M3 18th May 11C.

Q1

a) $F = mv\frac{dv}{dx}$ $\quad v = \frac{12}{x+3}$ $\quad \frac{dv}{dx} = -\frac{12}{(x+3)^2}$

$= \frac{1}{2}\left(\frac{12}{x+3}\right)\left(-\frac{12}{(x+3)^2}\right) = \frac{-72}{(x+3)^3}$. $\quad x = 3$

$F = \frac{-72}{(6)^3} = -\frac{1}{3}N$ c.o.

Magnitude $\frac{1}{3}N$ or $0.332N$ 3.s.f.

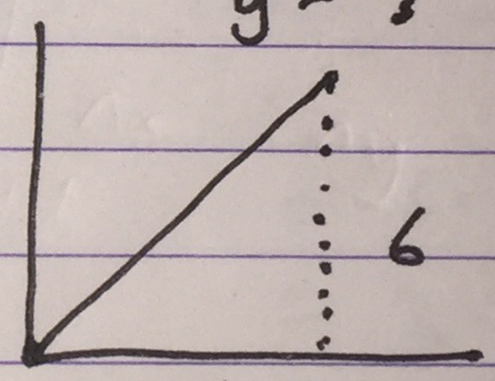
b) $\frac{dx}{dt} = \frac{12}{x+3} \Rightarrow \frac{(x+3)^2}{2} = 12t + c$

$\frac{7^2}{2} - 24 = c = \frac{1}{2}$

So $\frac{(x+3)^2}{2} = 12t + \frac{1}{2}$

$\frac{(13)^2}{2} - \frac{1}{2} = 12t = 84 \Rightarrow t = 7s$.

Q2 Put C at O.



So $M = \frac{1}{2}(6)(9)\rho = 27\rho$

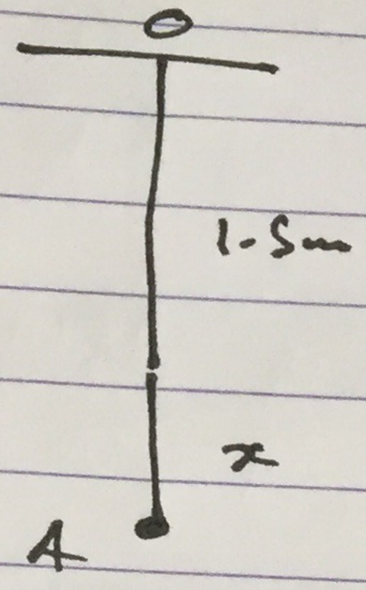
$27\rho\bar{x} = \rho\int_6^9 \frac{2}{3}x^2\,dx = \rho\left[\frac{2x^3}{9}\right]_0^9 = 162\rho$

$\Rightarrow \bar{x} = \frac{162\rho}{27\rho} = 6cm$ from C

So $9 - 6 = 3cm$ from AB.

Q3

a)



EPE at A $= \frac{14.7x^2}{3} = 4.9x^2$

GPE at O $= 0.6g(1.5+x)$

So $\frac{4.9}{2}x^2 = 0.6g(1.5+x)$

$x^2 = 1.2(1.5+x)$

$x^2 = 1.2x - 1.8 = 0$

$x = \frac{1.2 \pm \sqrt{1.2^2 + 4(1.8)}}{2} = \frac{3 \pm 3\sqrt{6}}{5}$ $x \neq \frac{3-3\sqrt{6}}{5}$

So $x = \frac{3+3\sqrt{6}}{5} \Rightarrow OA = \frac{3+3\sqrt{6}}{5} + 1.5$

$\simeq 3.5196938\ldots$

$= \underline{3.57\text{m}}$.

b) $|T - mg| = m|a|$ Modulus signs not required

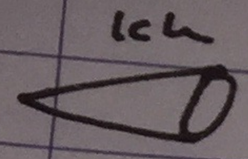
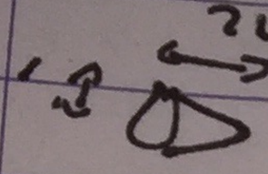
$\frac{14.7}{1.5}\left(\frac{3+3\sqrt{6}}{5}\right) - 0.6g = 0.6a$

$0.6a = 14.402999769$.

$a = 24.004999\ldots\text{ms}^{-2}$

$a = \underline{24.0\text{ms}^{-2}\ (3\text{s.f})}$

Q4

Object	Mass	Dist. of C.O.M.	Moment
c) 	$\frac{1}{3}\pi(r)^2 1ch\,\rho$	$\frac{1ch}{4}$	$\frac{\pi r^2 h^2}{12}\rho(k^2)$
	$\frac{1}{3}\pi(r^2)(2h)\rho$	$-\frac{h}{2}$	$\frac{-\pi r^2 h^2}{3}\rho$
$\frac{\pi r^2 \rho h}{3}(k+2)$	$\bar{x}$	$\pi^2 h^3 \rho\left(\frac{k^2}{12}-\frac{1}{3}\right)$	

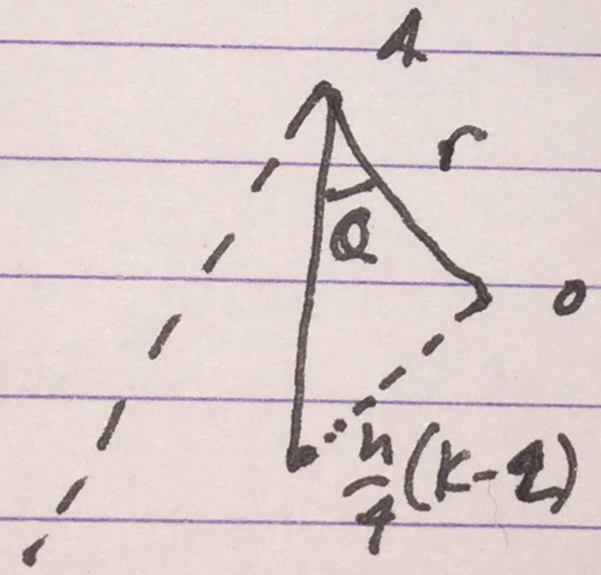
$$\text{So } \frac{\pi r^2 \rho h}{3}(k+2)\,\bar{x} = \pi r^2 h^2 \rho\left(\frac{k^2-4}{12}\right)$$

$$\frac{h}{3}(k+2)\,\bar{x} = h^2\left(\frac{(k-2)(k+2)}{12}\right)$$

$$\frac{h}{3}\bar{x} = h^2\left(\frac{k-2}{12}\right)$$

$$\bar{x} = \frac{3}{12}h(k-2) = \underline{\underline{\tfrac{h}{4}(k-2)}}.$$

b)



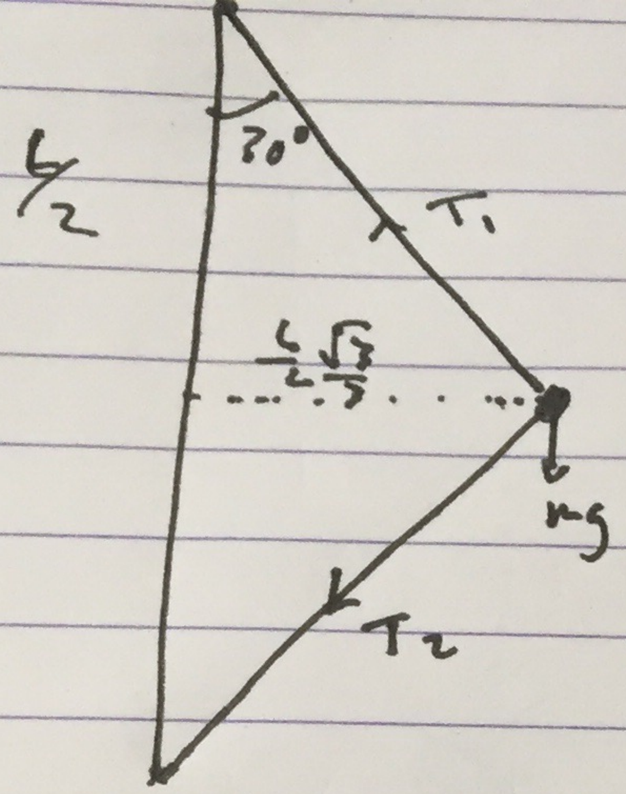
$$\frac{h}{4}(6-2) = h = \frac{1}{3}r$$

$$\tan\theta = \frac{\frac{h}{4}(k-2)}{r} = \frac{\frac{1}{3}r}{r} = \frac{1}{3}$$

$$\theta = 18.4^\circ \quad (3\text{s.f.})$$

$$\underline{\underline{\qquad\qquad}}.$$

Q5



$r = \frac{L}{2} \cdot \frac{\sqrt{3}}{3} = \frac{L\sqrt{3}}{6}$

$R(\uparrow)$ $\quad T_1 \cos 30 - T_2 \cos 30 = mg$

$T_1 - T_2 = \frac{2}{\sqrt{3}} mg \quad (1)$

$R(\leftarrow)$ $\quad T_1 \sin 30 + T_2 \sin 30 = m\left(\frac{L\sqrt{3}}{6}\right)\omega^2$

$T_1 + T_2 = \frac{mL\sqrt{3}}{3}\omega^2 \quad (2)$

i)

Adding (1)+(2) $\quad 2T_1 = \frac{2}{\sqrt{3}} mg + \frac{mL\sqrt{3}}{3}\omega^2$

$T_1 = \frac{mg}{\sqrt{3}} + \frac{mL\sqrt{3}}{6}\omega^2$

$T_1 = \frac{m\sqrt{3}}{6}\left(L\omega^2 + 2g\right)$

ii) Subtracting (2)−(1) $\Rightarrow$ $2T_1 = \frac{mL\sqrt{3}\omega^2}{6} - \frac{mg}{\sqrt{3}}$

$= \frac{m\sqrt{3}}{6}\left(L\omega^2 - 2g\right)$

i) $T_1 > 0 \qquad L\omega^2 - 2g > 0 \qquad \omega^2 > \frac{2g}{L}$

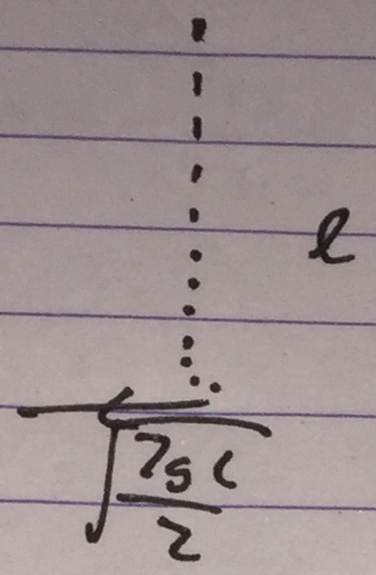
$\omega > \sqrt{\frac{2g}{L}} \qquad \frac{2\pi}{T} > \sqrt{\frac{2g}{L}} \qquad 2\pi > T\sqrt{\frac{2g}{L}}$

$T < \frac{2\pi}{\sqrt{\frac{2g}{L}}} = 2\pi\sqrt{\frac{L}{2g}}$ 2 into the sqrt

$= \pi\sqrt{\frac{4L}{2g}} = \pi\sqrt{\frac{2L}{g}}$

(Q8) Mass: $\underline{\underline{2m}}$

a)

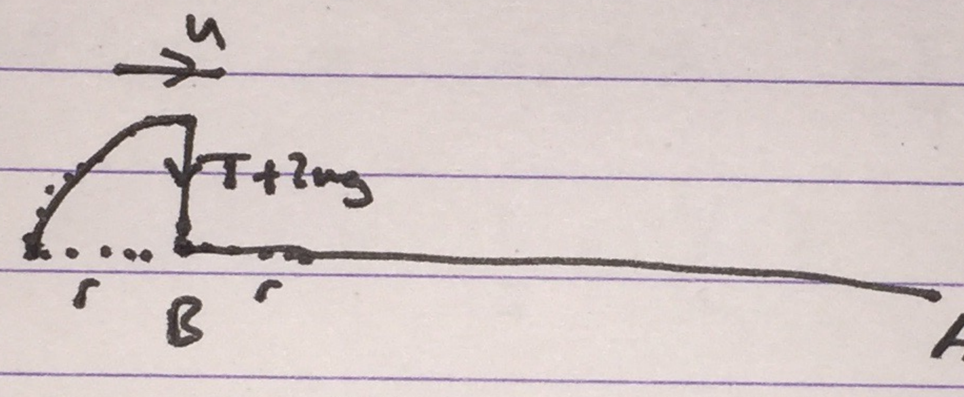


$$2mgl + \tfrac{1}{2}(2m)v^2 = \tfrac{1}{2}(2m)\left(\tfrac{3gl}{2}\right)$$

$$gl + \frac{v^2}{2} = \frac{3gl}{4} \quad \Rightarrow \quad \frac{v^2}{2} = \frac{3gl}{4} \quad v^2 = \frac{3gl}{2}$$

$$v = \sqrt{\frac{3gl}{2}}$$

b)



$AB = l - r$

At top $T \geq 0$

$$T + 2mg = \frac{2m(u^2)}{r}$$

By conservation of energy $\tfrac{1}{2}(2m)u^2 + 2mgr = \tfrac{1}{2}(2m)\left(\tfrac{3gl}{2}\right)$

$$\frac{u^2}{2} = \frac{3gl}{4} - gr \qquad u^2 = \frac{3gl}{2} - 2gr$$

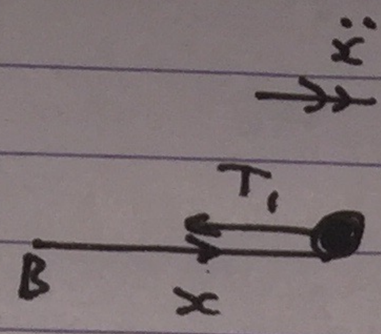
So $T + 2mg = \dfrac{2m\left(\frac{3gl}{2} - 2gr\right)}{r} = \dfrac{3mgl - 4mgr}{r} = \dfrac{3mgl}{r} - 4mg$

$$T = \frac{3mgl}{r} - 6mg \geq 0 \quad \Rightarrow \quad \frac{3l}{r} \geq 6 \quad r \leq l/2$$

So $-r \geq -l/2$ Adding l

$$-l/2 + l = l/2 \leq -r + l = AB$$

Q7



a) i) $R(\rightarrow)$ $\quad m\ddot{x} = -\frac{15x}{1.2} = -\frac{25}{2}x$

$\ddot{x} = -25x$ $\quad \therefore$ SHM $\quad \omega^2 = 25$

$\omega = 5$

ii) $T = \frac{2\pi}{\omega} = \frac{2\pi}{5}$

b) $v = a\omega = 5(0.8) = 4\,ms^{-1}$.

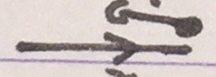
c) $DB = 0.6m$

So $\quad x = 0.8\cos(5t)$

D behind B $\quad x = -0.6 \quad -\frac{3}{4} = \cos(5t)$

$t = \frac{1}{5}\cos^{-1}\left(-\frac{3}{4}\right) = 0.484s$ (3sf)

d) $m = 0.8kg$ $\quad T_1$ is unchanged.

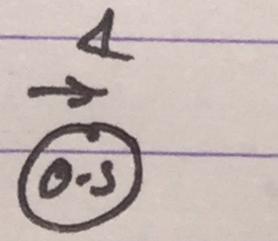


$0.8\ddot{x} = \frac{-15}{1.2}x = -\frac{25}{2}x$

$\ddot{x} = -\frac{125}{8}x$ $\quad \therefore$ SHM. $\quad \omega^2 = \frac{125}{8}$

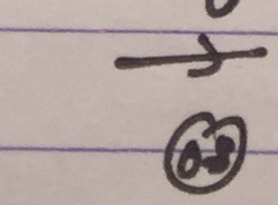
$\omega = \frac{\sqrt{\frac{125}{8}}}{2}$

e) COLM:



$0.8v = 0.5 \times 4$

$v = 2.5\,ms^{-1}$



So $\quad 2.5 = \sqrt{\frac{125}{8}}\,a$

$a = \frac{\sqrt{16}}{5}m = 0.632m$ (3sf)