S2 IAL June 2016 Model Answers
Kerime2

1. During a typical day, a school website receives visits randomly at a rate of 9 per hour.

The probability that the school website receives fewer than $v$ visits in a randomly selected one hour period is less than 0.75

(a) Find the largest possible value of $v$ **(1)**

(b) Find the probability that in a randomly selected one hour period, the school website receives at least 4 but at most 11 visits. **(2)**

(c) Find the probability that in a randomly selected 10 minute period, the school website receives more than 1 visit. **(3)**

(d) Using a suitable approximation, find the probability that in a randomly selected 8 hour period the school website receives more than 80 visits. **(5)**

1. Let $V$ = no. of visits in a one hr period

$V \sim Po(9)$

(a) $P(V \le 10) = 0.7060$
$P(V \le 11) = 0.8030$
$P(V < 11) = 0.7060$

$\therefore$ ~~$V_{max} = 10$~~ $V_{max} = 11$

(b) $P(4 \le V \le 11) = P(V \le 11) - P(V \le 3)$

$= 0.8030 - 0.0212 = 0.7818$

(c) Let $X$ = no. of visits received in 10 mins

$X \sim Po(1.5)$

**Question 1 continued**

$P(X > 1) = 1 - P(X \leq 1)$

$= 1 - 0.5578$

$= 0.4422$

(d) Let $Y$ = # of visits in 8hrs

$Y \sim Po(72)$

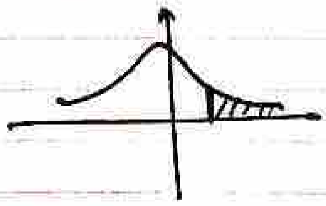
Let $Y' \approx$ # of visits in 8hrs

$Y' \sim N(72, 72)$

$P(Y > 8) \approx P(Y' \geq 80.5)$

$P(Y' \geq 80.5) = P(Z \geq 1.00)$

$= 1 - \Phi(1.00)$



$= 1 - 0.8413$

$= 0.1587$

$P(Y > 80) \approx 0.1587$

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2. The random variable $X \sim B(10, p)$

(a) (i) Write down an expression for $P(X = 3)$ in terms of $p$

(ii) Find the value of $p$ such that $P(X = 3)$ is 16 times the value of $P(X = 7)$

**(4)**

The random variable $Y \sim Po(\lambda)$

(b) Find the value of $\lambda$ such that $P(Y = 3)$ is 5 times the value of $P(Y = 5)$

**(3)**

The random variable $W \sim B(n, 0.4)$

(c) Find the value of $n$ and the value of $a$ such that $W$ can be approximated by the normal distribution, $N(32, a)$

**(3)**

2(a) (i) $P(X=3) = 120p^3(1-p)^7$

(ii) $P(X=3) = 16P(X=7)$

$\therefore \cancel{120}p^3(1-p)^7 = 16 \times \cancel{120}p^7(1-p)^3$

$\therefore p^3(1-p)^7 = 16p^7(1-p)^3$

$\div p^7(1-p)^3 \Rightarrow \dfrac{(1-p)^4}{p^4} = 16$

$\therefore (1-p)^4 = 16p^4$

$\therefore 1-p = 2p \Rightarrow p = \dfrac{1}{3}$

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**Question 2 continued**

(b) $P(Y=3) = 5P(Y=5)$

$\therefore \frac{\cancel{e^{-\lambda}} \lambda^3}{6} = 5 \times \frac{\cancel{e^{-\lambda}} \lambda^5}{120}$

$\frac{\lambda^3}{6} = \frac{\lambda^5}{24} \Rightarrow 24\lambda^3 = 6\lambda^5$

$\therefore \frac{1}{4}\lambda^2 = 1 \Rightarrow \lambda^2 = 4$

$\therefore \underline{\underline{\lambda = 2}}$

(c) $W \sim B(n, 0.4)$

$np = 0.4n = 32 \Rightarrow \underline{\underline{n = 80}}$

$np(1-p) = 0.4n(1-0.4) = 0.24n = 19.2$

$\therefore \underline{\underline{x = 19.2}}$

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3. A single observation $x$ is to be taken from $X \sim B(12, p)$

This observation is used to test $H_0: p = 0.45$ against $H_1: p > 0.45$

(a) Using a 5% level of significance, find the critical region for this test. **(2)**

(b) State the actual significance level of this test. **(1)**

The value of the observation is found to be 9

(c) State the conclusion that can be made based on this observation. **(1)**

(d) State whether or not this conclusion would change if the same test was carried out at the

(i) 10% level of significance,

(ii) 1% level of significance. **(2)**

3(a) $P(X \geq c.v) \leq 0.05$

$1 - P(X \leq c.v - 1) \leq 0.05$

$\therefore P(X \leq c.v - 1) \geq 0.95$

$\Rightarrow c.v - 1 = 8 \Rightarrow c.v = 9$

$\therefore$ CR is $\{x \geq 9\}$ $(12 \geq x \geq 9)$

~~$0 \leq x \leq$~~

(b) 0.0356 (3.56 %)

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**Question 3 continued**

(c) $H_1$ can be accepted.

$H_0$ can be rejected.

(d) (i) Conclusion in (c) remains the same.

(ii) Conclusion in (c) changes.

$H_0$ now accepted

$H_1$ now rejected.

4. The waiting times, in minutes, between flight take-offs at an airport are modelled by the continuous random variable $X$ with probability density function

$$f(x) = \begin{cases} \frac{1}{5} & 2 \leqslant x \leqslant 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Write down the name of this distribution.

**(1)**

A randomly selected flight takes off at 9 am

(b) Find the probability that the next flight takes off before 9.05 am

**(1)**

(c) Find the probability that at least 1 of the next 5 flights has a waiting time of more than 6 minutes.

**(3)**

(d) Find the cumulative distribution function of $X$, for all $x$

**(3)**

(e) Sketch the cumulative distribution function of $X$ for $2 \leqslant x \leqslant 7$

**(2)**

On foggy days, an extra 2 minutes is added to each waiting time.

(f) Find the mean and variance of the waiting times between flight take-offs on foggy days.

**(3)**

4(a) Uniform distribution.

(b) $P(X \leq 5) = P(2 \leq X \leq 5) = \frac{3}{5}$

(c) $P(X > 6) = \frac{1}{5}$

Let $Y$ = no. of flights with waiting time greater than 6 mins

$Y \sim B(5, \frac{1}{5})$ $P(Y \geq 1) = 1 - P(Y = 0)$

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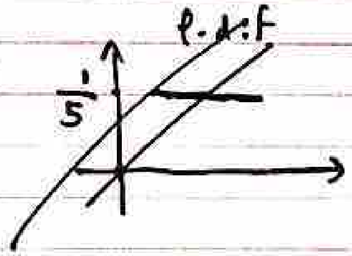
**Question 4 continued**

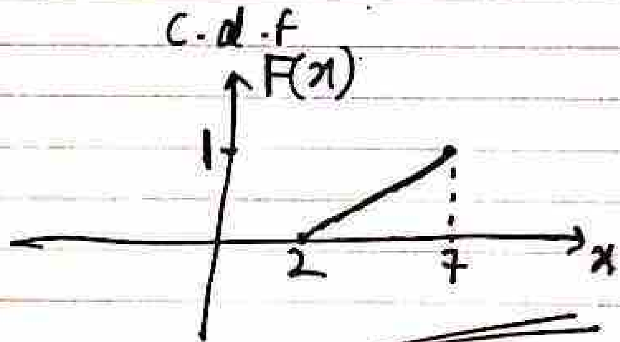
$$P(Y \geq 1) = 1 - \left(\frac{4}{5}\right)^5 = 0.672 \text{ (3sf)}$$

(d)

$$F(x) = \begin{cases} 0 & , \quad x < 2 \\ \frac{x-2}{5} & , \quad 2 \leq x \leq 7 \\ 1 & , \quad x > 7 \end{cases}$$

(e)




(f) New waiting time = $X+2$

$\cancel{E(X)}$ $E(X+2) = E(X) + 2 = 4.5 + 2$

$= 6.5$

$Var(X+2) = Var(X) = \frac{1}{12}(5)^2 = \frac{25}{12}$

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5. A bag contains a large number of coins. It contains only 1p, 5p and 10p coins. The fraction of 1p coins in the bag is $q$, the fraction of 5p coins in the bag is $r$ and the fraction of 10p coins in the bag is $s$.

Two coins are selected at random from the bag and the coin with the highest value is recorded. Let $M$ represent the value of the highest coin.

The sampling distribution of $M$ is given below

| $m$ | 1 | 5 | 10 |
|---|---|---|---|
| $P(M = m)$ | $\frac{1}{25}$ | $\frac{13}{80}$ | $\frac{319}{400}$ |

(a) List all the possible samples of two coins which may be selected. **(2)**

(b) Find the value of $q$, the value of $r$ and the value of $s$ **(7)**

5 (a) (1, 1) (1, 5) (5, 1)

(5, 5) (1, 10) (10, 1)

(10, 10) (5, 10) (10, 5)

(b)

| Coin | 1 | 5 | 10 |
|---|---|---|---|
| Probability | $q$ | $r$ | $s$ |

$q + r + s = 1$

(1, 1)

$P(M = 1) = q^2 = \frac{1}{25} \Rightarrow q = \frac{1}{5}$

(5, 5)
(1, 5)
(5, 1)

$P(M = 5) = r^2 + 2\left(\frac{1}{5}\right)(r) = \frac{13}{80}$



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**Question 5 continued**

$\therefore \quad r^2 + \frac{2}{5}r - \frac{13}{80} = 0$

$\therefore \quad 80r^2 + 32r - 13 = 0$

$(4r-1)(20r+13) = 0$

$\Rightarrow r = \frac{1}{4} \qquad r \neq -\frac{13}{20}$

$s = 1 - \frac{1}{4} - \frac{1}{5} = \frac{11}{20}$

$\therefore \quad q = \frac{1}{5} \qquad r = \frac{1}{4} \qquad s = \frac{11}{20}$

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6. A continuous random variable $X$ has probability density function

$$f(x) = \begin{cases} ax - bx^2 & 0 \leqslant x \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

Given that the mode is 1

(a) show that $a = 2b$ **(2)**

(b) Find the value of $a$ and the value of $b$ **(5)**

(c) Calculate F(1.5) **(2)**

(d) State whether the upper quartile of $X$ is greater than 1.5, equal to 1.5, or less than 1.5
Give a reason for your answer. **(2)**

6(a) $f'(x) = a - 2bx$

$f'(x) = 0 \Rightarrow a - 2bx = 0$

Mode is 1 $\Rightarrow f'(1) = 0$

$\therefore a - 2b = 0$

$\Rightarrow a = 2b$ as required.

(b) $F(2) = 1$

$\Rightarrow \int_0^2 2bx - bx^2 \, dx$

$= \left[ bx^2 - \frac{b}{3}x^3 \right]_0^2$

$= \frac{4}{3}b = 1$

$\Rightarrow b = 3/4$

$\Rightarrow a = 3/2$

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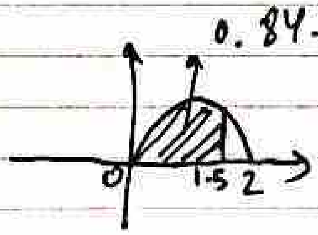
**Question 6 continued**

(c) $F(1.5) = \int_0^{1.5} \frac{3}{2}x - \frac{3}{4}x^2 \, dx$

$= \left[ \frac{3}{4}x^2 - \frac{1}{4} x^3 \right]_0^{1.5}$

$= \frac{27}{32}$

(d) 



Upper quartile is less than 1.5

because $P(X \leq 1.5) > 0.75$

and $P(X \leq 1) = 0.5$ (mode)

$\therefore \quad 1 < U.Q < 1.5$

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7. Last year 4% of cars tested in a large chain of garages failed an emissions test.

A random sample of $n$ of these cars is taken. The number of cars that fail the test is represented by $X$

Given that the standard deviation of $X$ is 1.44

(a) (i) find the value of $n$

(ii) find $E(X)$

**(4)**

A random sample of 20 of the cars tested is taken.

(b) Find the probability that all of these cars passed the emissions test.

**(1)**

Given that at least 1 of these cars failed the emissions test,

(c) find the probability that exactly 3 of these cars failed the emissions test.

**(4)**

A car mechanic claims that more than 4% of the cars tested at the garage chain this year are failing the emissions test. A random sample of 125 of these cars is taken and 10 of these cars fail the emissions test.

(d) Using a suitable approximation, test whether or not there is evidence to support the mechanic's claim. Use a 5% level of significance and state your hypotheses clearly.

**(6)**

7(a)(i) $X \sim B(n, 0.04)$

$np(1-p) = \frac{24}{625} n$

$\therefore \sqrt{\frac{24}{625} n} = 1.44$

$n = 1.44^2 \times \frac{625}{24}$

$\therefore n = 54$

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**Question 7 continued**

(ii) $E(X) = np = 54 \times 0.04$

$= 2.16$

(b) None fail, all pass

$\therefore \quad 0.96^{20} = 0.442$

$P(\text{all pass}) = 0.442$

(c) $X \sim B(20, 0.04)$

$P(X \geq 1) = 1 - P(X=0) = 1 - 0.442\ldots$

$= 0.5579\ldots$

$P(X=3 \mid X \geq 1) = \dfrac{P(X=3)}{P(X \geqslant 1)}$

$= \dfrac{0.03644\ldots}{0.5579\ldots} = 0.0653$ (3sf)

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(d) $X \sim B(125, 0.04)$

Let $Y \sim$ # of cars that fail test

~~$Y \sim Po$~~ $Y \sim Po(5)$

~~$P(X$~~ $H_0: \lambda = 5$

$H_1: \lambda > 5$

$P(X \geq 10) \approx P(Y \geq 10)$

$P(Y \geq 10) = 1 - P(Y \leq 9)$

$= 1 - 0.9682$

$= 0.0318$

$0.0318 < 0.05$

$\therefore$ 10 is in the critical region.

Accept $H_1$ reject $H_0$

There is sufficient evidence to support the mechanic's claim.

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