M1 IAL S14

1. Two small smooth balls A and B have mass 0.6 kg and 0.9 kg respectively. They are moving in a straight line towards each other in opposite directions on a smooth horizontal floor and collide directly. Immediately before the collision the speed of A is v m s^{-1} and the speed of B is 2 m s^{-1}. The speed of A is 2 m s^{-1} immediately after the collision and B is brought to rest by the collision.

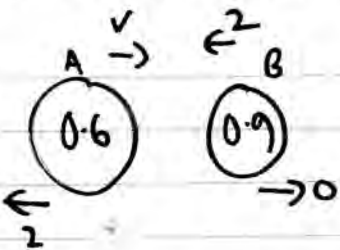
Find

(a) the value of v,

(3)

(b) the magnitude of the impulse exerted on A by B in the collision.

(2)



CLM $0.6v + 0.9(-2) = 0.6(-2) + 0$

$0.6v - 1.8 = -1.2$

$0.6v = 0.6$

$\underline{v = 1}$

mom B before $= 2(-0.9) = -1.8$

mom B after $= 0$

$\therefore$ Impulse $= 1.8$ Ns

2. A ball is thrown vertically upwards with speed 20 m s^{-1} from a point A, which is h metres above the ground. The ball moves freely under gravity until it hits the ground 5 s later.

(a) Find the value of h.

(3)

A second ball is thrown vertically downwards with speed w m s^{-1} from A and moves freely under gravity until it hits the ground.

The first ball hits the ground with speed V m s^{-1} and the second ball hits the ground with speed $\frac{3}{4}V$ m s^{-1}.

(b) Find the value of w.

(5)

a) $s = -h$ $\quad S = ut + \frac{1}{2}at^2$

$u = 20 \uparrow$ $\quad -h = 20t - 4.9 \times t^2 \quad t = 5 \quad \Rightarrow -h = -22.5$

v

$a = -9.8$ $\quad \therefore h = 22.5m$

$t = 5$

b) ① $v = u + at \quad v = 20 - 9.8(5) = -29 \quad 29 \downarrow$

② $S = 22.5$ $\quad V^2 = u^2 + 2as$

$\downarrow$ $u = w$

$v = 21.75$ $\quad 21.75^2 = w^2 + 2(9.8)(22.5)$

$a = 9.8$ $\quad \therefore w^2 = 32.0625$

t

$\therefore w = 5.66$

3. A particle P of mass 1.5 kg is placed at a point A on a rough plane which is inclined at 30° to the horizontal. The coefficient of friction between P and the plane is 0.6

(a) Show that P rests in equilibrium at A.

(5)

A horizontal force of magnitude X newtons is now applied to P, as shown in Figure 1. The force acts in a vertical plane containing a line of greatest slope of the inclined plane.

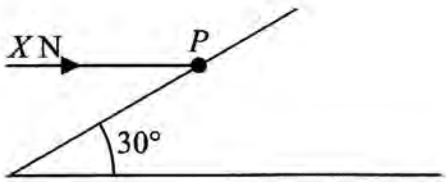


Figure 1

The particle is on the point of moving up the plane.

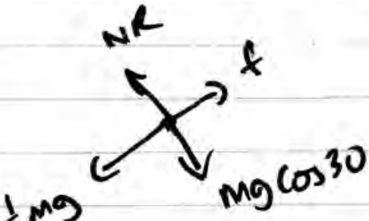
(b) Find

(i) the magnitude of the normal reaction of the plane on P,

(ii) the value of X.

(7)

a)



$NR = 1.5g\cos 30$

$f_{max} = \mu NR = \frac{3}{5}(1.5g\cos 30)$

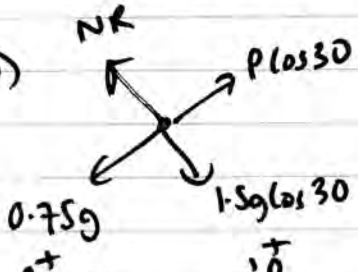
$f_{max} = 0.9g\cos 30$

It will rest in equilibrium if $f < f_{max}$

$f = \frac{1}{2}mg$ (if in equilibrium) $= 0.5mg = 0.75g$

$f_{max} = 0.7794g$ $\therefore f_{max} > f$ $\therefore$ equilibrium

b)



$NR = \frac{1}{2}P + 1.5g\cos 30$

$\therefore f_{max} = 0.3P + 0.9g\cos 30$

$\therefore P\cos 30 = 0.75g + 0.3P + 0.9g\cos 30$

$(\cos 30 - 0.3)P = 0.75g + 0.9g\cos 30$

$\therefore P = \dfrac{0.75g + 0.9g\cos 30}{\cos 30 - 0.3} \Rightarrow P = X = 26.5\text{N}$

$\therefore NR = 26.0\text{N}$

4.

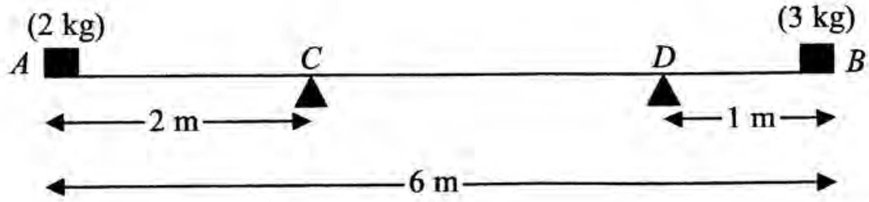


Figure 2

A plank AB, of length 6 m and mass 4 kg, rests in equilibrium horizontally on two supports at C and D, where $AC = 2$ m and $DB = 1$ m. A brick of mass 2 kg rests on the plank at A and a brick of mass 3 kg rests on the plank at B, as shown in Figure 2. The plank is modelled as a uniform rod and all bricks are modelled as particles.

(a) Find the magnitude of the reaction exerted on the plank

(i) by the support at C,

(ii) by the support at D.

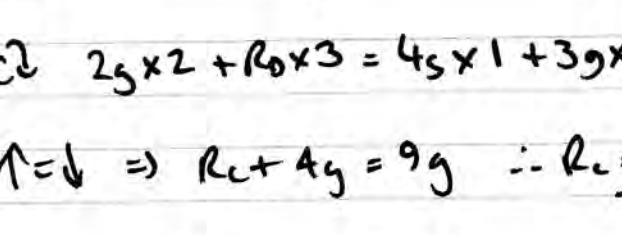
(6)

The 3 kg brick is now removed and replaced with a brick of mass x kg at B. The plank remains horizontal and in equilibrium but the reactions on the plank at C and at D now have equal magnitude.

(b) Find the value of x.

(4)

a)



$C\circlearrowright$ $2g\times 2 + R_D\times 3 = 4g\times 1 + 3g\times 4 \Rightarrow 4g + 3R_D = 16g$

$\uparrow = \downarrow \Rightarrow R_c + 4g = 9g \quad \therefore R_c = \underline{5g}$ $\quad 3R_D = 12g \quad R_D = \underline{4g\text{N}}$

b)

$B\circlearrowright$ $R\times 1 + R\times 4 = 4g\times 3 + 2g\times 6$

$5R = 24g \quad \therefore R = 4.8g$

$\uparrow = \downarrow \quad 2R = (6+x)g \qquad 9.6g = (6+x)g$

$\therefore x = \underline{3.6}$

5. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O.*]

A boy B is running in a field with constant velocity $(3\mathbf{i} - 2\mathbf{j})$ m s^{-1}. At time $t = 0$, B is at the point with position vector $10\mathbf{j}$ m.

Find

(a) the speed of B,

(2)

(b) the direction in which B is running, giving your answer as a bearing.

(3)

At time $t = 0$, a girl G is at the point with position vector $(4\mathbf{i} - 2\mathbf{j})$ m. The girl is running with constant velocity $\left(\frac{5}{3}\mathbf{i} + 2\mathbf{j}\right)$ m s^{-1} and meets B at the point P.

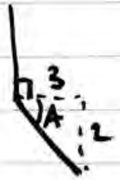
(c) Find

(i) the value of t when they meet,

(ii) the position vector of P.

(6)

a) Speed $= \sqrt{3^2+2^2} = \sqrt{13} = 3.61 \text{ ms}^{-1}$

b) 

bearing $= 90 + \tan^{-1}\left(\frac{2}{3}\right) = 123.7°$

2

c) $G = \begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} 5/3 \\ 2 \end{pmatrix} t = \begin{pmatrix} 0 \\ 10 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} t = B$

$\therefore 4 + \frac{5}{3}t = 3t \quad \therefore 4 = \frac{4}{3}t \quad \therefore t = 3$

2

$G = B$ at $\begin{pmatrix} 9 \\ 4 \end{pmatrix}$

6. A car starts from rest at a point A and moves along a straight horizontal road. The car moves with constant acceleration 1.5 m s^{-2} for the first 8 s. The car then moves with constant acceleration 0.8 m s^{-2} for the next 20 s. It then moves with constant speed for T seconds before slowing down with constant deceleration 2.8 m s^{-2} until it stops at a point B.

(a) Find the speed of the car 28 s after leaving A.
(3)

(b) Sketch, in the space provided, a speed–time graph to illustrate the motion of the car as it travels from A to B.
(2)

(c) Find the distance travelled by the car during the first 28 s of its journey from A.
(4)

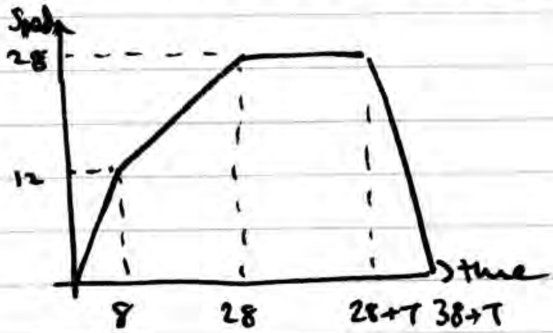
The distance from A to B is 2 km.

(d) Find the value of T.
(4)

a) $v = u + at \quad v = 0 + 1.5(8) = 12$

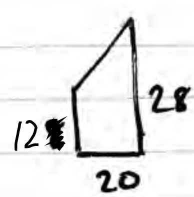
$v = u + at \quad v = 12 + 0.8(20) = 12 + 16 = 28 \text{ ms}^{-1}$

b)



c)

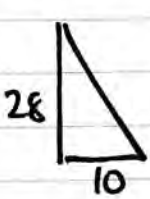
$= \frac{1}{2}(8)(12) = 48$



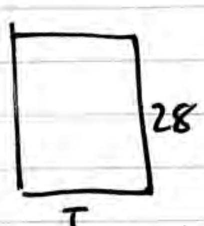
$= \frac{1}{2}(20)(12+28)$

$= 400$

$= 448$



$\frac{1}{2}(10)(28) = 140$



$\therefore 28T = 2000 - 448 - 140 = 1412$

$\therefore T = 50.4 \text{ s}$

7.

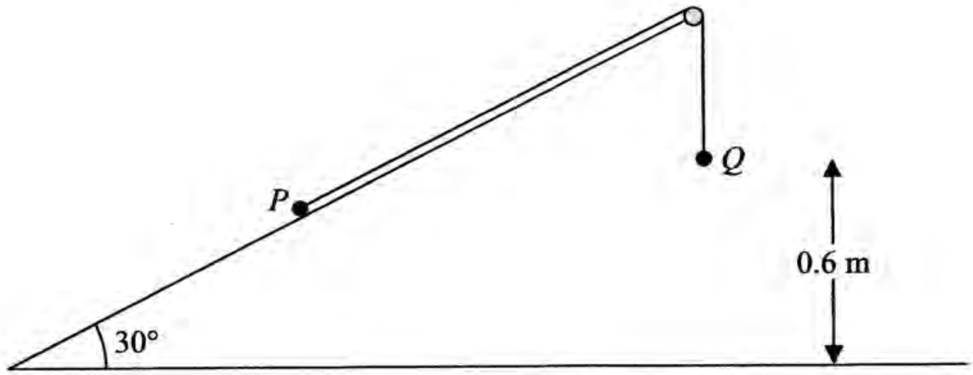


Figure 3

Two particles P and Q, of mass 2 kg and 3 kg respectively, are connected by a light inextensible string. Initially P is held at rest on a fixed smooth plane inclined at 30° to the horizontal. The string passes over a small smooth fixed pulley at the top of the plane. The particle Q hangs freely below the pulley and 0.6 m above the ground, as shown in Figure 3. The part of the string from P to the pulley is parallel to a line of greatest slope of the plane. The system is released from rest with the string taut.

For the motion before Q hits the ground,

(a) (i) show that the acceleration of Q is $\frac{2g}{5}$,

(ii) find the tension in the string.

(8)

On hitting the ground Q is immediately brought to rest by the impact.

(b) Find the speed of P at the instant when Q hits the ground.

(2)

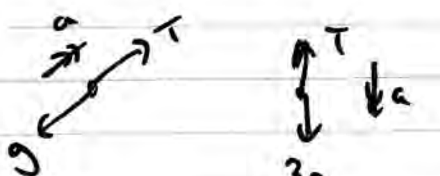
In its subsequent motion P does not reach the pulley.

(c) Find the total distance moved up the plane by P before it comes to instantaneous rest.

(4)

(d) Find the length of time between Q hitting the ground and P first coming to instantaneous rest.

(2)



$T - g = 2a$

$3g - T = 3a$ +

$2g = 5a$

i) $\therefore a = \frac{2}{5}g$

ii) $T - g = 2a$

$T = \frac{4}{5}g + g = \frac{9}{5}g$

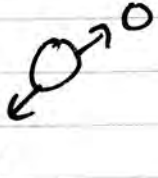
b) $s = 0.6$
$u = 0$
v
$a = \frac{2}{5}g$
t

$v^2 = u^2 + 2as$

$v^2 = 0 + \frac{12}{25}g$

$\therefore v = 2.17 ms^{-1}$

c)



$\nearrow$ $R(f) = ma$ $\quad -g = 2a \quad \therefore \quad a = -\frac{1}{2}g$

$s =$
$u = \sqrt{\frac{12g}{25}}$
$v = 0$
$a = -\frac{1}{2}g$
t

$v^2 = u^2 + 2as$

$0 = \frac{12}{25}g - gs$

$\therefore \cancel{g}s = \frac{12}{25}\cancel{g}$

$s = 0.48\text{m}$

total distance $= 1.08\text{m}$

$v = u + at$

$0 = \sqrt{\frac{12g}{25}} - \frac{1}{2}gt$

$\therefore \quad t = \frac{\sqrt{\frac{12g}{25}}}{\frac{1}{2}g} = 0.44\text{sec}$