Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number

Candidate Number

Further Pure Mathematics F2

Advanced/Advanced Subsidiary

Wednesday 8 June 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. (a) Express $\frac{1}{4r^2 - 1}$ in partial fractions. **(1)**

(b) Hence prove that

$$\sum_{r=1}^{n} \frac{1}{4r^2 - 1} = \frac{n}{2n+1}$$

(3)

(c) Find the exact value of

$$\sum_{r=9}^{25} \frac{5}{4r^2 - 1}$$

(2)

(a) $\frac{1}{4r^2-1} = \frac{1}{(2r)^2 - 1^2} = \frac{1}{(2r-1)(2r+1)}$

$= \frac{1}{2(2r-1)} - \frac{1}{2(2r+1)}$

(b) $\sum_{r=1}^{n} \frac{1}{4r^2-1} = \frac{1}{2} - \frac{1}{6}$

$+ \frac{1}{6} - \frac{1}{10}$

$+$

$\vdots$

$+ \frac{1}{2(2(n-1)-1)} - \frac{1}{2(2(n-1)+1)}$

$+ \frac{1}{2(2n-1)} - \frac{1}{2(2n+1)}$

$= \frac{1}{2} - \frac{1}{2(2n+1)}$

$= \frac{2n+1-1}{2(2n+1)}$

$= \frac{2n}{\cancel{2}(2n+1)}$

$= \frac{n}{2n+1}$ QED

(c) $\sum_{r=9}^{25} \frac{5}{4r^2-1} = 5\left\{\sum_{r=1}^{25} \frac{1}{4r^2-1} - \sum_{r=1}^{8} \frac{1}{4r^2-1}\right\} = 5\left[\frac{25}{2(25)+1} - \frac{8}{2(8)+1}\right]$

$= \frac{5}{51}$

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2. Use algebra to find the set of values of x for which

$$|x^2 - 9| < |1 - 2x|$$

(6)

$|x^2 - 9| = |1 - 2x|$

$x^2 - 9 = 1 - 2x$

$x^2 + 2x - 10 = 0$

$x^2 + 2(1)x + 1^2 - 1^2 - 10 = 0$

$(x+1)^2 = 11$

$x + 1 = \pm\sqrt{11}$

$x = -1 \pm \sqrt{11}$

$-(x^2 - 9) = 1 - 2x$

$x^2 - 9 = 2x - 1$

$x^2 - 2x - 8 = 0$

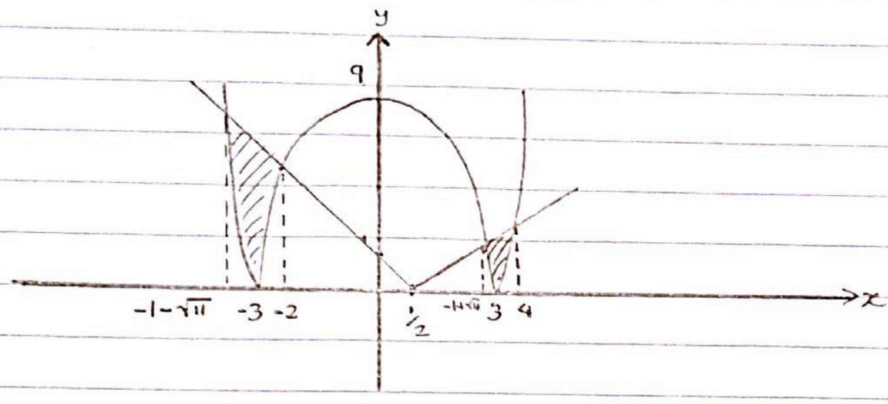
$x^2 - 4x + 2x - 8 = 0$

$x(x-4) + 2(x-4) = 0$

$(x+2)(x-4) = 0$

$x = -2$ or $x = 4$

$\therefore$ Critical values: $x = -1 \pm \sqrt{11}, -2, 4$



$\therefore -1-\sqrt{11} < x < -2$ OR $-1+\sqrt{11} < x < 4$

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3. Find, in terms of k, where k is a positive integer, the general solution of the differential equation

$$(1+x)\frac{dy}{dx} + ky = x^{\frac{1}{2}}(1+x)^{2-k}, \quad x > 0$$

giving your answer in the form $y = f(x)$.

(6)

$$\frac{dy}{dx} + \frac{ky}{1+x} = x^{\frac{1}{2}}(1+x)^{1-k}$$

Integrating factor: $\exp\left(\int \frac{k}{1+x}dx\right)$

$$= \exp\left(k\int\frac{dx}{1+x}\right)$$

$$= e^{k\ln(1+x)}$$

$$= e^{\ln(1+x)^k}$$

$$= (1+x)^k$$

$$y(1+x)^k = \int (1+x)^k x^{1/2}(1+x)^{1-k}\,dx$$

$$= \int x^{1/2}(1+x)\,dx$$

$$= \int x^{1/2} + x^{3/2}\,dx$$

$$= \frac{2}{3}x^{3/2} + \frac{2}{5}x^{5/2} + A$$

$$= \frac{10x^{3/2} + 6x^{5/2} + B}{15}$$

$$= \frac{2(3x^{5/2} + 5x^{3/2} + C)}{15}$$

$$\therefore y = \frac{2(3x^{5/2} + 5x^{3/2} + C)}{15(1+x)^k}$$

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4.

$$f(x) = \sin\left(\frac{3}{2}x\right)$$

(a) Find the Taylor series expansion for f(x) about $\frac{\pi}{3}$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$ up to and including the term in $\left(x - \frac{\pi}{3}\right)^4$ **(6)**

(b) Hence obtain an estimate of $\sin\frac{1}{2}$, giving your answer to 4 decimal places. **(2)**

(a) $f(x) = \sin\left(\frac{3}{2}x\right) \Rightarrow f\left(\frac{\pi}{3}\right) = 1$

$f'(x) = \frac{3}{2}\cos\left(\frac{3}{2}x\right) \Rightarrow f'\left(\frac{\pi}{3}\right) = 0$

$f''(x) = -\frac{9}{4}\sin\left(\frac{3}{2}x\right) \Rightarrow f''\left(\frac{\pi}{3}\right) = -\frac{9}{4}$

$f'''(x) = -\frac{27}{8}\cos\left(\frac{3}{2}x\right) \Rightarrow f'''\left(\frac{\pi}{3}\right) = 0$

$f^{IV}(x) = \frac{81}{16}\sin\left(\frac{3}{2}x\right) \Rightarrow f^{IV}\left(\frac{\pi}{3}\right) = \frac{81}{16}$

$f(x) = f(\pi/3) + \left(x - \frac{\pi}{3}\right)f'\left(\frac{\pi}{3}\right) + \frac{1}{2!}\left(x - \frac{\pi}{3}\right)^2 f''(\pi/3) + \cdots$

$\therefore \sin\left(\frac{3}{2}x\right) = 1 - \frac{9}{8}\left(x - \frac{\pi}{3}\right)^2 + \frac{27}{128}\left(x - \frac{\pi}{3}\right)^4 + \cdots$

(b) $\frac{3}{2}x = \frac{1}{2} \Rightarrow x = \frac{1}{3}$

$\Rightarrow \sin\frac{1}{2} \approx 1 - \frac{9}{8}\left(\frac{1}{3} - \frac{\pi}{3}\right)^2 + \frac{27}{128}\left(\frac{1}{3} - \frac{\pi}{3}\right)^4$

$\approx 0.481476\ldots$

$\therefore \sin\left(\frac{1}{2}\right) \approx 0.4815$ (4 DP)

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5. The transformation T from the z-plane to the w-plane is given by

$$w = \frac{2z - 1}{z + 3}, \quad z \neq -3$$

The circle in the z-plane with equation $x^2 + y^2 = 1$, where $z = x + iy$, is mapped by T onto the circle C in the w-plane.

Find the centre and the radius of C.

(7)

$x^2+y^2=1 \Rightarrow |z|=1$

$wz+3w=2z-1$

$1+3w=2z-wz$

$=z(2-w)$

$z=\dfrac{3w+1}{2-w}$

$|z|=\left|\dfrac{3w+1}{2-w}\right|$

$1=\dfrac{|3w+1|}{|w-2|}$

$|w-2|=|3w+1|$

$|u+iv-2|=|3(u+iv)+1|$

$|(u-2)+iv|=|(3u+1)+3iv|$

$(u-2)^2+v^2=(3u+1)^2+9v^2$

$u^2-4u+4+v^2=9u^2+6u+1+9v^2$

$8u^2+10u+8v^2=3$

$u^2+\frac{5u}{4}+v^2=\frac{3}{8}$

$u^2+2(\frac{5}{8})u+(\frac{5}{8})^2-(\frac{5}{8})^2+v^2=\frac{3}{8}$

$(u+\frac{5}{8})^2+v^2=\frac{49}{64} \Rightarrow$ centre: $(-\frac{5}{8},0)$

radius $=\frac{7}{8}$

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6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3x^2 + 2x + 1$$

(9)

(b) Find the particular solution of this differential equation for which $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$

(5)

(a) Auxiliary equation:

$\alpha^2 + 3\alpha + 2 = 0$

$\alpha^2 + 2\alpha + \alpha + 2 = 0$

$\alpha(\alpha+2) + 1(\alpha+2) = 0$

$(\alpha+2)(\alpha+1) = 0$

$\Rightarrow \alpha = -2$ or $\alpha = -1$

Complementary function:

$y = Ae^{-2x} + Be^{-x}$

Particular integral:

$y = \lambda x^2 + \mu x + \omega$

$y' = 2\lambda x + \mu$

$y'' = 2\lambda$

$2\lambda + 3(2\lambda x + \mu) + 2(\lambda x^2 + \mu x + \omega) = 3x^2 + 2x + 1$

$2\lambda x^2 + (2\mu + 6\lambda)x + (2\lambda + 3\mu + 2\omega) = 3x^2 + 2x + 1$

comparing coefficients:

x^2: $2\lambda = 3 \Rightarrow \lambda = 3/2$

x: $2\mu + 6\lambda = 2$

$\mu + 3\lambda = 1$

$\mu = 1 - 3\lambda \Rightarrow \mu = -\frac{7}{2}$

constants: $2\lambda + 3\mu + 2\omega = 1$

$2(3/2) + 3(-7/2) + 2\omega = 1$

$\Rightarrow \omega = \frac{17}{4}$

$\therefore\ y = Ae^{-2x} + Be^{-x} + \frac{3}{2}x^2 - \frac{7}{2}x + \frac{17}{4}$

(b) $0 = A + B + 0 - 0 + \frac{17}{4}$

$\Rightarrow A + B = -\frac{17}{4}$ (1)

$$\frac{dy}{dx} = -2Ae^{-2x} - Be^{-x} + 3x - \frac{7}{2}$$

$$0 = -2A \quad - B \quad + 0 \quad - \frac{7}{2}$$

$$\Rightarrow \quad -2A - B = \frac{7}{2} \quad (ii)$$

$$(1) \Rightarrow \quad A \quad + \quad B = -\frac{17}{4}$$

$$(ii) \Rightarrow \quad -2A \quad + \quad B = \frac{7}{2}$$

$$(+) \quad -A \qquad = -3/4$$

$$\Rightarrow A = 3/4$$

Sub. A in (1)

$$3/4 + B = -17/4$$

$$B = -17/4 - 3/4$$

$$B = -5$$

$$\Rightarrow \quad y = \frac{3}{4}e^{-2x} - 5e^{-x} + \frac{3}{2}x^2 - \frac{7}{2}x + \frac{17}{4}$$

$$\therefore y = \frac{1}{4}\left(3e^{-2x} - 20e^{-x} + 6x^2 - 14x + 17\right)$$

7.

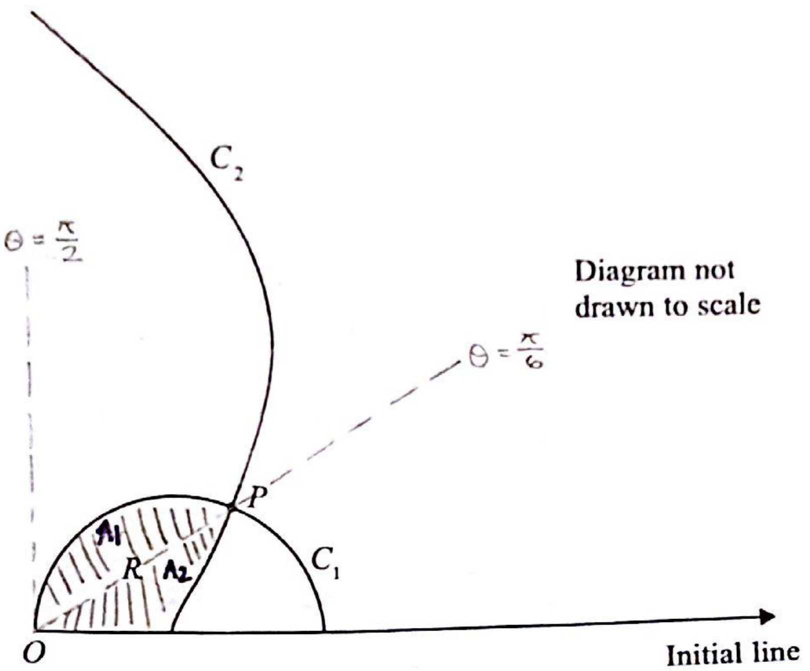

Figure 1

Figure 1 shows a sketch of the curves C_1 and C_2 with polar equations

$$C_1 : r = \frac{3}{2}\cos\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

$$C_2 : r = 3\sqrt{3} - \frac{9}{2}\cos\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

The curves intersect at the point P.

(a) Find the polar coordinates of P. **(3)**

The region R, shown shaded in Figure 1, is enclosed by the curves C_1 and C_2 and the initial line.

(b) Find the exact area of R, giving your answer in the form $p\pi + q\sqrt{3}$ where p and q are rational numbers to be found. **(8)**

(a) $\frac{3}{2}\cos\theta = 3\sqrt{3} - \frac{9}{2}\cos\theta$

$6\cos\theta = 3\sqrt{3}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \arccos\left(\sqrt{3}/2\right)$

$\therefore \theta = \frac{\pi}{6}$

$r = \frac{3\sqrt{3}}{4}$ $\qquad \therefore P\left(\frac{3\sqrt{3}}{4}, \frac{\pi}{6}\right)$

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$$A_1 = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{9}{2}\cos\theta\right)^2 d\theta$$

$$= \frac{9}{8}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta \, d\theta$$

$$= \frac{9}{8}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos 2\theta + 1}{2} \, d\theta$$

$$= \frac{9}{16}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\cos 2\theta + 1) \, d\theta$$

$$= \frac{9}{16}\left[\frac{1}{2}\sin 2\theta + \theta\right]_{\pi/6}^{\pi/2}$$

$$= \frac{9}{32}\left[\sin 2\theta + 2\theta\right]_{\pi/6}^{\pi/2}$$

$$= \frac{9}{32}\left[0 + \pi - \frac{\sqrt{3}}{2} - \frac{\pi}{3}\right]$$

$$= \frac{9}{32}\left(\frac{2}{3}\pi - \frac{\sqrt{3}}{2}\right)$$

$$A_1 = \frac{3\pi}{16} - \frac{9\sqrt{3}}{64}$$

$$A_2 = \frac{1}{2}\int_0^{\pi/6} \left(3\sqrt{3} - \frac{9}{2}\cos\theta\right)^2 d\theta$$

$$= \frac{1}{2}\int_0^{\pi/6} \left(27 - 27\sqrt{3}\cos\theta + \frac{81}{4}\cos^2\theta\right) d\theta$$

$$= \frac{1}{2}\int_0^{\pi/6} \left(27 - 27\sqrt{3}\cos\theta + \frac{81}{8}\cos 2\theta + \frac{81}{8}\right) d\theta$$

$$= \frac{1}{2}\int_0^{\pi/6} \frac{297}{8} - 27\sqrt{3}\cos\theta + \frac{81}{8}\cos 2\theta \Big) \, d\theta$$

$$= \frac{1}{2}\left[\frac{297\theta}{8} - 27\sqrt{3}\sin\theta + \frac{81}{16}\sin 2\theta\right]_0^{\pi/6}$$

$$= \frac{1}{2}\left[\frac{99\pi}{16} - \frac{27\sqrt{3}}{2} + \frac{81\sqrt{3}}{32} - 0\right]$$

$$= \frac{1}{2}\left(\frac{99\pi}{16} - \frac{351}{32}\sqrt{3}\right)$$

$$A_2 = \frac{99\pi}{32} - \frac{351}{64}\sqrt{3}$$

Shaded area $= A_1 + A_2$

$$= \frac{3\pi}{16} - \frac{9\sqrt{3}}{64} + \frac{99\pi}{32} - \frac{351}{64}\sqrt{3}$$

$\therefore$ Shaded area $= \left(\frac{105}{32}\pi - \frac{45}{8}\sqrt{3}\right)$ units^2

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8. (a) Use de Moivre's theorem to show that

$$\cos^5\theta \equiv p\cos 5\theta + q\cos 3\theta + r\cos\theta$$

where p, q and r are rational numbers to be found.

(6)

(b) Hence, showing all your working, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^5\theta \, d\theta$$

(4)

(a) $(2\cos\theta)^5 = (z+z^{-1})^5$

$\Rightarrow 32\cos^5\theta = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$

$= (z^5 + z^{-5}) + 5(z^3 + z^{-3}) + 10(z + z^{-1})$

$2(16\cos^5\theta) = 2\cos 5\theta + 5(2\cos 3\theta) + 10(2\cos\theta)$

$16\cos^5\theta = \cos 5\theta + 5\cos 3\theta + 10\cos\theta$

$\therefore \cos^5\theta = \frac{1}{16}\cos 5\theta + \frac{5}{16}\cos 3\theta + \frac{5}{8}\cos\theta$

(b) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^5\theta \, d\theta = \frac{1}{16}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\cos 5\theta + 5\cos 3\theta + 10\cos\theta)\, d\theta$

$= \frac{1}{16}\left[\frac{1}{5}\sin 5\theta + \frac{5}{3}\sin 3\theta + 10\sin\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= \frac{1}{16}\left[-\frac{\sqrt{3}}{10} + 0 + 5\sqrt{3} - \frac{1}{10} - \frac{5}{3} - 5\right]$

$= \frac{1}{16}\left(\frac{49\sqrt{3}}{10} - \frac{203}{30}\right)$

$= \frac{1}{160}\left(49\sqrt{3} - \frac{203}{3}\right)$

$= \frac{1}{160}\left(\frac{147\sqrt{3} - 203}{3}\right)$

$= \frac{1}{480}(147\sqrt{3} - 203)$

$\therefore \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^5\theta \, d\theta = \frac{7}{480}(21\sqrt{3} - 29)$

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9. The complex number z is represented by the point P in an Argand diagram.

Given that $\arg\left(\frac{z-5}{z-2}\right) = \frac{\pi}{4}$

(a) sketch the locus of P as z varies,

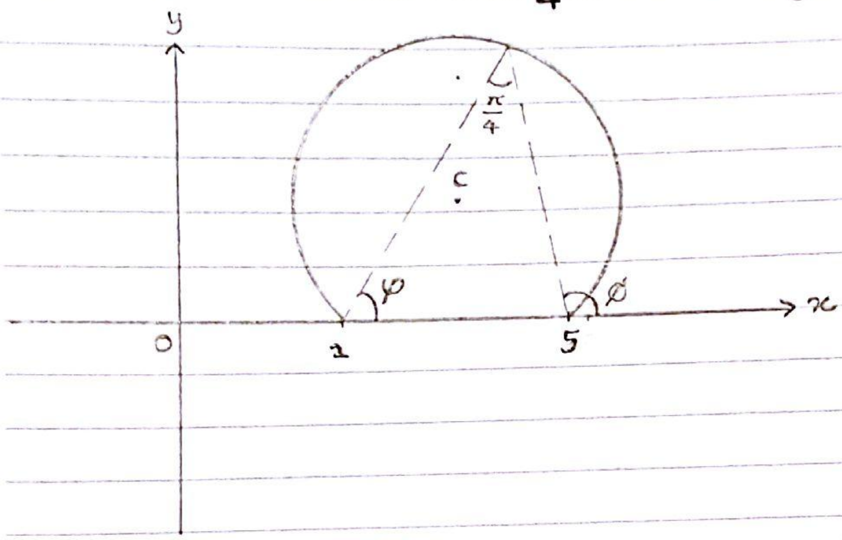
(3)

(b) find the exact maximum value of $|z|$.

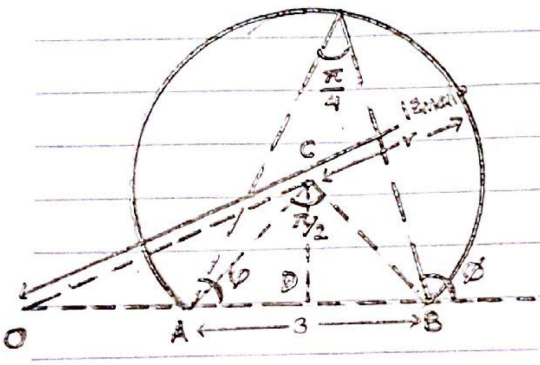
(4)

(a) $\arg(z-5) - \arg(z-2) = \frac{\pi}{4}$ $\quad$ [let $\arg(z-5) = \phi$ and $\arg(z-2) = \varphi$]

$\phi - \varphi = \frac{\pi}{4}$



(b)



ΔABC is a right angled triangle.

$AC = BC =$ radius

$\Rightarrow 3^2 = r^2 + r^2$

$9 = 2r^2$

$\frac{9}{2} = r^2$

$\therefore r = \frac{3}{2}\sqrt{2}$

$AC^2 = AD^2 + CD^2$

$\frac{9}{2} = \left(\frac{3}{2}\right)^2 + CD^2$

$\Rightarrow CD = \frac{3}{2}$

Centre, C, is at $(3.5, 1.5)$.

$|z_{max}| = \sqrt{3.5^2 + 1.5^2} + r$

$= \sqrt{\left(\frac{29}{2}\right)} + \frac{3}{2}\sqrt{2}$

$\therefore |z_{max}| = \frac{\sqrt{58} + 3\sqrt{2}}{2}$

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