Physics 5

3.5.1 Radioactivity

Evidence for the nucleus

Qualitative study for the Rutherford scattering.

Rutherford directed a **narrow beam of α-particles**, all of the **same kinetic energy**, in **an evacuated container** at a **thin gold film**, in order to identify the structure of an atom:

- The α-particles need to be of the same kinetic energy otherwise slow α-particles would be deflected more than faster α-particles on the same initial path.
- The container needs to be evacuated as otherwise the α-particles would be stopped by air particles.

Observations:

- **Most α-particles passed through**
- **A few α-particles deflected – some more than 90° (bounce back)**

Interpretation:

- Most of the atom's mass is concentrated in a small region, the **nucleus**, at the centre of the atom.
- The nucleus is positively charged because it **repels** α-particles
- The nucleus itself is small since very few α-particles are deflected are deflected by much.

α, β and ϒ radiation

Their properties and experimental identification using simple absorption experiments; applications e.g. to relative hazards of exposure to humans.

Radiation	Range	Ionising	Speed	Penetrating Power	Application
Alpha- α	100mm	Strongly	Slow	Absorbed by paper or a few cm of air	Fire Alarms
Beta- β^-	1m	Weakly	Fast	Absorbed by approximately 3mm of aluminium	Thickness of paper, fill up of bottle
Gamma- ϒ	Follows inverse law	Very Weakly	Speed of Light	Absorbed by many cm of lead, or several m of concrete	Diagnostic techniques is medicine, sterilising equipment

Alpha: $${}^{A}_{Z}X \rightarrow {}^{4}_{2}\alpha + {}^{A-4}_{Z-2}Y$$

Beta (-): $${}^{A}_{Z}X \rightarrow {}^{0}_{-1}\beta + {}_{Z+1}^{A}Y + \bar{\Upsilon}_e$$

Beta (+): $${}^{A}_{Z}X \rightarrow {}^{0}_{+1}\beta + {}_{Z-1}^{A}Y + \mathrm{V}_e$$

Gamma: **no change**

Electron Capture: $${}^{A}_{Z}X + {}^{0}_{-1}e \rightarrow {}_{Z-1}^{A}Y + \mathrm{V}_e$$

Hazards of exposure to humans:

Affects living cells because:

- **It can destroy cell membranes which cause cells to die.**
- **Damage DNA.**

As a result of ionising radiation, living cells die or grow uncontrollably or mutate. This can affect patient's health, possibly affecting future generation because of genetic disorders. Mutations can lead to cancer.

The Inverse Square Law for γ radiation, including its experimental verification; application, e.g. to safe handling of radioactive sources.

1. A gamma source emits gamma radiation in all directions.
2. Radiation spreads out with distance from the source.
3. However, the **amount of radiation per unit area (intensity)** will decrease with distance from the source.
4. The intensity decreases with the square of the distance:

$$\boldsymbol{I = \frac{kI_0}{d^2}} \quad I = intensity, k = constant, d = distance, I_0 = intensity\ at\ source$$

- The verify the inverse square law for a ϒ source, use a Geiger counter to measure the count rate, C, at different measured distances, d, from the tube and the background count rate, C_0, without the source present.
- The **corrected count rate** is C- C_0 is proportional to the **intensity** of the radiation.
- Using inverse square law: $C - C_0 = \frac{constant}{(d+d_0)^2}$.
- Graph of 'd' against $\frac{1}{(C-C_0)^{1/2}}$ should give a straight line with a negative intercept $-d_0$.

Background radiation; examples of its origins and experimental elimination from calculations.

- **Experimental elimination = inverse square law for gamma radiation.**

Sources of Background radiation: air, medical, ground and buildings, food and drink, cosmic rays, nuclear weapons, air travel and nuclear power.

Storage of radioactive materials:

- In lead-lined containers.
- Lead lining must be thick enough to reduce the gamma radiation from the sources in the container to about background level.
- Containers under 'lock and key'.

Using radioactive materials:

- No source is allowed to come into contact with the skin.
- Solid sources transferred using handling tools (tongs, robots etc). The source needs to be far enough so that the person is beyond the range of α and β radiation from the source.
- Liquid, gas and solid (powder) sources should be in sealed containers. This ensures that the radioactive gas cannot be breathed in and liquid cannot be splashed on the skin.
- Source should not be used for longer than is necessary.

Radioactive Decay

Random nature of radioactive decay.

- **Radioactive decay is completely random.**
- **Any sample of a particular isotope has the same rate of decay, i.e. the same proportion of the atoms will decay in a given time.**

Use of activity $A = \lambda N$.

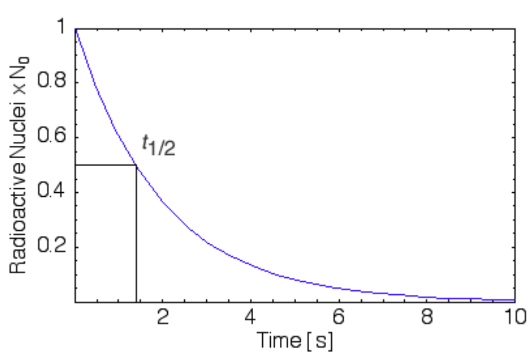
- The **activity** of a sample is **the number of atoms that decay each second**. It is **proportional to the size of the sample**.
- The **decay constant (λ) is the probability of an individual nucleus decaying per second.** Its unit is s^{-1}.

$$\text{Activity} = \text{decay constant} \times \text{number of atoms}$$
$$A = \lambda N$$

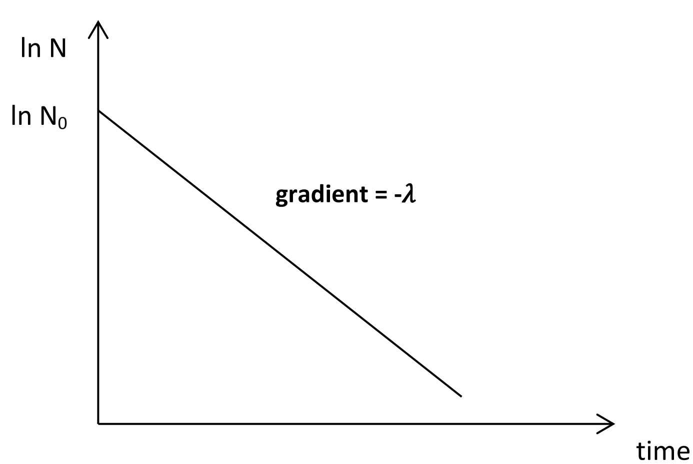
- **Activity** is measured in **becquerels (Bq): $1\ Bq = 1\ decay\ per\ second\ (s^{-1})$**

Half-life, $T_{1/2} = \frac{ln2}{\lambda}$; determination f graphical decay data including decay curves and log graphs; applications e.g. relevance to storage of radioactive waste, radioactive dating.

- The **half-life** ($T_{1/2}$) of a radioactive isotope is the **average time taken for the mass of the isotope to decrease to half its initial mass**.
- The **half-life** ($T_{1/2}$) of a radioactive isotope is **the average time taken for the number of undecayed atoms to halve.**



When measuring the **activity** and **half-life** of a source, remember to **subtract background radiation from activity readings to give the source activity.**



Half-life can be calculated using the equation:

$$T_{1/2} = \frac{ln2}{\lambda} \cong \frac{0.693}{\lambda}$$

Constant decay probability of a given nucleus;

$\frac{\Delta N}{\Delta t} = -\lambda N, \; N = N_0 e^{-\lambda t}$

The number of radioactive atoms remaining, N, depends on the number originally present, N_0. The **number of radioactive atoms remaining** can be calculated using the equation:

$$N = N_0 e^{-\lambda t}$$

This is represented by the decay graph.

As a sample decays, its activity decreases. The **activity** of a sample as it decays can be calculated using the equation:

$$A = A_0 e^{-\lambda t}$$

The corrected count rate, C, due to a sample of radioactive isotope at a fixed distance from the Geiger tube is proportional to the activity of the source. Therefore, the count rate decreases with time in accordance with the equation: $C = C_0 e^{-\lambda t}$, where C_0 is the count rate at time = 0.

- The **decay constant (λ) is the probability of an individual nucleus decaying per second.** Its unit is s^{-1}.

Radioactive Isotopes have many uses:

Radiocarbon Dating

- Isotope **carbon-14** is used in **radiocarbon dating**, formed as a result of cosmic rays knocking out neutrons from nuclei, which then collide with a nitrogen nuclei: ${}^{1}_{1}n + {}^{14}_{7}N \rightarrow {}^{14}_{6}C + {}^{1}_{1}p$
- Plants take in carbon dioxide from the atmosphere for photosynthesis, including the radioactive isotope carbon-14.
- When they die, the activity of carbon-14 in the plants starts to fall, with a half-life of around 5730 years.
- Samples of living material can be tested for current amount of carbon-14 in them, and so they can be dated.

Argon Dating

- Ancient rocks contain trapped **argon** gas as a result of decay of the radioactive isotope of potassium, ${}^{40}_{19}K$, into the argon isotope, ${}^{40}_{18}Ar$. This happens when its nucleus captures an inner shell electron, therefore a proton in the nucleus changes to a neutron and a neutrino is emitted.
- ${}^{40}_{19}K + {}^{0}_{-1}e \rightarrow {}^{40}_{18}Ar + V_e$
- The potassium isotope can also decay by β^- emission to form the calcium isotope, ${}^{40}_{20}Ca$.
- ${}^{40}_{19}K \rightarrow {}^{0}_{-1}\beta + {}^{40}_{20}Ca + \bar{Y}_e$
- This process is 8 times more probable than electron capture.
- The effective half-life of the decay of ${}^{40}_{19}K$ is 1250 million years. The age of the rock (when it solidified) can be calculated by measuring the proportion of argon-40 to potassium-40.

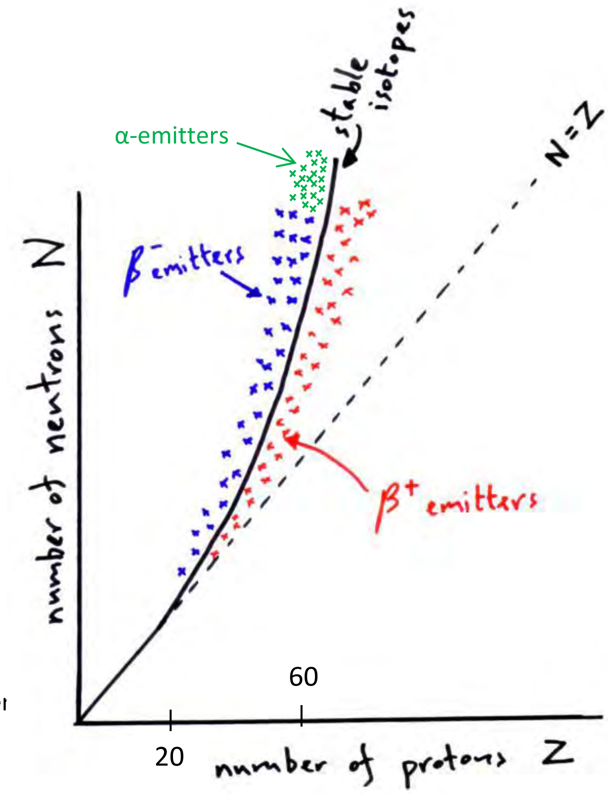
- For every N potassium-40 atoms now present, if there is 1 argon-40 atom, there must have been N+9 atoms of potassium-40 originally (i.e. 1 that decayed into argon-40 + 8 that decayed into calcium-40 + N remaining of potassium-40. This is N_0.)
- The radioactive decay equation $N = N_0 e^{-\lambda t}$ can be used to find the age of the sample.

Nuclear Instability

Graph of N against Z for stable nuclei.

N = number of neutrons

Z = proton number/atomic number



A nucleus will be **unstable** if it has:

- Too many neutrons.
- Too few neutrons.
- Too many nucleons – too heavy.
- Too much energy.

For light isotopes (Z= 0-20):

- The stable nuclei follow the straight line N=Z.

As Z increases beyond about 20:

- Stable nuclei have more neutrons than protons.
- The neutron/proton ratio increases.
- The extra neutrons help to bind the nucleons together as more protons would do.

Possible decay modes of unstable nuclei including α, β^+, β^- and electron capture.

Alpha-emitters occur beyond about Z=60:

- These nuclei have more neutrons than protons but they are too large to be stable. This is because the strong nuclear force between the nucleons is unable to overcome the electrostatic force of repulsion between the protons.

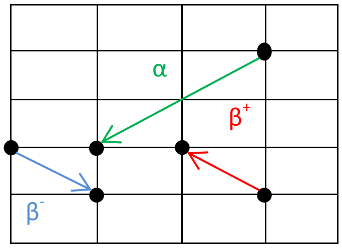
Beta-minus emitters:

- Occur to the left of the stability belt where the isotopes are neutron rich compared to stable isotopes.
- A neutron is converted to a proton and emitting a β^- particle (and an electron antineutrino).

Beta-plus emitters:

- Occur to the right of the stability belt where the isotopes are proton-rich compared to stable isotopes.
- A proton is converted to a neutron and emitting a β^+ particle (and an electron neutrino).

Changes of N and Z caused by radioactive decay and representation in simple decay equations.



Alpha: $^{A}_{Z}X \rightarrow ^{4}_{2}\alpha + ^{A-4}_{Z-2}Y$

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Electron Capture: $^{A}_{Z}X + ^{0}_{-1}e \rightarrow _{Z-1}^{A}Y + V_e$

In every nuclear reaction the following are conserved:

- **Energy**
- **Momentum**
- **Proton Number**
- **Charge**
- **Nucleon Number**

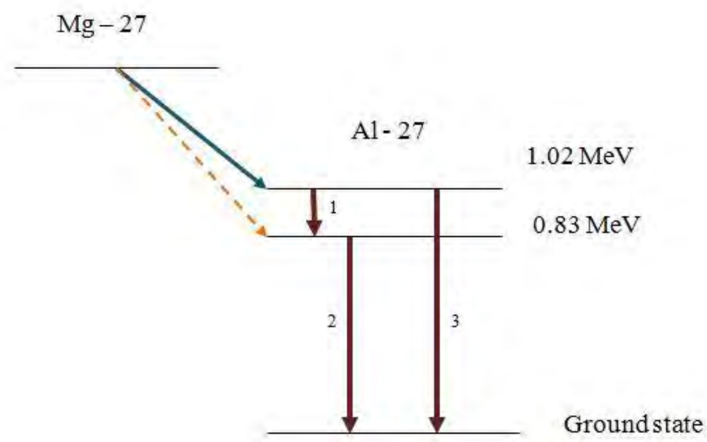
The mass is not conserved.:

- **The mass of an alpha particle is less than the individual masses of two protons and two neutrons.** This is called the **mass defect.**
- **Mass doesn't have to be conserved because of Einstein's equation: $E = mc^2$.**
- This shows that mass and energy are equivalent.
- The energy released when the nucleons bonded together accounts for the missing mass – so the $energy\ released = mass\ defect\ \times c^2$.

Existence of nuclear excited states; ϒ ray emission; application e.g. use of technetium-99m as a ϒ source in medical diagnosis.

- After an unstable nucleus emits an alpha or a beta particle or undergoes electron capture, it might **emit a ϒ photon**.
- Emission of a ϒ photon does not change the number of protons or neutrons in the nucleus, but it does allow the nucleus to lose energy.
- This happens when the **'daughter'** nucleus is in an excited state after it emits an α or a β particle or undergoes electron capture.
- The excited state is usually short lived and the nucleus moves to its lowest energy state, its ground state, either directly or via one or more lower-energy excited states.

- E.g.



1 = emission of 0.19 MeV ϒ photon.
2= emission of 0.83 MeV ϒ photon.
3= emission of 1.02 MeV ϒ photon.

Medical Tracers

- **Technetium-99m** is used in **medical tracers**. These are radioactive substances that are used to show tissue or organ function.
- The tracer is injected into or swallowed by the patient and then moves through the body to the region of interest.
- The radiation emitted is recorded and an image of inside the patient is produced.
- **Technetium-99m** is suitable for this use because it **emits ϒ-radiation**, has a **half-life of 6 hours** (long enough to be recorded but short enough to limit the radiation to an acceptable level) and **decays to a much more stable isotope.**

Nuclear Radius

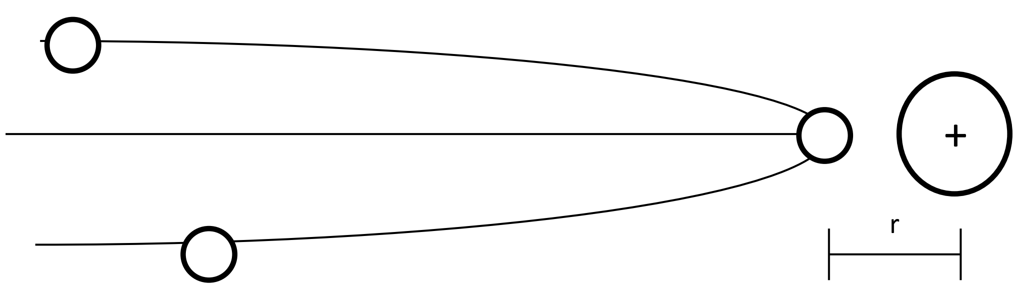
Estimate of radius from closest approach of alpha particles and determination of radius from electron diffraction; knowledge of typical values.

Estimating the closest approach of a scattered particle.

- We know the α-particles **initial kinetic energy**.
- An α-particle that 'bounces back' is deflected through 180° will have stopped a short distance from the nucleus. It does this at the point where **its electric potential = its kinetic energy**.

$$Initial\ K.E. = E_{elec} = \frac{Q_{gold} \times q_{alpha}}{4\pi\varepsilon_0 r}$$

$\varepsilon_0 = permittivity\ of\ free\ space, Q\ or\ q = charge$

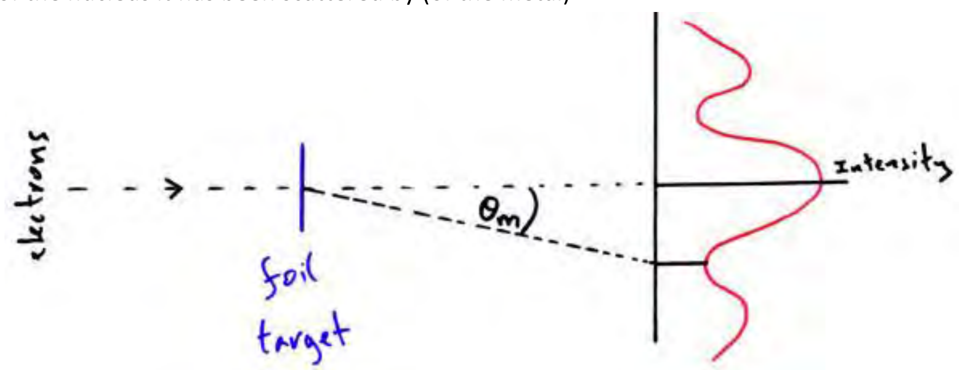


- It is estimated that the **diameter of an atom** is about **0.1nm ($1x10^{-10}$).**
- It is estimated that the **diameter of a nucleus** is about **1 fm ($2x10^{-15}$).**

High-energy electron diffraction

- Like other particles, electrons show wave-particle duality – so electron beams can be diffracted.
- A beam of moving electrons has an associated **de Broglie wavelength, λ**, which at high speeds is approximately: $\lambda \cong \frac{hc}{E}$.
- If a beam of high-energy electrons is directed onto a thin film of material in front of a screen, a diffraction pattern will be seen on the screen.
- As with light diffraction patterns, the **first minimum** appears where: $sin\theta \cong \frac{1.22\lambda}{d} = \frac{0.61\lambda}{R}$.

d = diameter of the nucleus it has been scattered by (of the metal)



Dependence of radius on nucleon number.

- The particles that make up the nucleus are called **nucleons**.
- The number of nucleons in an atom is called the **mass number, A**.
- As more nucleons are added to the nucleus, it gets bigger.

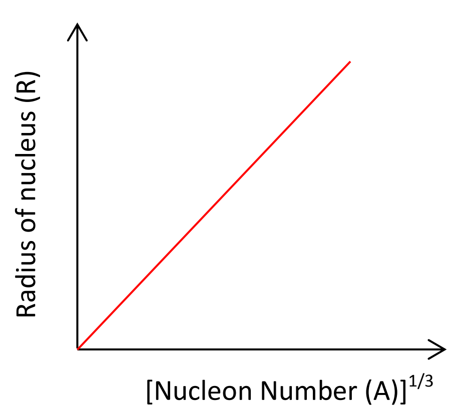
$R = r_0 A^{1/3}$ derived from experimental data.

The **nuclear radius** increases roughly as the cube root of the mass (nucleon) number.

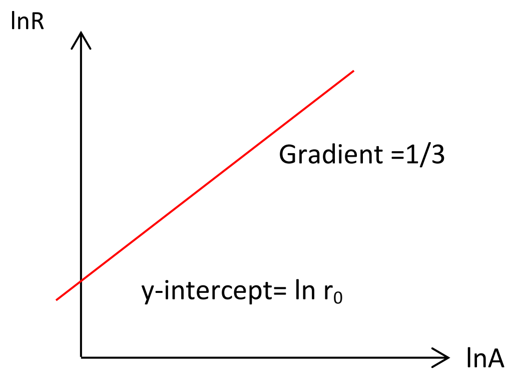
1. The **straight-line graph** shows the **nuclear radius (R)** is **directly proportional** to the **cube root of the nucleon number (A).**
2. This relationship can be writes as: $R \propto A^{1/3}$.
3. This can be made into an equation by introducing a constant, r_0, giving:

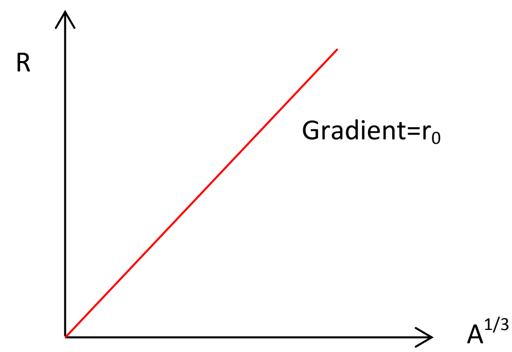
$$R = r_0 A^{1/3}$$

Where r_0 is the value of R when A=1, i.e. for a proton (hydrogen nucleus). The value of r_0 is about 1.4fm.



Nuclear Radius graphs





Calculation of nuclear density.

- Assuming that the nucleus is spherical, it **volume**, $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(r_0 A^{1/3})^3 = \frac{4}{3}\pi {r_0}^3 A$.
- The **nuclear volume, V**, is **proportional to the mass of the nucleus**.
- The **DENSITY** of the nucleus is **CONSTANT**, independent of the radius, and is the same throughout a nucleus.
- This shows that nucleons are separated by the same distance regardless of the size of the nucleus and are therefore evenly separated inside the nucleus.
- Calculating nuclear density:

$$density\ of\ nucleus = \frac{Au}{\frac{4}{3}\pi {r_0}^3 A} = \frac{1u}{\frac{4}{3}\pi {r_0}^3} = \frac{1.661 \times 10^{-27}}{\frac{4}{3}\pi(1.05 \times 10^{-15})} = 3.4 \times 10^{17} kgm^{-3}$$

m = Au, where 1u = 1 atomic mass unit = 1.661×10^{-27}.

Nuclear density should be calculated in the order $\times 10^{17}$.

Nuclear density is significantly larger than atomic density. This suggests:

- Most of the atom's mass is in in the nucleus.
- The nucleus is small compared to the atom.
- An atom must contain a lot of empty space.

3.5.2 Nuclear energy

Mass and energy

Appreciation that $E = mc^2$ applies to all energy changes.

- The equation $E = mc^2$ applies to all energy changes.

Simple calculations on mass difference and binding energy.

Atomic mass unit, u; conversion of units; 1u=931.3MeVu^{-1}.

- The mass of a nucleus is less than the mass of its constituent parts – the difference is called a **mass defect**.
- **The mass defect, Δm, of a nucleus is defined as the difference between the mass of the separated nucleons and the mass of the nucleus.**
- $E = mc^2$ suggests that the mass and energy are equivalent.
- As the nucleons join together, the total mass decreases – this 'lost' mass is converted into energy and released.
- The amount of energy lost is equivalent to the mass defect.
- If you had to pull the nucleus completely apart, the energy you'd have to use to do it would be the same as the energy released when the nucleus formed.

The energy needed to **separate** all of the nucleons in a nucleus is called the **binding energy** (measured in **MeV**), and it is equivalent to the **mass defect**.

- Calculating binding energy:

e.g. Calculate the binding energy of the nucleus of a lithium atom, ${}^{6}_{3}Li$, given that its mass defect is 0.0343u.

1. Convert atomic mass unit, u, into kg:

$$0.0343 \times (1.661 \times 10^{-27}) = 5.70 \times 10^{-27} kg$$

2. Use $E = mc^2$ to calculate the binding energy:

$$E = (5.70 \times 10^{-27}) \times (3.0 \times 10^{8})^2 = 5.13 \times 10^{-12} J$$

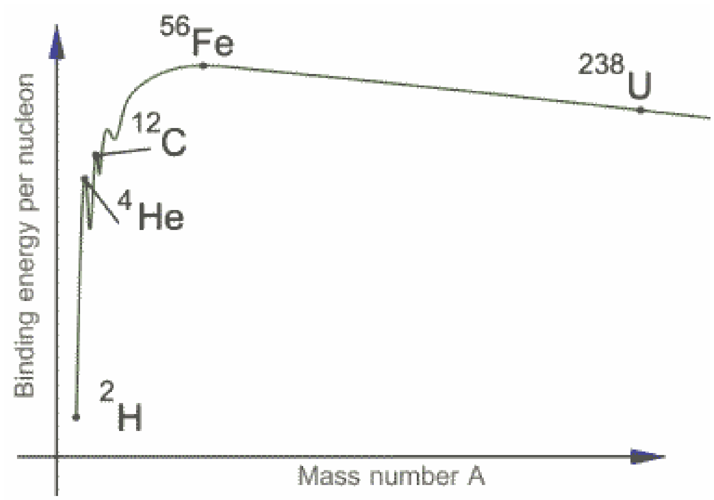
$$5.13 \times 10^{-12} J = \frac{5.13 \times 10^{-12} J}{1.6 \times 10^{-13} J} = 32 MeV$$

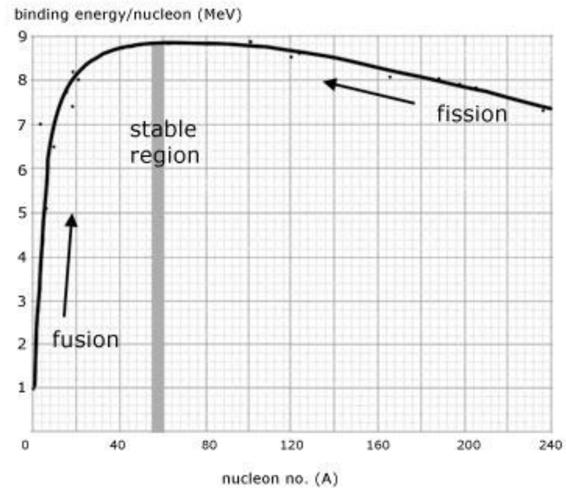
- The **binding energy per unit of mass defect** can be calculated (using example above):

$$\frac{binding\ energy}{mass\ defect} = \frac{32MeV}{0.0343u} \approx 931.3 MeVu^{-1}$$

- This means that the mass defect of **1u** is equivalent to about 931.3MeVu^{-1}.

Graph of average binding energy per nucleon against nucleon number.





$$Binding\ energy\ per\ nucleon\ (MeV) = \frac{Binding\ Energy\ (B)}{Nucleon\ Number\ (A)}$$

- **High binding energy per nucleon** means that **more energy is needed to remove nucleons from the nucleus.**
- **The most stable nuclei occur around the maximum point of the graph – which is at nucleon number 56 (Fe).**
- **Combining small nuclei is called fusion – this increases the binding energy per nucleon dramatically, which means a lot of energy is released during nuclear fusion.**
- **Fission** is when **large nuclei are split in two –** the **nucleon numbers** of the two **new nuclei** are **smaller** than the original nucleus, which means there is an **increase in the binding energy per nucleon. Energy is also released during fission.**
- **The change of binding energy per nucleon is about 0.5MeV in fission reaction and can be more than 10 times as much in fusion reactions.**

Fission and Fusion Processes.

Simple calculations from nuclear masses of energy released in fission and fusion reactions. (page190)

Induced fission

Induced fission by thermal neutrons; possibility of a chain reaction; critical mass.

The functions of the moderator, the control rods and the coolant in a thermal nuclear reactor; factors affecting the choice of materials for the moderator, the control rods and the coolant and examples of materials used; details of particular reactors are not required.

Safety aspects

Fuel used, shielding, emergency shut-down.

Production, handling and storage of radioactive waste materials.

Fission

- "splitting up into smaller parts"
- **Large nuclei**, with at least 83 neutrons (e.g. uranium), are **unstable** and some can **randomly split into two smaller nuclei – nuclear fission.**
- The process is spontaneous as it happens by itself, or induced if the reaction is encouraged.
- Energy is released during nuclear fission because the new, smaller nuclei have a higher binding energy per nucleon.
- The larger the nucleus, the more unstable it is – so large nuclei are more likely to spontaneously fission.
- This means that nuclear fission limits the number of nucleons a nucleus can have and so nuclear fission limits the number of possible elements.

Induced Fission: nuclear reactors

- **Nuclear reactors** use rods of **uranium** that are rich in $^{235}_{92}U$ as **'fuel'** for fission reactions.
- Fission can be induced by making a neutron enter a $^{235}_{92}U$ nucleus and making it very unstable.
- The fission reactions produce more neutrons which then induce other nuclei to fission – **chain reaction**.
- Only low energy neutrons can be captured by the uranium nuclei. Therefore the neutrons can only cause a chain reaction if they are slowed down – these slowed down neutrons are called **thermal neutrons**.
- **$^{235}_{92}U$ fuel rods** need to be in **a moderator (water)** to **slow down/absorb neutrons.** Choosing a **moderator that absorbs more neutrons the higher the temperature** will **decrease** the chance of **meltdown** if the reactor **overheats** – as it will naturally **slow down** the reaction.
- The **chain reaction** should be carried out at a **steady rate**, where one fission follows another. The amount of 'fuel' needed is called the **critical mass – any less than the critical mass (sub-critical mass)** and the reaction will die out. Nuclear reactors use a **supercritical mass** of fuel and **control the rate of fission using control rods.**
- **Control rods** control the rate of reaction by limiting the number of neutrons in the nuclear reactor. These **absorb neutrons** so that the rate of fission is controlled. Control rods are made of **boron**.
- In an **emergency** the reactor will be **shut down** by **release of the control rods** into the reactor, thus stopping the reaction as quickly as possible.
- A **coolant** is sent around the reactor to **remove heat** produce in the fission reactions – often the **same water** that is being used as the **moderator**.
- The **heat** from the reactor can then be used to make **steam** for powering **electricity-generating turbines**.
- The nuclear reactor is surrounded by a **thick concrete case**, which acts as **shielding**. This **prevents radiation escaping** and reaching the people working in the power station.

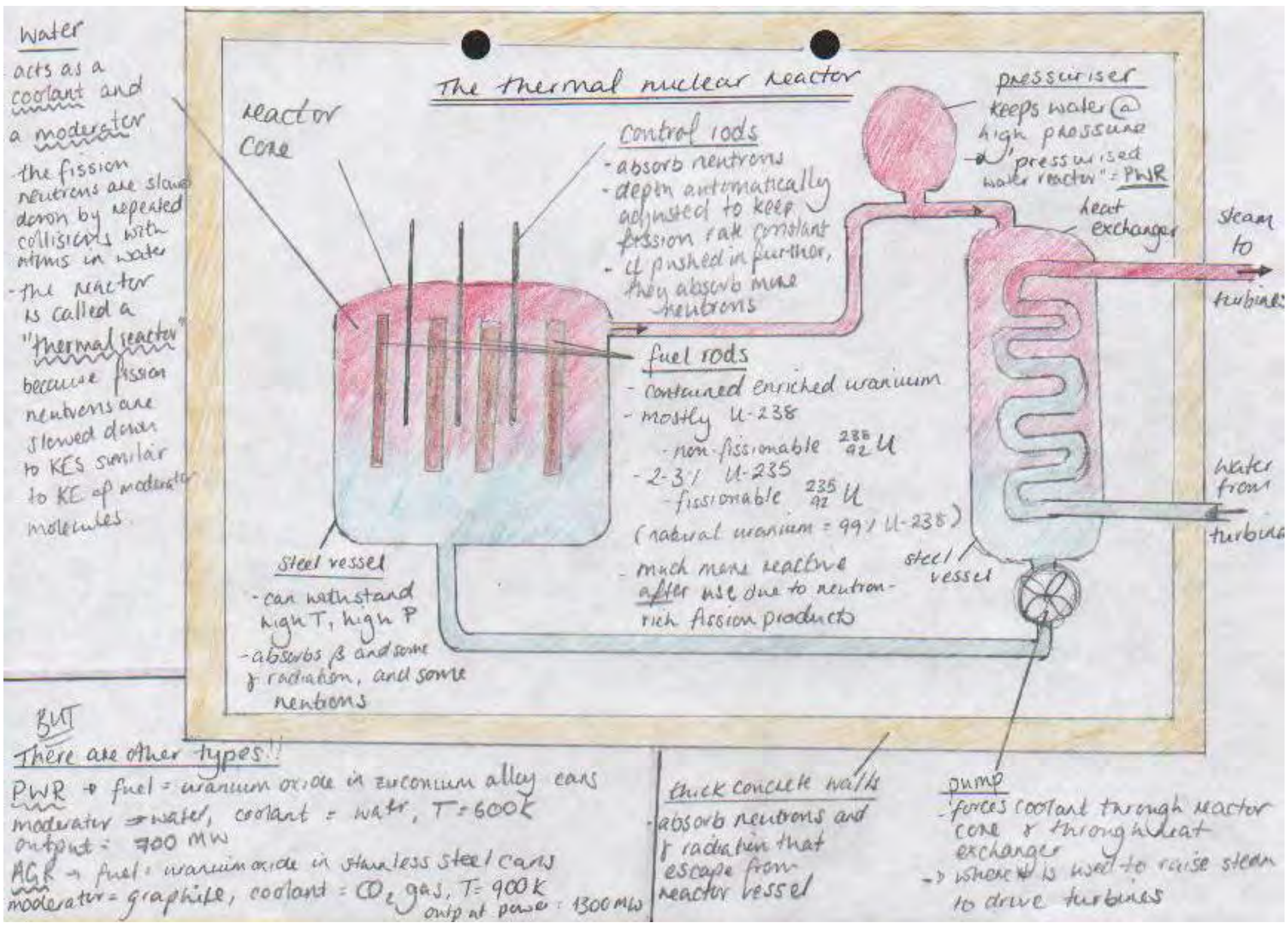
Waste products of nuclear fission need to be disposed of carefully:

- **Waste products** of **nuclear fission** usually have a **larger proportion of neutrons** than nuclei of a similar atomic number – this makes them **unstable** and **radioactive.**
- The products can be used in tracers for medical diagnosis.
- They may be highly **radioactive** and their **handling** and **disposal** needs great care.
- When material is removed from the reactor, it is **hot** and so is placed in a **cooling pond** to **decrease its temperature** to a safe level.
- Radioactive waste is then **stored underground** in **sealed containers** until its **activity has fallen** sufficiently

Fusion

- "joining nuclei together"

- **Two light nuclei** can **combine** together to form a larger nucleus – this is called **nuclear fusion**.
- A lot of **energy** is **released** because the **heavier nuclei have a higher binding energy per nucleon**.
- All **nuclei** are **positively charged** and so there is a great deal of **electrostatic repulsion** between them. Therefore, a lot of energy is required to overcome these electrostatic forces and get close enough for the attractive force of the **strong interaction** to hold them together.
- About **1 MeV** of KE is needed to fuse nuclei together.



3.5.3 Thermal Physics

Thermal Energy

Conditions involving change of energy

Energy between two objects takes place if:

- **One object exerts a force on the other and makes it move. (one object does work on the other object).**
- **One object is hotter than the other object so heat transfer takes place by means of conduction, convection or radiation. (energy is transferred due to temperature difference).**

The internal energy of an object is the sum of the random distribution of the kinetic and potential energies of its molecules.

For a change in temperature; $Q = mc\Delta T$, where c is the Specific Heat Capacity.

The Specific Heat Capacity, c, of a substance is the energy needed to raise the temperature of 1kg of that substance by 1K, without changing state.

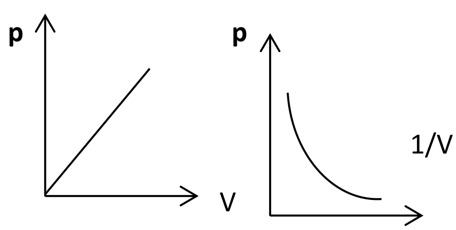
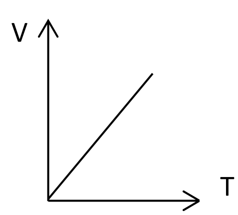
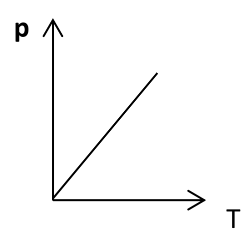
For a change of state; $Q = ml$, where l is the Specific Latent Heat.

The Specific Latent Heat of Fusion, l_f, of a substance is the energy needed to change the state of unit mass of the substance from solid to liquid without change of temperature.

The Specific Latent Heat of Vaporisation, l_v, of a substance is the energy needed to change the state of unit mass of the substance from liquid to gas without change of temperature.

Ideal Gases

Gas laws as experimental relationships between pressure (p), volume (V), temperature (T) and mass.

Law	Boyle's Law	Charles' Law	Gay-Lusac Law
Constant	Temperature (isothermal)	Pressure (adiabatic)	Volume
Variable	$pV = constant$ $p \propto \frac{1}{V}$ 	$\frac{V}{T} = constant$ $V \propto T$ 	$\frac{p}{T} = constant$ $p \propto T$ 

Temperature in degrees here!!!

- **Boyle's Law – at a constant temperature, the pressure and volume of the gas are inversely proportional.**
- **Charles' Law – at constant pressure, the volume of a gas is directly proportional to its absolute temperature.**
- **The Pressure Law/Gay-Lusac Law – at constant volume, the pressure of a gas is directly proportional to its absolute temperature.**

Concept of absolute zero of temperature.

- The **lowest possible temperature** is called **absolute zero** (**0K**, which equals **-273°C**).
- At **0K**, all **particles have the minimum possible kinetic energy** – the particles are **point masses**.

$$K = C + 273$$

Ideal gas equation $pV = nRT$ for n moles and as $pV = NkT$ for N molecules.
Avogadro constant N_A, molar gas constant R, Boltzmann constant k.

Ideal Gas Equation:

$$pV = nRT$$

p= Pressure (Pa, Nm^{-2})
V= Volume (cm^3, dm^3, m^3)
T= temperature (K, °C)
R= Molar gas constant = 8.31 $Jmol^{-1}K^{-1}$
n= Number of moles

Boltzmann's constant:

- $N_A = Avogadro's\ constant = 6.023 \times 10^{23}$
- $Number\ of\ particle\ in\ a\ mass\ of\ gas, N = nN_A$
- $Boltzmann's\ constant, k = \frac{R}{N_A} = \frac{8.31}{6.023 \times 10^{23}} = 1.38 \times 10^{-23} JK^{-1}$

- **Combine** $N = nN_A$ **and** $k = \frac{R}{N_A}$**:** $Nk = nR$

$$pV = NkT$$

Molar mass and molecular mass.

- The **molar mass** of a substance is the mass of **1 mole** of that substance.
- The **molecular mass** of a substance is the mass of **1 molecule** of that substance.

Molecular kinetic theory model

Explanation of relationships between p, V and T in terms of a simple molecular model.

Boyle's Law:

- The pressure of a gas at constant temperature is increased if the volume is reduced because the gas molecules travel less distance between impacts at the walls due to the reduced volume.
- Hence there are more impacts per second and so pressure is greater.

Pressure Law:

- The pressure of a gas at constant volume is increased by raising the temperature.
- The average speed of the molecules is increased by raising temperature so the impacts of the molecules on the container walls exert more force and are more frequent.
- Hence the pressure (F/A) increases.

Assumptions leading to and derivation of $pV = \frac{1}{3}Nm(c_{rms})^2$.

The **root mean squared** is the square root of the mean value of the square of the molecular speeds of the molecules of a gas: $c_{rms} = (\frac{(c_1{}^2 + c_2{}^2 + \cdots c_N{}^2)}{N})^{1/2}$.

<u>**Assumptions made:**</u>

- **Randomly moving particles – range of velocities and direction.**
- **Particles are "point masses" – they have no dimensions/their volume is negligible.**
- **Collisions with other particles and container are completely elastic (conservation of E_K and momentum).**
- **No intermolecular forces/ interaction between gas particles.**
- **Time of collision with container is less than time of flight between impacts.**

The **root mean squared** value is **equal** to the **root mean** value.

Average molecular kinetic energy: $\frac{1}{2}m(c_{rms})^2 = \frac{3}{2}kT = \frac{3RT}{2N_A}$.

$$\textit{Mean kinetic energy of a molecule of an ideal gas} = \frac{3}{2}kT$$

$$k = \frac{R}{N_A}$$

$$E_k = \frac{3RT}{2N_A}$$

- **Total kinetic energy of n moles of an ideal gas:** $N_A \times \frac{3}{2}kT = \frac{3}{2}RT$, $\frac{3}{2}RT \times n = \frac{3}{2}nRT$

$$\textit{Total kinetic energy of n moles of an ideal gas} = \frac{3}{2}nRT$$