C1 S14 UK

1. Find

$$\int (8x^3 + 4)\,\mathrm{d}x$$

giving each term in its simplest form.

(3)

$= \dfrac{8x^4}{4} + 4x + c \quad = \dfrac{2x^4}{2} + 4x + c$

2. (a) Write down the value of $32^{\frac{1}{5}}$

(1)

(b) Simplify fully $(32x^5)^{-\frac{2}{5}}$

(3)

a) $\sqrt[5]{32} = 2$ b) $(32x^5)^{\frac{1}{5}} = 2x$

$\Rightarrow (32x^5)^{\frac{2}{5}} = 4x^2 \Rightarrow (32x^5)^{-\frac{2}{5}} = \frac{1}{4x^2}$

3. Find the set of values of x for which

(a) $3x - 7 > 3 - x$

(2)

(b) $x^2 - 9x \leqslant 36$

(4)

(c) **both** $3x - 7 > 3 - x$ **and** $x^2 - 9x \leqslant 36$

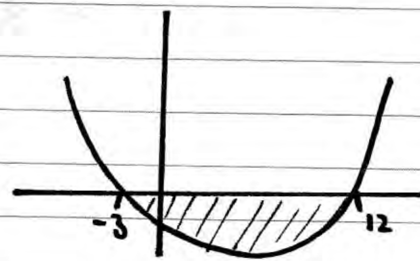
(1)

a) $4x > 10 \Rightarrow x > 2.5$

b) $x^2 - 9x - 36 \leq 0$

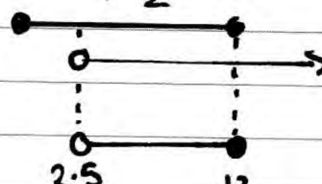
$(x-12)(x+3) \leq 0$

$12 \qquad -3$



$-3 \leq x \leq 12$

c) $2.5 < x \leq 12$



4.

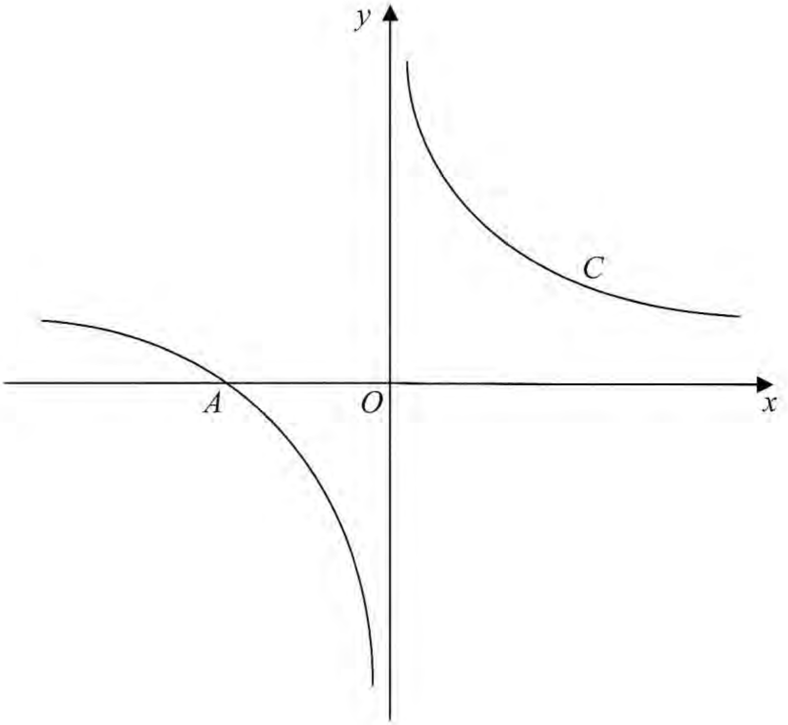


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{1}{x} + 1, \qquad x \neq 0$$

The curve C crosses the x-axis at the point A.

(a) State the x coordinate of the point A.

(1)

The curve D has equation $y = x^2(x - 2)$, for all real values of x.

(b) A copy of Figure 1 is shown on page 7.
On this copy, sketch a graph of curve D.
Show on the sketch the coordinates of each point where the curve D crosses the coordinate axes.

(3)

(c) Using your sketch, state, giving a reason, the number of real solutions to the equation

$$x^2(x - 2) = \frac{1}{x} + 1$$

(1)

a) $y=0 \Rightarrow \frac{1}{x}=-1 \Rightarrow \underline{x=-1}$ $\quad \underline{A(-1,0)}$

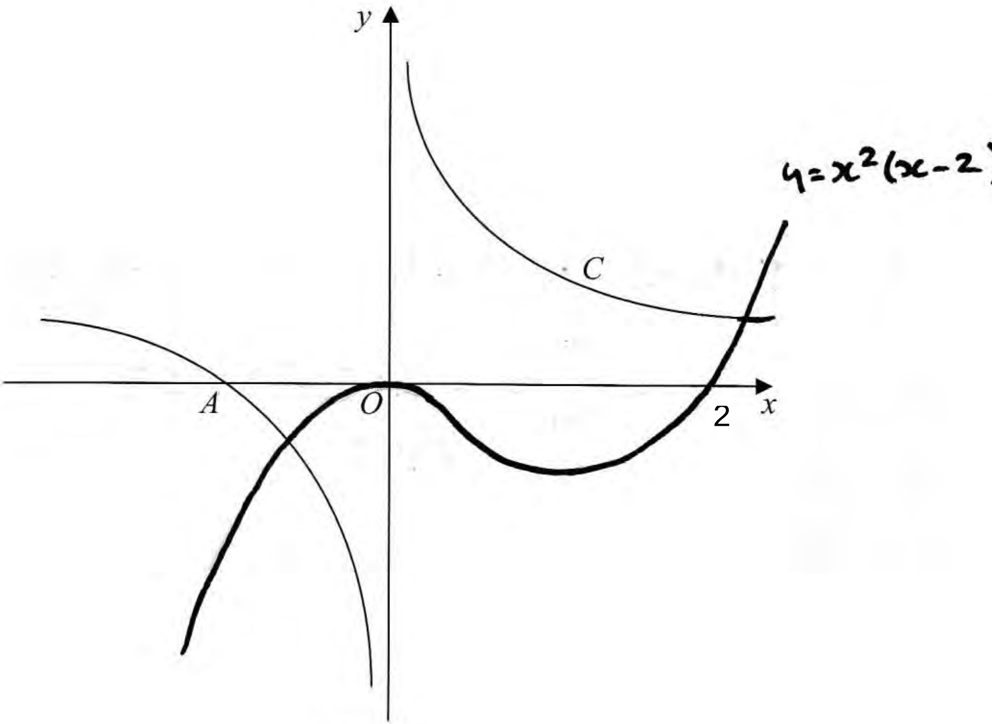


Figure 1

c) 2 solutions as curves intersect twice.

5. A sequence of numbers $a_1, a_2, a_3 \ldots$ is defined by

$$a_{n+1} = 5a_n - 3, \qquad n \geqslant 1$$

Given that $a_2 = 7$,

(a) find the value of a_1

(2)

(b) Find the value of $\sum_{r=1}^{4} a_r$

(3)

a) $a_2 = 5a_1 - 3 \rightarrow 7 = 5a_1 - 3 \quad \therefore \underline{a_1 = 2}$

$a_1 = 2$

$a_2 = 7$

$a_3 = 32$

$a_4 = 157$

$\therefore \sum_{r=1}^{4} a_r = 2 + 7 + 32 + 157$

$= \underline{198}$

6 (a) Write $\sqrt{80}$ in the form $c\sqrt{5}$, where c is a positive constant.

(1)

A rectangle R has a length of $(1 + \sqrt{5})$ cm and an area of $\sqrt{80}$ cm^2.

(b) Calculate the width of R in cm. Express your answer in the form $p + q\sqrt{5}$, where p and q are integers to be found.

(4)

a) $\sqrt{80} = \sqrt{16}\sqrt{5} = \underline{4\sqrt{5}}$

b) $\text{width} = \dfrac{\sqrt{80}}{1+\sqrt{5}} \times \dfrac{(1-\sqrt{5})}{(1-\sqrt{5})} = \dfrac{\sqrt{80} - \sqrt{400}}{-4}$

$= \dfrac{4\sqrt{5} - 20}{-4} = \underline{5 - \sqrt{5}}$ $\qquad \underline{p = 5} \quad \underline{q = -1}$

7. Differentiate with respect to x, giving each answer in its simplest form.

(a) $(1-2x)^2$ **(3)**

(b) $\dfrac{x^5 + 6\sqrt{x}}{2x^2}$ **(4)**

a) $\frac{d}{dx}(1-2x)^2 = \frac{d}{dx}(1-4x+4x^2) = -4+8x$

b) $\frac{d}{dx}\left(\frac{x^5+6\sqrt{x}}{2x^2}\right) = \frac{d}{dx}\left(\frac{1}{2}x^3+3x^{-\frac{3}{2}}\right) = \frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}}$

8. In the year 2000 a shop sold 150 computers. Each year the shop sold 10 more computers than the year before, so that the shop sold 160 computers in 2001, 170 computers in 2002, and so on forming an arithmetic sequence.

(a) Show that the shop sold 220 computers in 2007.

(2)

(b) Calculate the total number of computers the shop sold from 2000 to 2013 inclusive.

(3)

In the year 2000, the selling price of each computer was £900. The selling price fell by £20 each year, so that in 2001 the selling price was £880, in 2002 the selling price was £860, and so on forming an arithmetic sequence.

(c) In a particular year, the selling price of each computer in £s was equal to three times the number of computers the shop sold in that year. By forming and solving an equation, find the year in which this occurred.

(4)

2000 $a = u_1 = 150$ $a = 150$ u_8 = year 2007

2001 $u_2 = 160$ $d = 10$ $u_8 = a + 7d$

$= 150 + 70 = \underline{220}$

2

b) $S_{14} = \frac{1}{2}(14)[2(150) + 13 \times 10] = 7(300 + 130)$

$= 7 \times 430 = \underline{3010}$

2

c) $a = u_1 = 900$ $a = 900$

$u_2 = 880$ $d = -20$

Selling price in term n $= 900 + (n-1)(-20) = 920 - 20n$

Number of computes sold $= 150 + (n-1)(10) = 140 + 10n$

$\therefore 920 - 20n = 3 \times (140 + 10n) \Rightarrow 500 = 50n \therefore n = 10$

$\underline{2009}$

2

9.

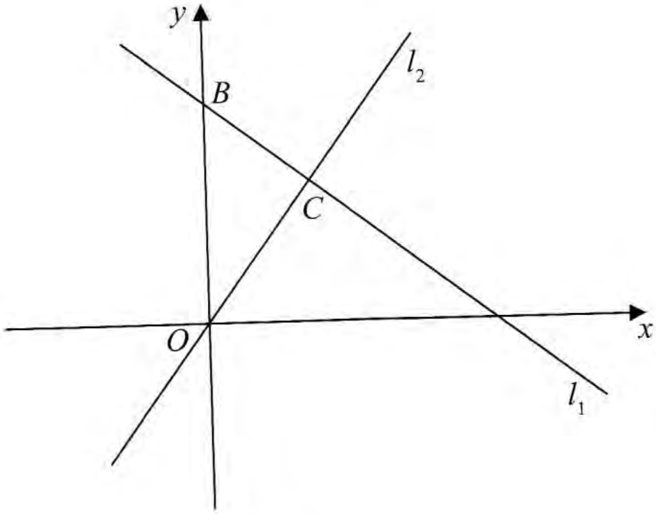


Figure 2

The line l_1, shown in Figure 2 has equation $2x + 3y = 26$

The line l_2 passes through the origin O and is perpendicular to l_1

(a) Find an equation for the line l_2 **(4)**

The line l_2 intersects the line l_1 at the point C.

Line l_1 crosses the y-axis at the point B as shown in Figure 2.

(b) Find the area of triangle OBC.

Give your answer in the form $\frac{a}{b}$, where a and b are integers to be determined. **(6)**

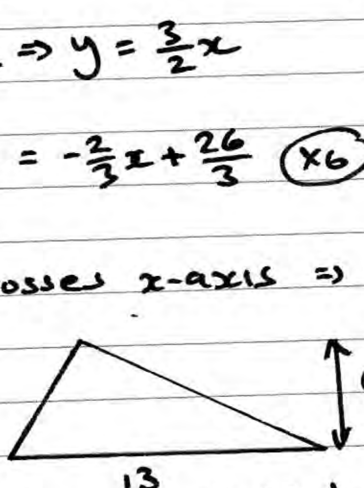
a) $l_1 \quad 3y = -2x + 26 \Rightarrow y = -\frac{2}{3}x + \frac{26}{3} \qquad m_{l_1} = -\frac{2}{3} \therefore m_{l_2} = \frac{3}{2}$

$\therefore l_2 \Rightarrow y = \frac{3}{2}x$

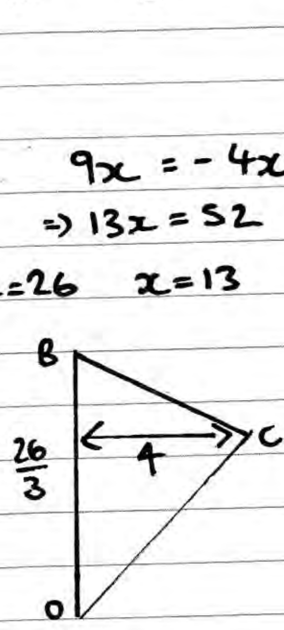
b) $\frac{3}{2}x = -\frac{2}{3}x + \frac{26}{3}$ (×6) $\qquad 9x = -4x + 52$

$\Rightarrow 13x = 52 \qquad \therefore x = 4 \quad y = 6$

l_1 crosses x-axis $\Rightarrow 2x = 26 \quad x = 13$



not needed Read Qu!



$$\text{Area} = \frac{\frac{26}{3} \times 4}{2}$$

$$= \frac{\frac{52}{3}}{2}$$

10. A curve with equation $y = f(x)$ passes through the point (4, 25).

Given that

$$f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, \qquad x > 0$$

(a) find $f(x)$, simplifying each term.

(5)

(b) Find an equation of the normal to the curve at the point (4, 25).

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

a) $f(x) = \dfrac{\frac{3}{8}x^3}{3} - \dfrac{10x^{\frac{1}{2}}}{\frac{1}{2}} + x + c \Rightarrow f(x) = \frac{1}{8}x^3 - 20x^{\frac{1}{2}} + x + c$

$25 = \frac{1}{8}(4)^3 - 20(4)^{\frac{1}{2}} + 4 + c \Rightarrow 25 = 8 - 40 + 4 + c$

$\therefore c = 53$

$\therefore f(x) = \frac{1}{8}x^3 - 20x^{\frac{1}{2}} + x + 53$

b) $x = 4 \quad m_t = \frac{3}{8}(4)^2 - 10(4)^{-\frac{1}{2}} + 1 = 6 - 5 + 1 = 2$

$\therefore m_n = -\frac{1}{2} \Rightarrow y - 25 = -\frac{1}{2}(x - 4)$

$2y - 50 = -x + 4 \quad \therefore x + 2y - 54 = 0$

11. Given that

$$f(x) = 2x^2 + 8x + 3$$

(a) find the value of the discriminant of $f(x)$.

(2)

(b) Express $f(x)$ in the form $p(x+q)^2 + r$ where p, q and r are integers to be found.

(3)

The line $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = f(x)$.

(c) Calculate the value of c.

(5)

a) $d = b^2 - 4ac = 64 - 4(2)(3) = \underline{40}$

b) $2\left(x^2 + 4x + \frac{3}{2}\right) = 2\left[(x+2)^2 - 4 + \frac{3}{2}\right]$

$= 2\left[(x+2)^2 - \frac{5}{2}\right] \quad \therefore f(x) = 2(x+2)^2 - 5$

c) gradient of line $= 4$.

$f'(x) = 4x + 8 \quad \therefore 4x + 8 = 4 \Rightarrow 4x = -4$

$\underline{x = -1}$

$y = 2(-1)^2 + 8(-1) + 3 = -3$

$y = 4x + c \qquad -3 = 4(-1) + c \qquad \therefore \underline{c = 1}$