S14 C1R

1. Factorise fully $25x - 9x^3$

(3)

$x(25 - 9x^2) = x(5 - 3x)(5 + 3x)$

2. (a) Evaluate $81^{\frac{3}{2}}$

(2)

(b) Simplify fully $x^2\left(4x^{-\frac{1}{2}}\right)^2$

(2)

a) $(81^{\frac{1}{2}})^3 = 9^3 = 729$

b) $x^2(16x^{-1}) = 16x$

3. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 4a_n - 3, \qquad n \geqslant 1$$

$$a_1 = k, \qquad \text{where } k \text{ is a positive integer.}$$

(a) Write down an expression for a_2 in terms of k.

(1)

Given that $\sum_{r=1}^{3} a_r = 66$

(b) find the value of k.

(4)

a) $a_2 = 4k - 3$

b) $a_3 = 4(4k-3) - 3$
$= 16k - 15$

$\sum_{1}^{3} a_r = $ k
$4k - 3$
$16k - 15$
$21k - 18 = 66$

$21k = 84$
$\underline{k = 4}$

4. Given that $y = 2x^5 + \dfrac{6}{\sqrt{x}}$, $x > 0$, find in their simplest form

(a) $\dfrac{dy}{dx}$

(3)

(b) $\int y \, dx$

(3)

$y = 2x^5 + 6x^{-\frac{1}{2}}$

a) $y' = 10x^4 - 3x^{-\frac{3}{2}}$

b) $\int y\,dx = \dfrac{2x^6}{6} + \dfrac{6x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{1}{3}x^6 + 12x^{\frac{1}{2}} + c$

5. Solve the equation

$$10 + x\sqrt{8} = \frac{6x}{\sqrt{2}}$$

Give your answer in the form $a\sqrt{b}$ where a and b are integers.

(4)

$\times\sqrt{2} \Rightarrow 10\sqrt{2} + 4x = 6x \quad \therefore 2x = 10\sqrt{2}$

$\therefore \underline{x = 5\sqrt{2}}$

8.

$$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x}, \quad x > 0$$

Given that $y = 37$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(7)

$y' = 6x^{-\frac{1}{2}} + x^{\frac{3}{2}}$

$y = \dfrac{6x^{\frac{1}{2}}}{\frac{1}{2}} + \dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$

$y = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + c$

$37 = 12(2) + \frac{2}{5}(2)^5 = 24 + 12.8 = 36.8$

$\therefore c = 0.2$

$\therefore y = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$

6.

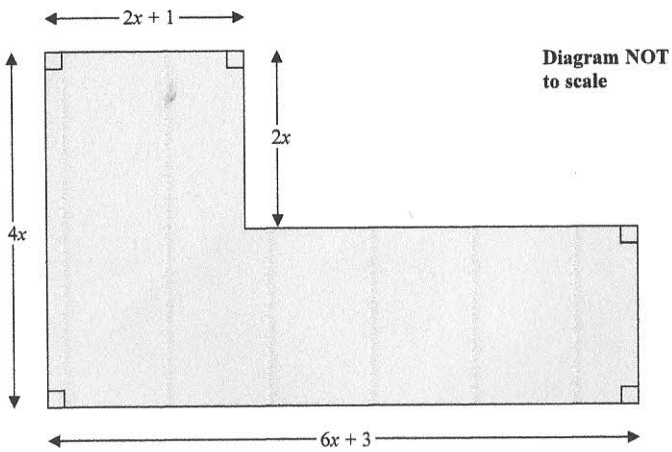


Figure 1

Figure 1 shows the plan of a garden. The marked angles are right angles.

The six edges are straight lines.

The lengths shown in the diagram are given in metres.

Given that the perimeter of the garden is greater than 40 m,

(a) show that $x > 1.7$ **(3)**

Given that the area of the garden is less than 120 m^2,

(b) form and solve a quadratic inequality in x. **(5)**

(c) Hence state the range of the possible values of x. **(1)**

a) $P = 2(4x + 6x + 3) = 20x + 6$

$\therefore 20x + 6 > 40 \Rightarrow 20x > 34 \quad \therefore \underline{x > 1.7}$

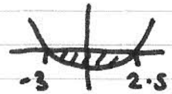
b) $A = (2x+1)(2x) + (6x+3)(2x)$ ✱

$= 2x(8x+4) = 16x^2 + 8x \quad < 120$

$16x^2 + 8x - 120 < 0 \Rightarrow 2x^2 + x - 15 < 0$

$(2x-5)(x+3) < 0$

$2.5 \qquad -3$



$-3 < x < 2.5$

$\Rightarrow \underline{1.7 < x < 2.5}$

7.

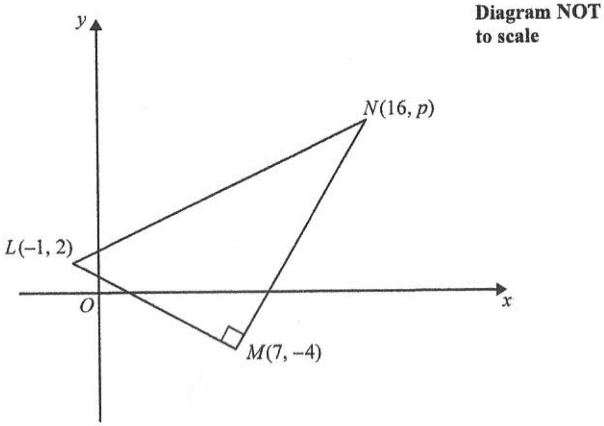


Figure 2

Figure 2 shows a right angled triangle LMN.

The points L and M have coordinates $(-1, 2)$ and $(7, -4)$ respectively.

(a) Find an equation for the straight line passing through the points L and M.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

Given that the coordinates of point N are $(16, p)$, where p is a constant, and angle $LMN = 90°$,

(b) find the value of p. **(3)**

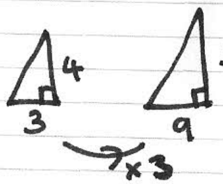
Given that there is a point K such that the points L, M, N, and K form a rectangle,

(c) find the y coordinate of K. **(2)**

a) $m_{LM} = \frac{-6}{8} = \frac{-3}{4} \qquad y - 2 = \frac{-3}{4}(x+1)$

$4y - 8 = -3x - 3 \quad \therefore 3x + 4y - 5 = 0$

b) $m_{MN} = \frac{4}{3}$ (perp to LM)



$\therefore 4 \times 3 = 12 \qquad \therefore p = -4 + 12$

$\underline{p = 8}$

c) $\times k \rightarrow 8$ $\quad \therefore \underline{k(8, 14)}$

$\downarrow 6 = N(16, 8)$

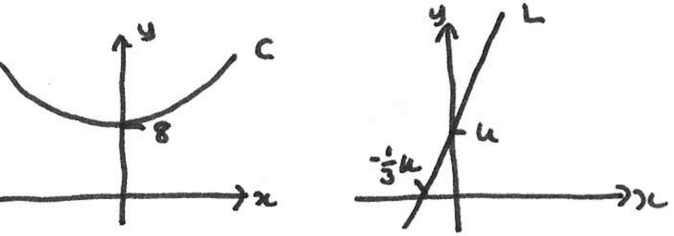
9. The curve C has equation $y = \frac{1}{3}x^2 + 8$

The line L has equation $y = 3x + k$, where k is a positive constant.

(a) Sketch C and L on separate diagrams, showing the coordinates of the points at which C and L cut the axes. **(4)**

Given that line L is a tangent to C,

(b) find the value of k. **(5)**



b) $y' = \frac{2}{3}x$ = gradient of tangent $= 3$

$\therefore\ 2x = 9 \quad x = 4.5,\ y = \frac{1}{3}\left(\frac{9}{2}\right)^2 + 8$

$y = \frac{81}{12} + 8 = \frac{27}{4} + \frac{32}{4} = \frac{59}{4}$

$\therefore\ \frac{59}{4} = 3x + k \quad k = \frac{59}{4} - 3\left(\frac{9}{2}\right) = \frac{59}{4} - \frac{54}{4} = \frac{5}{4}$

10. Xin has been given a 14 day training schedule by her coach.

Xin will run for A minutes on day 1, where A is a constant.

She will then increase her running time by $(d + 1)$ minutes each day, where d is a constant.

(a) Show that on day 14, Xin will run for

$$(A + 13d + 13) \text{ minutes.}$$

(2)

Yi has also been given a 14 day training schedule by her coach.

Yi will run for $(A - 13)$ minutes on day 1.

She will then increase her running time by $(2d - 1)$ minutes each day.

Given that Yi and Xin will run for the same length of time on day 14,

(b) find the value of d. **(3)**

Given that Xin runs for a total time of 784 minutes over the 14 days,

(c) find the value of A. **(3)**

a) $u_{14} = a + 13d = A + 13(d+1) = A + 13d + 13$ #

b) $u_{14} = (A - 13) + 13(2d - 1) = A + 26d - 26$

$A + 26d - 26 = A + 13d + 13 \Rightarrow 13d = 39 \therefore d = 3$

c) $S_n = \frac{1}{2}n(A + L)$

$S_{14} = 7(A + A + 13(3) + 13) = 784$

$\Rightarrow 2A + 52 = 112 \Rightarrow 2A = 60 \quad \therefore A = 30$

11.

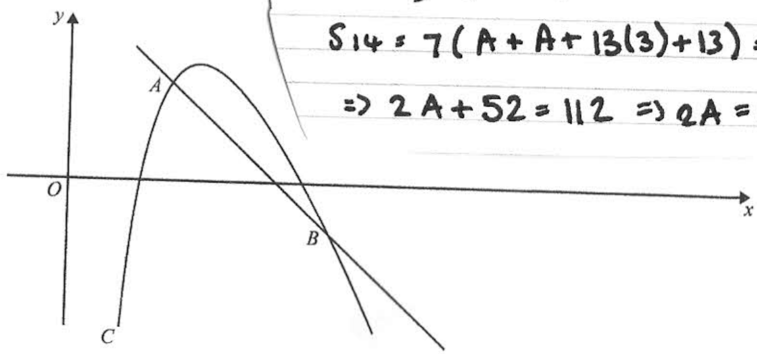


Figure 3

A sketch of part of the curve C with equation

$$y = 20 - 4x - \frac{18}{x}, \quad x > 0$$

is shown in Figure 3.

Point A lies on C and has an x coordinate equal to 2

(a) Show that the equation of the normal to C at A is $y = -2x + 7$ **(6)**

The normal to C at A meets C again at the point B, as shown in Figure 3.

(b) Use algebra to find the coordinates of B. **(5)**

$y' = -4 + 18x^{-2}$ $(x=2)$ $m_t = \frac{18}{2^2} - 4 = \frac{1}{2}$

$\therefore m_n = -2 \qquad (x=2)\ y = 20 - 8 - 9 = 3$

$y - 3 = -2(x - 2) \therefore y - 3 = -2x + 4 \therefore y = -2x + 7$ #

b) $-2x + 7 = 20 - 4x - \frac{18}{x} \Rightarrow 2x + \frac{18}{x} - 13 = 0$

$(\times x)\ 2x^2 - 13x + 18 = 0 \qquad (x - 2)(2x - 9) = 0$

$\therefore x = 4.5 \quad y = -2$