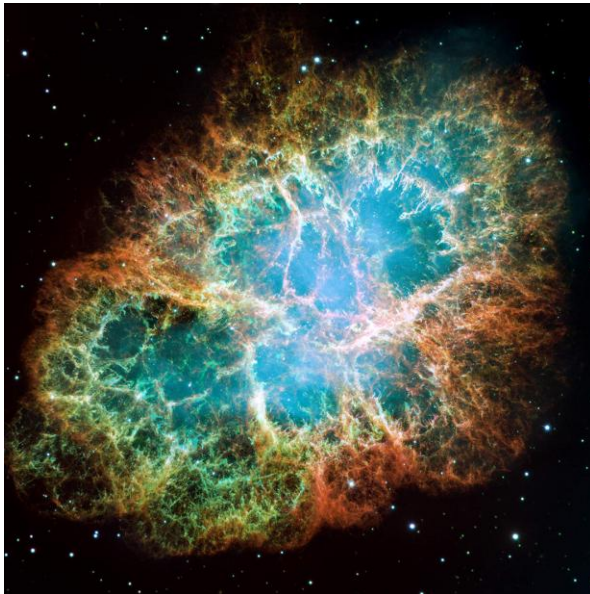

Figure 1 *The Crab Nebula.*

Figure 1 shows a supernova remnant of a star that exploded in AD 1054, about 2000 parsecs away. It is now about 2–3 parsecs in width.

How science works

Novae and supernovae

A nova is a star that suddenly becomes brighter, often having been too dim to be visible so it appears as a 'new' star. This can happen if a white dwarf draws matter from an invisible companion star and suddenly overheats and expels the excess matter as a result. Supernovae are, as described above, exploding stars that scatter much of their matter into space. Although astronomers discover many supernovae in distant galaxies every year using large telescopes and other detectors, supernovae visible to the unaided eye are rare. The last such supernova seen in the Milky Way occurred in 1604. A supernova observed in 1987 in a nearby galaxy became visible to the unaided eye and has been studied extensively since.

A supernova is typically a thousand million times more luminous than the Sun. Its absolute magnitude is therefore between −15 and −20. In comparison, the absolute magnitude of the Sun is +4.8. This increase of luminosity occurs within about 24 hours. Measurements of their subsequent luminosity show a gradual decrease on a time scale of the order of years. Thus the tell-tale sign of a supernova is a sudden and very large increase in luminosity of the star corresponding to a change of about 20 magnitudes in its absolute magnitude.

A supernova explosion throws the matter surrounding the core into space at high speeds. Elements heavier than iron are formed by nuclear fusion in a supernova explosion. Such fusion reactions occur as the shock wave travels through the layers of matter surrounding the neutron-filled core. The supernova explosion scatters the matter surrounding the core into space. Thus the supernova remnants in space contain all the naturally occurring elements. Note that helium is formed from hydrogen in fusion reactions in main sequence stars. Other elements as heavy as iron are formed progressively in fusion reactions in red giant stars. As explained earlier, elements heavier than iron cannot be formed in main sequence and red giant stars. Their existence in the Earth tells us that the Solar System formed from the remnants of a supernova.

A supernova explosion also causes an intense outflow of neutrinos and gamma photons. Neutrinos from supernova 1987A were detected three hours before light was detected from it. The light seen from the explosion was produced when the shock wave hit the outer layers of the star. In contrast, the neutrinos produced by nuclear fusion as the shock wave made its way through the interior travelled much faster than the shock wave, reaching the surface hours before the shock wave.

More about supernovae

Supernovae are classified into several types according to their line absorption spectra.

- **Type I** supernovae have no strong hydrogen lines present and are further subdivided into three groups.
  - **Type Ia** supernovae show a strong absorption line due to silicon. They rapidly reach peak luminosity of about 10^9 times the Sun's luminosity then decrease smoothly and gradually. They are thought to occur when a white dwarf star in a binary system attracts matter from a companion giant star causing fusion reactions to restart in which carbon nuclei form silicon nuclei. The fusion process becomes unstoppable as further matter is drawn from the giant star and the white dwarf explodes.
  - **Type Ib** supernovae show a strong absorption line due to helium; these are thought to occur when a supergiant star without hydrogen in its outer layers collapses. After reaching peak luminosity, their light output decreases steadily and gradually.

- **Type Ic** supernovae lack the strong lines present in types Ia and Ib; these are thought to occur when a supergiant without hydrogen or helium in its outer layers collapses. After reaching peak luminosity, their light output also decreases steadily and gradually.
- **Type II** supernovae have strong hydrogen lines; these are thought to occur when a supergiant which has retained the hydrogen or helium in outer layers collapses. Their peak luminosity is not as high as type Ia supernovae and their light output decreases gradually but unsteadily.

Table 1 summarises the characteristics of the different types of supernovae.

Type	Spectrum	Light output	Origin
Ia	no hydrogen lines; strong silicon line	decreases steadily	white dwarf attracts matter and explodes
Ib	no hydrogen lines; strong helium line	decreases steadily	supergiant collapses then explodes
Ic	no hydrogen; no helium lines	decreases steadily	supergiant collapses then explodes
II	strong hydrogen and helium lines	decreases unsteadily	supergiant collapses then explodes

Table 2 *Types of supernova*

Type Ia supernovae reach a known peak luminosity and are characterised by the presence of a strong silicon absorption line, so they are used to find the distance to their host galaxy. A supernova can temporarily outshine its host galaxy, so the detection of a type Ia supernova in a galaxy at unknown distance enables the distance to the galaxy to be found. This method of measuring distances to distant galaxies has led to the prediction of a new form of energy referred to as **dark energy**.

Neutron stars and black holes

A **neutron star** is the core of a supernova after all the surrounding matter has been thrown off into space. A neutron star is extremely small in size compared with a star such as the Sun. If its mass was the same as that of the Sun:

- its diameter would be about 30 km
- its surface gravity would be over two thousand million times stronger than at the surface of the Sun.

The first evidence for neutron stars came with the discovery in 1967 of pulsating radio stars or **pulsars**. The radio pulses are at frequencies of up to about 30 Hz. A typical pulsar is less than 100 km in diameter and has a mass of about two solar masses. From their observations including the discovery of extremely strong magnetic fields in pulsars, astronomers deduced that pulsars are rapidly rotating neutrons stars that produce beams of radio waves. These sweep round the sky as the neutron star rotates like the light beam from a lighthouse.

Application

What causes the radio beams from a pulsar?

Each time the beam sweeps over the Earth we receive a pulse of radio waves. The radio beams are thought to be generated by charged particles spiralling in the intense magnetic field above the magnetic poles of the neutron star. The magnetic axis and the rotation axis are different, so the radio beams sweep round as the star spins about its rotation axis.
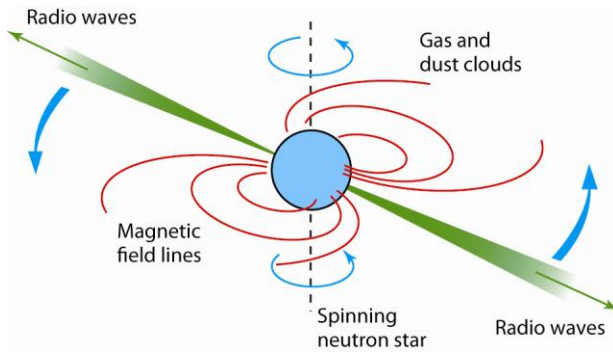
Figure 2 *Radio waves from a pulsar*

A **black hole** is an object so dense that not even light can escape from it. A supernova core contains neutrons only but if its mass is greater than about three solar masses, the neutrons are unable to withstand the immense forces pushing them together. The core collapses on itself and becomes so dense that not even light can escape from it. The object is then a black hole. It can't emit any photons and it absorbs any photons that are incident on it.

The **event horizon** of a black hole is a sphere surrounding the black hole from which nothing can ever emerge. The radius of this sphere is called the Schwarzschild radius, R_S, of the black hole. Einstein's general theory of relativity gives the following equation for the Schwarzschild radius of a black hole of mass M

$$R_S = \frac{2GM}{c^2}$$

where G is the universal constant of gravitation and c is the speed of light in free space.

What happens inside a black hole can not be observed. A black hole attracts and traps any surrounding matter, increasing its mass as a result. Matter falling towards a black hole radiates energy until it falls within the event horizon. Inside the black hole, matter is drawn with ever-increasing density towards a singularity at its centre, a point where the laws of physics as we know them may not apply.

The key characteristic of a black hole is its mass. It may also be charged and it may or may not be rotating. Matter that falls into a black hole contributes its mass, its charge if any and its rotational motion if any to the black hole. Any other property carried by infalling matter is lost. For example, the properties of a black hole are unaffected by the chemical elements in the matter dragged into the black hole. This information about the infalling matter is lost in the black hole.

Evidence for black holes

Evidence for black holes formed from collapsed neutron stars was found in 1971 using the first satellite-mounted X-ray telescope. The satellite pinpointed an X-ray source, labelled Cygnus X-1, in the same location as a supergiant star 2500 parsecs away. The intensity of the X-rays varied irregularly on a time scale of the order of 0.01 seconds, indicating a source diameter of the order of 3000 km (= speed of light × 0.01 s) which is smaller than the Earth. When the position of the supergiant was found to vary slightly, it was realised the supergiant and the X-ray source must be orbiting each other as a binary system. The mass of the X-ray source was estimated at about 7 solar masses or a quarter of the mass of the supergiant. Its mass is above the upper limit of 3 solar masses for a neutron star, so astronomers think that Cygnus X-1 is a black hole which attracts matter from the supergiant. As the matter falls towards the black hole, it becomes so hot that it emits X-rays.
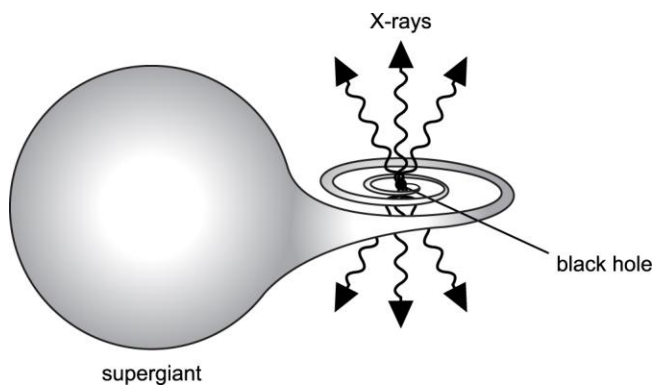
Figure 3 *Evidence for a black hole*

Further similar evidence for black holes has been found from several other X-ray sources. These findings indicate that black holes may form in binary systems where one of the stars explodes as a supernova, leaving a core of mass greater than about three solar masses that collapsed to become a black hole. The other star may not have reached or gone beyond the giant stage as in the above examples.

A further possibility is that a white dwarf or a neutron star might have pulled matter off a binary companion star and turned into a black hole when its mass exceeded three solar masses or both stars in a binary system might have become neutron stars and merged to become a black hole.

Supermassive black holes

Supermassive black holes of almost unimaginable mass are thought to exist at the centre of many galaxies. At the centre of a galaxy, stars are much closer together than they are at the edges of the galaxy. A supermassive black hole at the centre could pull millions of millions of stars in. Such black holes can therefore gain enormous quantities of matter and are referred to as supermassive black holes. Strong evidence now exists that there are supermassive black holes at the centre of many galaxies.

- The Andromeda galaxy, M31: detailed observations of the central region of the Andromeda galaxy, the nearest large galaxy to the Milky Way, show that stars near the galactic centre are orbiting the centre at speeds of the order of $100 \, km \, s^{-1}$ at distances of no more than about 5 parsecs from the centre. These stars must therefore be orbiting a central object of diameter less than 5 parsecs. Applying satellite theory to this object gives a central mass of about 10 million solar masses which is thought to be a supermassive black hole.
- The Milky Way Galaxy: images using infrared radiation and radio waves from the centre of the Milky Way indicate stars there that are orbiting the galactic centre at speeds of more than $1500 \, km \, s^{-1}$ orbiting at distances of about 2 parsecs from the galactic centre. This information indicates a supermassive black hole of mass equal to about 2.6 million solar masses. Strong evidence has also been found of other local galaxies that have a supermassive black hole at the centre.
- Distant galaxies have also yielded evidence of a supermassive black hole at each centre. The Sombrero galaxy, M104, has fast-moving stars in orbits close to its centre, indicating a supermassive black hole of mass equal to 1000 million solar masses.

Topic 3.3 of these notes will return to the subject of supermassive black holes.

Summary questions

$G = 6.67 \times 10^{-11} \, N \, m^2 \, kg^{-2}$, $c = 3.0 \times 10^8 \, m \, s^{-1}$

1 a What change in a giant star causes its core to collapse?

b Why does infalling matter rebound when the core of a giant star collapses?

2 a A neutron star is made of neutrons. State two other characteristics of a neutron star.

b Explain why a neutron star has a mass which is:

i more than 1.4 solar masses

ii less than about 3 solar masses.

3 a What astronomical observation indicates that a supernova has occurred?

b What astronomical observation indicates that a particular supernova is due to an explosion of a white dwarf star rather than the collapse of a red giant star?

4 a i What is a black hole and what are its physical properties?

ii Where should astronomers look to locate a supermassive black hole?

b For a black hole of the same mass as the Sun, which is 2.0×10^{30} kg, calculate:

i its Schwarzschild radius

ii the mean density inside its event horizon.

c By carrying out appropriate calculations, compare the density of a supermassive black hole of mass 10 million solar masses with your answer to **b ii**.