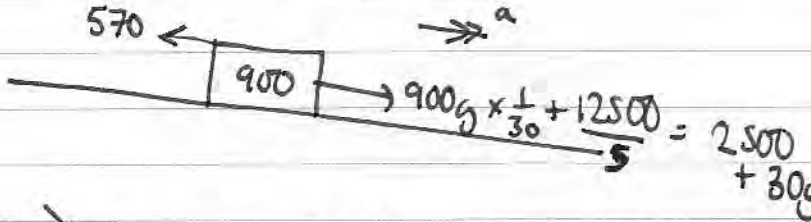
S15 M2

1. A van of mass 900 kg is moving down a straight road that is inclined at an angle θ to the horizontal, where $\sin\theta = \dfrac{1}{30}$. The resistance to motion of the van has constant magnitude 570 N. The engine of the van is working at a constant rate of 12.5 kW.

At the instant when the van is moving down the road at 5 m s^{-1}, the acceleration of the van is a m s^{-2}.

Find the value of a.

(5)



$RF^{\searrow} = ma \qquad 2794 - 570 = 900a$

$\therefore a = 2.47$

2

2.

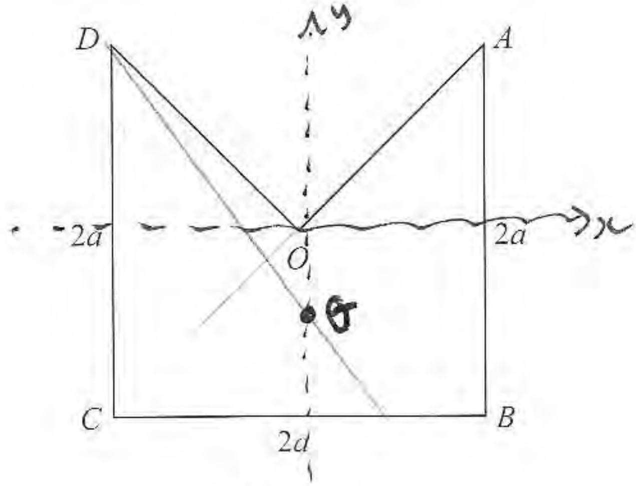


Figure 1

The uniform lamina $OABCD$, shown in Figure 1, is formed by removing the triangle OAD from the square $ABCD$ with centre O. The square has sides of length $2a$.

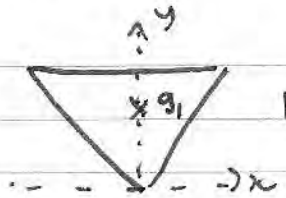
(a) Show that the centre of mass of $OABCD$ is $\frac{2}{9}a$ from O.

(4)

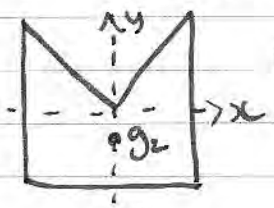
The mass of the lamina is M. A particle of mass kM is attached to the lamina at D to form the system S. The system S is freely suspended from A and hangs in equilibrium with AO vertical.

(b) Find the value of k.

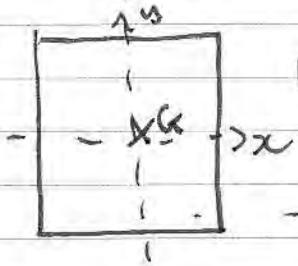
(4)



$M_1 = a \times a = a^2 \quad g_1\left(0, \frac{2}{3}a\right)$



$M_2 = 3a^2 \qquad g_2(0, \bar{y})$



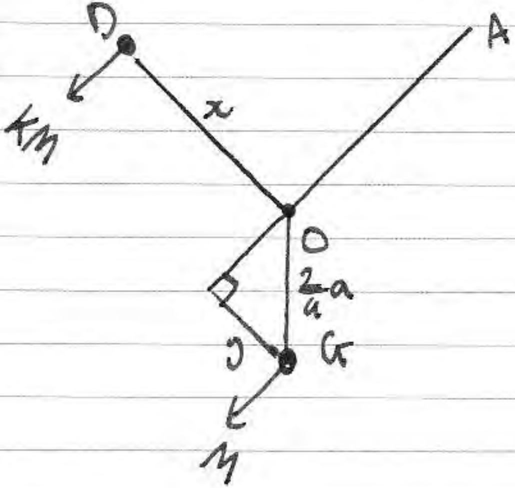
$M_3 = 4a^2 \quad g_3(0,0)$

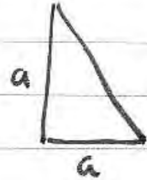
$a^2 \times \frac{2}{3}a + 3a^2 \times \bar{y} = 4a^2 \times 0$

$\frac{2}{3}a^{\cancel{3}} = -3\cancel{a^2}\,\bar{y} \qquad \bar{y} = -\frac{2}{9}a$

$\therefore \frac{2}{9}a$ below O.

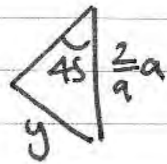
b)





$a^2 + a^2 = 2a^2$

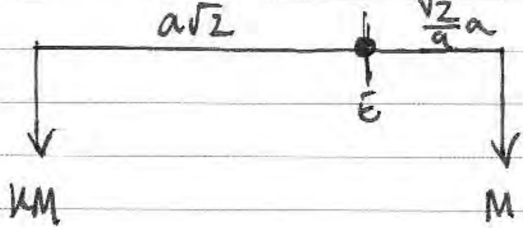
$x = a\sqrt{2}$



$y = \frac{2}{9} a \sin 45$

$y = \frac{\cancel{2}}{9} a \frac{\sqrt{2}}{\cancel{2}}$

$y = \frac{\sqrt{2}}{9} a$



E ↺ $k\cancel{M} \times \cancel{a\sqrt{2}} = \frac{\cancel{\sqrt{2}}}{9} \cancel{M} \cancel{a}$

$\therefore k = \frac{1}{9}$

3. A particle P of mass 0.75 kg is moving with velocity $4\mathbf{i}$ m s^{-1} when it receives an impulse $(6\mathbf{i} + 6\mathbf{j})$ N s. The angle between the velocity of P before the impulse and the velocity of P after the impulse is $\theta°$.

Find

(a) the value of θ,

(5)

(b) the kinetic energy gained by P as a result of the impulse.

(3)

a) Mom Before $= \frac{3}{4}\begin{pmatrix}4\\0\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix}$

Impulse $= \begin{pmatrix}6\\6\end{pmatrix}$ $\therefore$ Mom after $\begin{pmatrix}9\\6\end{pmatrix} = mv$

$\therefore \frac{3}{4}v = \begin{pmatrix}9\\6\end{pmatrix} \Rightarrow v = \begin{pmatrix}12\\8\end{pmatrix}$

$\theta = \tan^{-1}\left(\frac{8}{12}\right) = 33.7°$

b) Initial KE $= \frac{1}{2}m\begin{pmatrix}4\\0\end{pmatrix}^2 = \frac{1}{2}\left(\frac{3}{4}\right) \times 4^2 = 6$

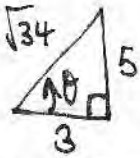
final KE $= \frac{1}{2}\left(\frac{3}{4}\right)\left(\sqrt{208}\right)^2 = 78$ final $v = \sqrt{12^2+8^2}$ $= \sqrt{208}$

$\therefore$ KE gain $= 72$ J.

4. A ladder AB, of weight W and length $2l$, has one end A resting on rough horizontal ground. The other end B rests against a rough vertical wall. The coefficient of friction between the ladder and the wall is $\frac{1}{3}$. The coefficient of friction between the ladder and the ground is μ. Friction is limiting at both A and B. The ladder is at an angle θ to the ground, where $\tan\theta = \frac{5}{3}$. The ladder is modelled as a uniform rod which lies in a vertical plane perpendicular to the wall.

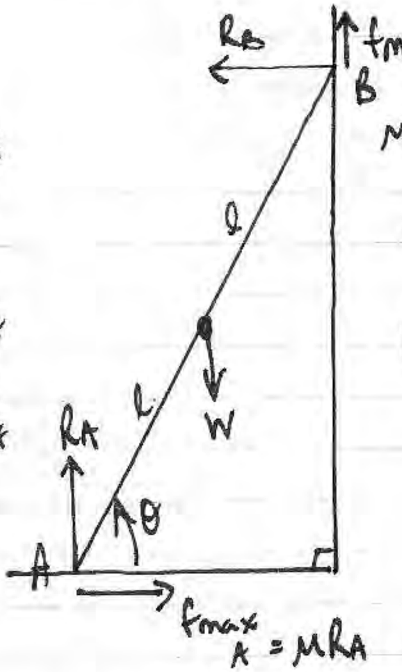
Find the value of μ.

(9)



$\cos\theta = \frac{3}{\sqrt{34}}$

$\sin\theta = \frac{5}{\sqrt{34}}$



$R\!\uparrow\!f = 0 \quad \frac{1}{3}R_B + R_A = W \quad (\mathrm{I})$

$\overrightarrow{Rf} = 0 \quad \mu R_A = R_B \quad (\mathrm{II})$

$\therefore \mu R_A + R_A = W$

$(\mu + 1)R_A = W$

$A \circlearrowleft \quad W \times \cancel{l}\cos\theta = R_B \times 2\cancel{l}\sin\theta + \frac{1}{3}R_B \times 2\cancel{l}\cos\theta$

$\frac{3}{\cancel{\sqrt{34}}}W = \frac{10}{\cancel{\sqrt{34}}}R_B + \frac{2}{\cancel{\sqrt{34}}}R_B \Rightarrow 3W = 12R_B$

$W = 4R_B$

$(\mathrm{I}) \quad \frac{1}{3}R_B + R_A = 4R_B \Rightarrow R_A = \frac{11}{3}R_B$

$(\mathrm{II}) \quad \mu \times \frac{11}{3}\cancel{R_B} = \cancel{R_B} \quad \therefore \mu = \frac{3}{11}$

5.

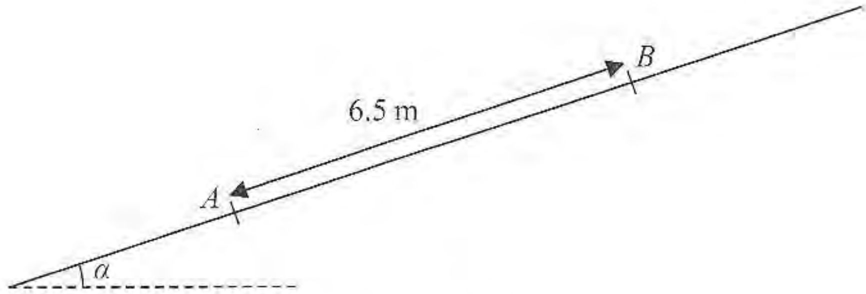


Figure 2

A particle P of mass 10 kg is projected from a point A up a line of greatest slope AB of a fixed rough plane. The plane is inclined at angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$ and $AB = 6.5$ m, as shown in Figure 2. The coefficient of friction between P and the plane is μ. The work done against friction as P moves from A to B is 245 J.

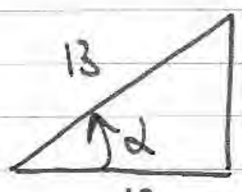
(a) Find the value of μ.

(5)

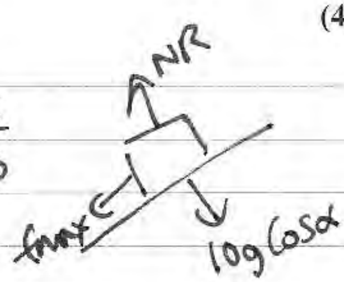
The particle is projected from A with speed 11.5 m s^{-1}. By using the work-energy principle,

(b) find the speed of the particle as it passes through B.

(4)



$\cos\alpha = \frac{12}{13}$ $\sin\alpha = \frac{5}{13}$



$f_{max} = \mu \times 10g \times \frac{12}{13}$

Wd against friction $= f_{max} \times 6.5$

$= 10g \times \frac{12}{13} \times \frac{13}{2}\mu = 245$

$\therefore 60g\,\mu = 245$

$\mu = \frac{5}{12}$

b) $KE_A = \frac{1}{2}(10) \times 11.5^2 = 661.25$

$-$ gain in PE $\quad - 10g \times 6.5 \times \frac{5}{13} = -245$

$-$ Wd against friction $\quad -245$

$\therefore KE_B = 171.25 = \frac{1}{2}(10)v^2 \qquad v = 5.85\,\text{ms}^{-1}$

6. A particle P moves on the positive x-axis. The velocity of P at time t seconds is $(2t^2 - 9t + 4)\ \mathrm{m\,s^{-1}}$. When $t = 0$, P is 15 m from the origin O.

Find

(a) the values of t when P is instantaneously at rest,

(3)

(b) the acceleration of P when $t = 5$

(3)

(c) the total distance travelled by P in the interval $0 \leqslant t \leqslant 5$

(5)

$v = 2t^2 - 9t + 4$

$a = \frac{dv}{dt} = 4t - 9$ $\qquad s = \int v\,dt = \frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t + c$

a) $2t^2 - 9t + 4 = 0$

$(2t-1)(t-4) = 0$

$t = \frac{1}{2}$ $\quad t = 4$

b) $a = 4t - 9$

$a = \frac{11}{2}$

c) $\int_4^5 v\,dt = \left[\frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t\right]_4^5$

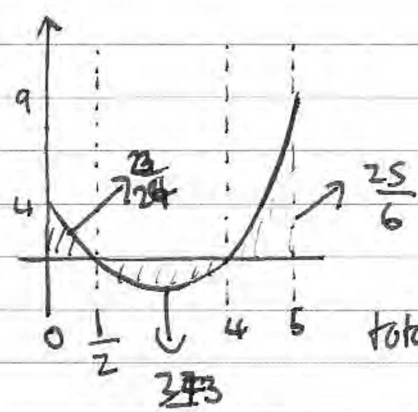
$= \left(\frac{-55}{6}\right) - \left(\frac{-40}{3}\right) = \frac{25}{6}$

$\int_{\frac{1}{2}}^4 v\,dt = \left[\frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t\right]_{\frac{1}{2}}^4$

$= \left(\frac{-40}{3}\right) - \left(\frac{23}{24}\right) = -\frac{343}{24}$

$\int_0^{\frac{1}{2}} v\,dt = \left[\frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t\right]_0^{1/2}$

$\left(\frac{23}{24}\right) - (0) = \frac{23}{24}$



total $= \frac{23}{24} + \frac{343}{24} + \frac{25}{6}$

$= \frac{233}{12}$

7.

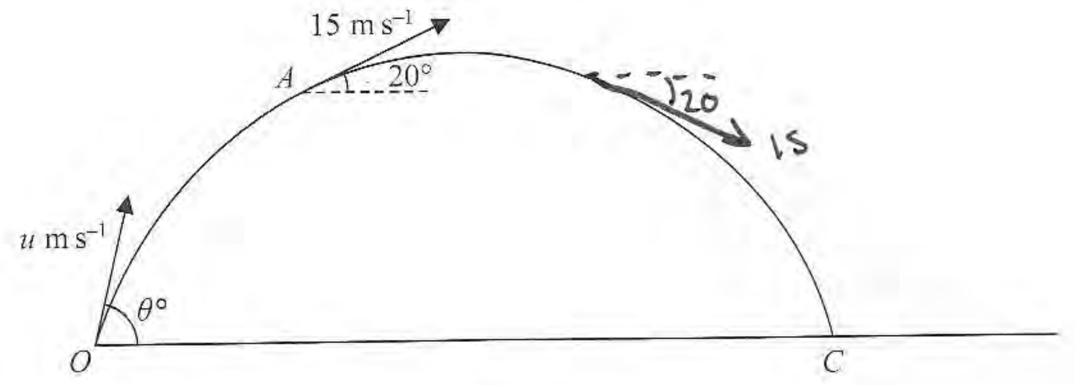


Figure 3

At time $t = 0$, a particle is projected from a fixed point O on horizontal ground with speed u m s^{-1} at an angle $\theta°$ to the horizontal. The particle moves freely under gravity and passes through the point A when $t = 4$ s. As it passes through A, the particle is moving upwards at 20° to the horizontal with speed 15 m s^{-1}, as shown in Figure 3.

(a) Find the value of u and the value of θ.

(7)

At the point B on its path the particle is moving downwards at 20° to the horizontal with speed 15 m s^{-1}.

(b) Find the time taken for the particle to move from A to B.

(2)

The particle reaches the ground at the point C.

(c) Find the distance OC.

(3)

at $t=0$ $\vec{H}$ vel = $u\cos\theta$ $\quad v\uparrow = u\sin\theta$

at $t=4$ $\vec{H}$ Vel = $15\cos 20$ $\quad v\uparrow$ vel = $15\sin 20$

$15\cos 20 = u\cos\theta$

$u\cos\theta = 14.095...$

s
u = $u\sin\theta$
v = $15\sin 20$
a = -9.8
t = 4

$v = u + at$

$15\sin 20 = u\sin\theta - 39.2$

$\therefore u\sin\theta = 44.3303$

$\frac{u\sin\theta}{u\cos\theta} = \tan\theta = \frac{44.3303...}{14.095...}$ $\quad \therefore \theta = 72.361...$ $\quad \theta = 72.4°$

2

$15\cos 20 = u\cos 72.4...$ $\quad \therefore u = 46.5$

2

b) s
$u = 15\sin 20$
$\uparrow$ $v = -15\sin 20$
$a = -9.8$
t

$v = u + at$

$-15\sin 20 = 15\sin 20 - 9.8t$

$\therefore \frac{30\sin 20}{9.8} = t = 1.05$

2

c) at $t=0$ $v\uparrow = u\sin\theta$

$= 46.5..\sin 72.361..$ $v\uparrow = 44.3303..$

$\therefore$ at C $v\uparrow = -44.3303$

$\overrightarrow{AC}$ s
$u = 15\sin 20$
$\uparrow$ $v = -44.3303...$
$a = -9.8$
t

$v = u + at$

$-44.3303... = 15\sin 20 - 9.8t$

$t = 5.047$

$\therefore$ total time $= 9.047$ sec

$\overrightarrow{H}$ Speed $= 15\cos 20$

Distance OC $= 15\cos 20 \times 9.047.. = 127$ m

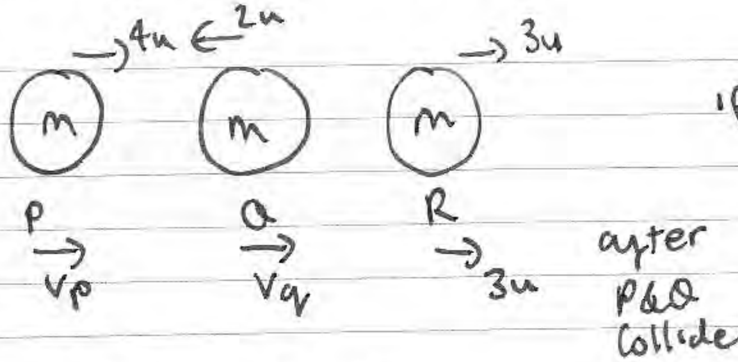
2

8. Three identical particles P, Q and R, each of mass m, lie in a straight line on a smooth horizontal plane with Q between P and R. Particles P and Q are projected directly towards each other with speeds $4u$ and $2u$ respectively, and at the same time particle R is projected along the line away from Q with speed $3u$. The coefficient of restitution between each pair of particles is e. After the collision between P and Q there is a collision between Q and R.

(a) Show that $e > \dfrac{2}{3}$ **(7)**

It is given that $e = \dfrac{3}{4}$

(b) Show that there will not be a further collision between P and Q. **(6)**



if Q and R collide

$V_q > 3u$. (2)

$e = \dfrac{\text{sep}}{\text{app}} = \dfrac{V_q - V_p}{6u} \Rightarrow V_q - V_p = 6eu$ (1)

PQ

Mom before $4mu - 2mu = 2mu$ CLM $2\cancel{m}u = \cancel{m}V_p + \cancel{m}V_q$

Mom after $mV_p + mV_q$

$V_q + V_p = 2u \Rightarrow V_p = 2u - V_q$ sub in (1)

$V_q - 2u + V_q = 6eu \Rightarrow 2V_q - 2u = 6eu$

$V_q - u = 3eu$

$V_q = 3eu + u$ sub in (2)

$3eu + u > 3u$

$3e\cancel{u} > 2\cancel{u}$ $\therefore e > \frac{2}{3}$ #

b) $e = \frac{3}{4} \Rightarrow V_q - V_p = \frac{9u}{2}$ $2V_q - 2V_p = 9u$

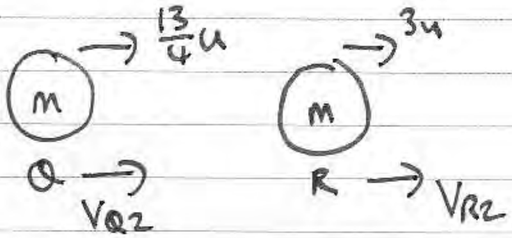
$V_q = 3eu + u \Rightarrow V_q = \frac{9}{4}u + u = \frac{13}{4}u$

$\therefore \frac{13}{2}u - 2V_p = 9u \qquad 2V_p = -\frac{5}{2}u \qquad V_p = -\frac{5}{4}u$

Speed is $\frac{5}{4}u$ away from Q after the 1st collison

$\therefore$ If Q does not collide with P again

$V_{q2} > -\frac{5}{4}u$



$e = \dfrac{V_{R2} - V_{Q2}}{\frac{1}{4}u}$

$eu = 4V_{R2} - V_{Q2}$

$\frac{3}{4}u = 4V_{R2} - 4V_{Q2}$

$3u = 16V_{R2} - 16V_{Q2}$

Mom Before $= \frac{13}{4}mu + 3mu$

$= \frac{25}{4}mu$

Mom after $= mV_{q2} + mV_{R2}$ CLM

$\cancel{m}V_{q2} + \cancel{m}V_{R2} = \frac{25}{4}\cancel{m}u \qquad 4V_{Q2} + 4V_{R2} = 25u$

$\therefore 16V_{R2} + 16V_{q2} = 100u$

$16V_{R2} - 16V_{q2} = 3u$

$32V_{q2} = 97u$

$V_{q2} = \frac{97}{32}u$

$\therefore$ Q is moving away from P after the 2nd collison

$\therefore$ they will not collide.