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Surname	Other names
aymanzayedmannan	

Pearson Edexcel
International Advanced Level

Centre Number

Candidate Number

Mechanics M2

Advanced/Advanced Subsidiary

Thursday 28 January 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
WME02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. A car of mass 900 kg is travelling up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{25}$. The car is travelling at a constant speed of 14 m s^{-1} and the resistance to motion from non-gravitational forces has a constant magnitude of 800 N. The car takes 10 seconds to travel from A to B, where A and B are two points on the road.

(a) Find the work done by the engine of the car as the car travels from A to B.

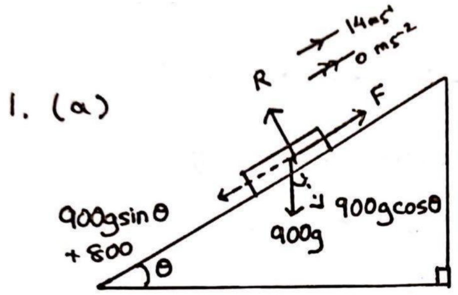
(4)

When the car is at B and travelling at a speed of 14 m s^{-1} the rate of working of the engine of the car is suddenly increased to P kW, resulting in an initial acceleration of the car of 0.7 m s^{-2}. The resistance to motion from non-gravitational forces still has a constant magnitude of 800 N.

(b) Find the value of P.

(4)

1. (a)



$R(\nearrow): F - 900g\sin\theta - 800 = 0$

$F = 900(9.8)(\frac{1}{25}) + 800$

$= 1152.8 \text{ N}$

speed = $\frac{\text{distance}}{\text{time taken}}$

$14 = \frac{\text{distance}}{10}$

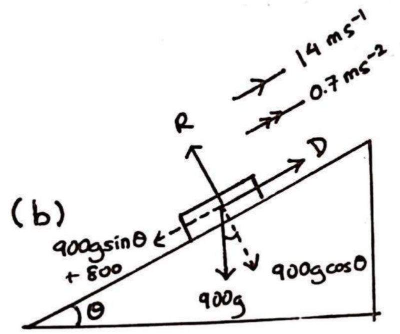
distance = 140 m

Work done by the engine = 1152.8×140

$= 161392$

≈ 160000 J (2 SF)

(b)



$R(\nearrow): D - 900g\sin\theta - 800 = 900(0.7)$

$D = 900(9.8)(\frac{1}{25}) + 800 + 630$

$D = 1782.8 \text{ N}$

$P = Dv$

$= 1782.8 \times 14$

$= 24959.2$

≈ 25 kW (2SF)

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2. A particle P of mass 0.7 kg is moving in a straight line on a smooth horizontal surface. The particle P collides with a particle Q of mass 1.2 kg which is at rest on the surface. Immediately before the collision the speed of P is $6\,\mathrm{m\,s^{-1}}$. Immediately after the collision both particles are moving in the same direction. The coefficient of restitution between the particles is e.

(a) Show that $e < \dfrac{7}{12}$

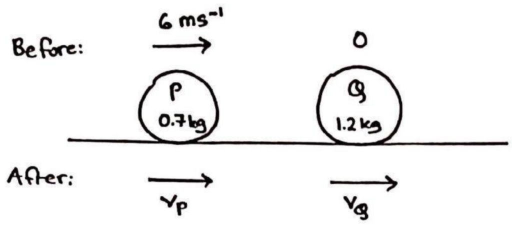
(7)

Given that $e = \dfrac{1}{4}$

(b) find the magnitude of the impulse exerted on Q in the collision.

(3)

2.



(a) PCLM (→):

$$6(0.7) + 0(1.2) = 0.7v_p + 1.2v_q$$

$$4.2 = 0.7v_p + 1.2v_q$$

$$\frac{21}{5} = \frac{7v_p}{10} + \frac{6v_q}{5}$$

$$42 = 7v_p + 12v_q \quad \text{——— (i)}$$

NEL (→):

$$e = \frac{v_q - v_p}{6}$$

$$6e = v_q - v_p \quad \text{——— (ii)}$$

$$\text{(i)} \Rightarrow 7v_p + 12v_q = 42$$

$$\text{(ii)} \times 12 \Rightarrow -12v_p + 12v_q = 72e$$

$$(-) \quad 19v_{\cancel{p}} = 42 - 72e$$

$$v_p = \frac{1}{19}(42 - 72e)$$

Both particles move in the same direction $\Leftrightarrow v_p > 0$

$$\frac{1}{19}(42 - 72e) > 0$$

$$42 - 72e > 0$$

$$-72e > -42$$

$$-e > -\frac{42}{72}$$

$$\therefore e < \frac{7}{12}$$

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Question 2 continued

(b) $v_P = \frac{1}{19}\left(42 - 72\left(\frac{1}{4}\right)\right) = \frac{24}{19} \text{ ms}^{-1}$

$I = mv - mu$

$= m(v-u)$

$= 0.7\left(\frac{24}{19} - 6\right)$

$= 0.7\left(-\frac{90}{19}\right)$

$= -\frac{63}{19}$

$\therefore |I| = \frac{63}{19} \text{ Ns} \approx 3.32 \text{ Ns (3 SF)}$

Q2

(Total 10 marks)

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3. At time t seconds ($t \geqslant 0$) a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = (6t^2 + 6t)\mathbf{i} + (3t^2 + 24)\mathbf{j}$$

When $t = 0$ the particle P is at the origin O. At time T seconds, P is at the point A and $\mathbf{v} = \lambda(\mathbf{i} + \mathbf{j})$, where λ is a constant.

Find

(a) the value of T,

(3)

(b) the acceleration of P as it passes through the point A,

(3)

(c) the distance OA.

(5)

(a) At Ts,

$\underset{\sim}{v} = \lambda(\underset{\sim}{i} + \underset{\sim}{j})$

$(6t^2+6t)\underset{\sim}{i} + (3t^2+24)\underset{\sim}{j} = \lambda\underset{\sim}{i} + \lambda\underset{\sim}{j}$

Comparing $\underset{\sim}{i}$:

$6t^2+6t = \lambda$

Comparing $\underset{\sim}{j}$:

$3t^2+24 = \lambda$

$\therefore \; 6t^2+6t = 3t^2+24$

$3t^2+6t-24=0$

$t^2+2t-8=0$

$t^2+4t-2t-8=0$

$t(t+4)-2(t+4)=0$

$(t-2)(t+4)=0$

$\therefore\; T=2$ or $T=-4$

$\therefore\; T=2 \;\; [\because t \geqslant 0]$

(b) $\underset{\sim}{a} = \dfrac{d\underset{\sim}{v}}{dt} = (12t+6)\underset{\sim}{i} + 6t\underset{\sim}{j}$ ms^{-2}

when $t=2$,

$\underset{\sim}{a} = (12(2)+6)\underset{\sim}{i} + 6(2)\underset{\sim}{j}$

$\therefore\; \underset{\sim}{a} = 30\underset{\sim}{i} + 12\underset{\sim}{j}$ ms^{-2}

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Question 3 continued

c) $\underset{\sim}{x} = \int x \, dt$

$= \int [(6t^2+6t)\underset{\sim}{i} + (3t^2+24)\underset{\sim}{j}] \, dt$

$= \left(\frac{6t^3}{3} + \frac{6t^2}{2}\right)\underset{\sim}{i} + \left(\frac{3t^3}{3} + \frac{24t}{1}\right)\underset{\sim}{j} + C$

$= (2t^3+3t^2)\underset{\sim}{i} + (t^3+24t)\underset{\sim}{j} + C$

when $t=0$,

$\underset{\sim}{x} = 0$

$C = 0$

$\underset{\sim}{x} = (2t^3+3t^2)\underset{\sim}{i} + (t^3+24t)\underset{\sim}{j}$ m

when $t=2$,

$\underset{\sim}{x} = (2(2)^3+3(2)^2)\underset{\sim}{i} + (2^3+24(2))\underset{\sim}{j}$

$= 28\underset{\sim}{i} + 56\underset{\sim}{j}$ m

$|OA| = \sqrt{28^2+56^2} = 28\sqrt{5} = 62.6099\ldots \approx 62.6$ m (3 SF)

Q3

(Total 11 marks)

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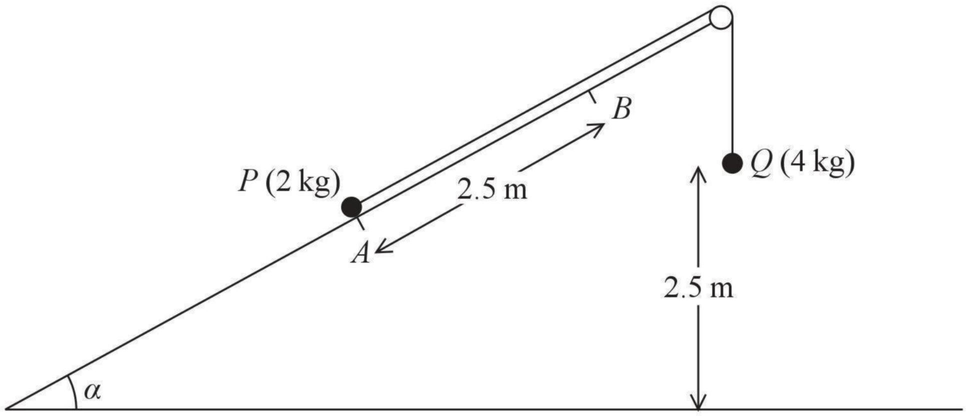

Figure 1

Two particles P and Q, of mass 2 kg and 4 kg respectively, are connected by a light inextensible string. Initially P is held at rest at the point A on a rough fixed plane inclined at α to the horizontal ground, where $\sin\alpha = \frac{3}{5}$. The string passes over a small smooth pulley fixed at the top of the plane. The particle Q hangs freely below the pulley and 2.5 m above the ground, as shown in Figure 1. The part of the string from P to the pulley lies along a line of greatest slope of the plane. The system is released from rest with the string taut. At the instant when Q hits the ground, P is at the point B on the plane. The coefficient of friction between P and the plane is $\frac{1}{4}$.

(a) Find the work done against friction as P moves from A to B.

(4)

(b) Find the total potential energy lost by the system as P moves from A to B.

(3)

(c) Find, using the work-energy principle, the speed of P as it passes through B.

(4)

(a) $R(\nwarrow)$: $R = 2g\cos\alpha$

$= 2g\left(\frac{4}{5}\right)$

$R = \frac{8g}{5}$ N

$F = \mu R$

$= \frac{1}{4} \times \frac{8g}{5}$

$F = \frac{2g}{5}$ N

Work done against friction $= \frac{2g}{5} \times 2.5$

$= g$

$= 9.8$ J

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Question 4 continued

(b) PE lost by Q = $4g(2.5)$

$= 10g$ J

PE gained by P = $2g(2.5 \times \sin\alpha)$

$= 2g \times \frac{5}{2} \times (\frac{3}{5})$

$= 3g$

PE lost by the system = PE lost by Q − PE gained by P

$= 10g - 3g$

$= 7g$

$= 68.6$ J

At the point when Q hits the ground, P and Q travel at the same speed

$KE_P = \frac{1}{2}(2)v^2$

$= v^2$

$KE_Q = \frac{1}{2}(4)v^2$

$= 2v^2$

KE gained by the system = $2v^2 + v^2 = 3v^2$

PE lost by the system = Work done against friction + KE gained by the system

$7g = g + 3v^2$

$6g = 3v^2$

$2g = v^2$

$v = \sqrt{(2g)} = 4.42718\ldots \approx 4.43$ m s^{-1} (3SF)

Q4

(Total 11 marks)

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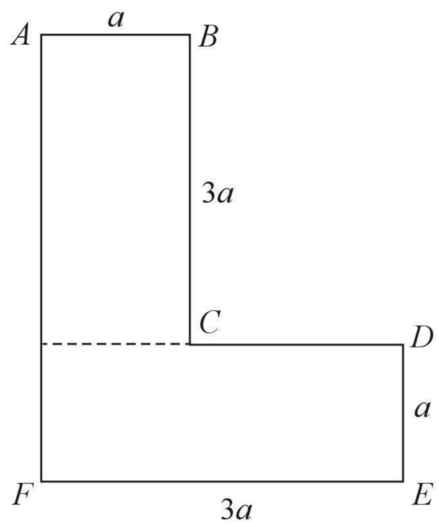

Figure 2

The uniform lamina $ABCDEF$, shown in Figure 2, consists of two identical rectangles with sides of length a and $3a$. The mass of the lamina is M. A particle of mass kM is attached to the lamina at E. The lamina, with the attached particle, is freely suspended from A and hangs in equilibrium with AF at an angle θ to the downward vertical.

Given that $\tan\theta = \frac{4}{7}$, find the value of k.

(10)

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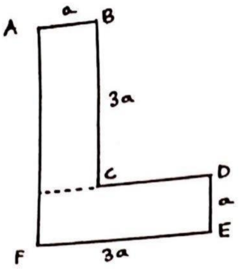
Question 5 continued

5.



let mass/area be m

Shape	Mass	Mass ratio	Distance of COM from AF	Distance of COM from FE
ABC	$3a^2m$	1	$a/2$	$5a/2$
FDE	$3a^2m$	1	$3a/2$	$a/2$
L	$6a^2m$	2	$\bar{x}$	$\bar{y}$

$M(F): \ 2\bar{x} = \frac{a}{2} + \frac{3a}{2}$

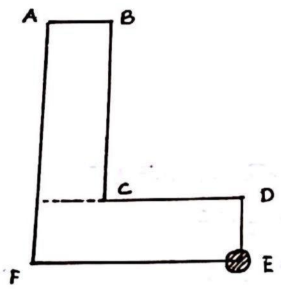
$= 2a$

$\bar{x} = a$

$M(F): \ 2\bar{y} = \frac{5a}{2} + \frac{a}{2}$

$\bar{y} = \frac{3a}{2}$

COM of the lamina is $(a, \frac{3a}{2})$.



Shape	Mass	Mass ratios	Distance of COM from AF	Distance of COM from FE
L	M	1	a	$3a/2$
particle	kM	k	3a	0
L with particle	$(1+k)M$	$k+1$	$\bar{x}_1$	$\bar{y}_1$

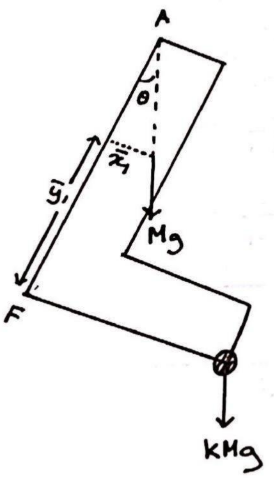
$M(F): \ (k+1)\bar{x}_1 = a + 3ak$

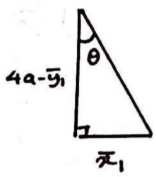
$= a(1+3k)$

$\bar{x}_1 = \frac{a(1+3k)}{k+1}$

$M(F): (k+1)\bar{y}_1 = \frac{3a}{2} + 0$

$\bar{y}_1 = \frac{3a}{2(k+1)}$





$\tan\theta = \frac{\bar{x}_1}{4a - \bar{y}_1}$

$= \frac{\frac{a(1+3k)}{k+1}}{4a - \frac{3a}{2(k+1)}}$

$= \frac{\frac{a(1+3k)}{k+1}}{\frac{8a(k+1) - 3a}{2(k+1)}}$

$= \frac{\cancel{a}(1+3k)}{\cancel{k+1}} \times \frac{2\cancel{(k+1)}}{\cancel{a}(8k+8-3)}$

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Question 5 continued

$$\tan\theta = \frac{2+6k}{8k+5}$$

$$\frac{4}{7} = \frac{2+6k}{8k+5}$$

$$32k + 20 = 14 + 42k$$

$$6 = 10k$$

$$k = \frac{6}{10}$$

$$\therefore k = \frac{3}{5}$$

Q5

(Total 10 marks)

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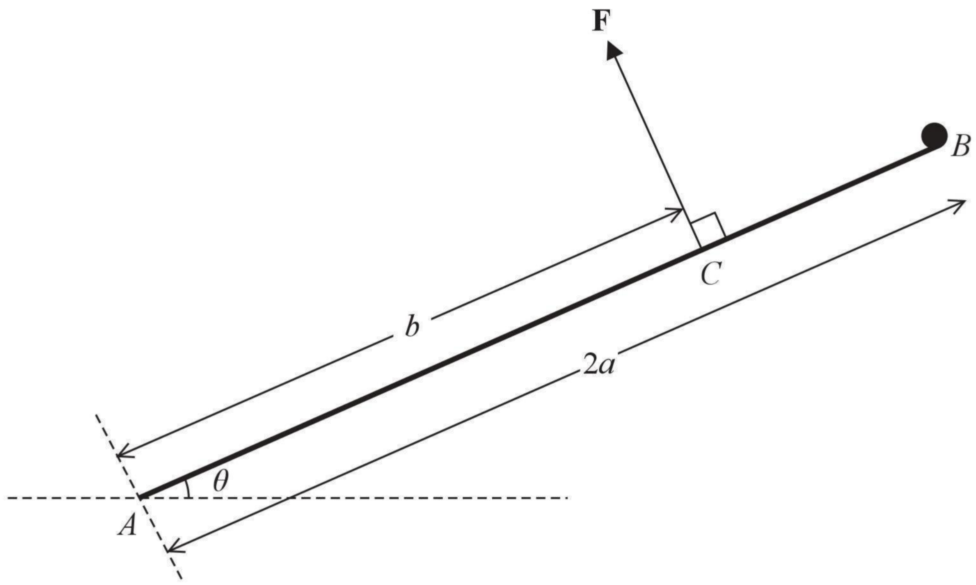

Figure 3

A uniform rod AB, of mass $3m$ and length $2a$, is freely hinged at A to a fixed point on horizontal ground. A particle of mass m is attached to the rod at the end B. The system is held in equilibrium by a force $\mathbf{F}$ acting at the point C, where $AC = b$. The rod makes an acute angle θ with the ground, as shown in Figure 3. The line of action of $\mathbf{F}$ is perpendicular to the rod and in the same vertical plane as the rod.

(a) Show that the magnitude of $\mathbf{F}$ is $\dfrac{5mga}{b}\cos\theta$

(4)

The force exerted on the rod by the hinge at A is $\mathbf{R}$, which acts upwards at an angle ϕ above the horizontal, where $\phi > \theta$.

(b) Find

(i) the component of $\mathbf{R}$ parallel to the rod, in terms of m, g and θ,

(ii) the component of $\mathbf{R}$ perpendicular to the rod, in terms of a, b, m, g and θ.

(5)

(c) Hence, or otherwise, find the range of possible values of b, giving your answer in terms of a.

(2)

(a) $M_{(A)}\underset{+ve}{\circlearrowleft}$: $bF = 3amg\cos\theta + 2a(mg\cos\theta)$

$bF = 5amg\cos\theta$

$\therefore\ F = \dfrac{5amg}{b}\cos\theta$

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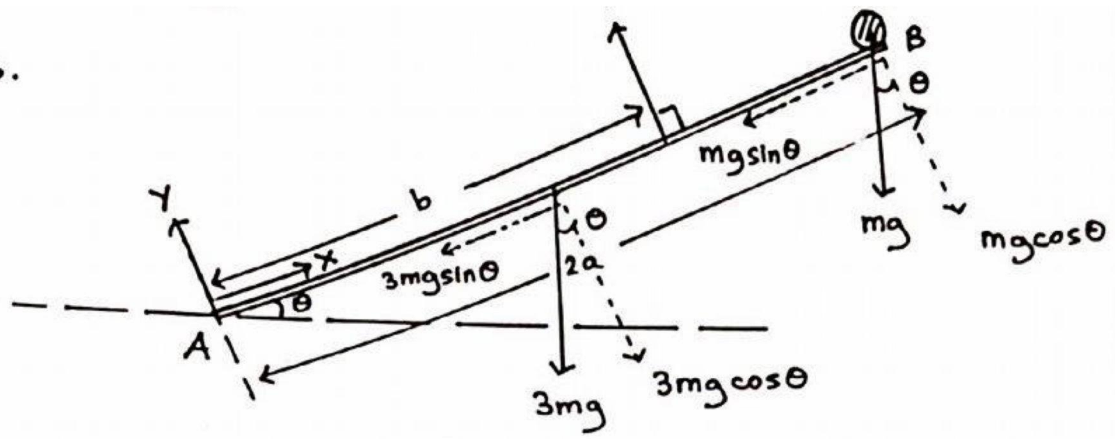


(b)(i) $R(\nearrow)$: $X - 3mg\sin\theta - mg\sin\theta = 0$

$X = 4mg\sin\theta$

(ii) $R(\nwarrow)$: $Y + F - 3mg\cos\theta - mg\cos\theta = 0$

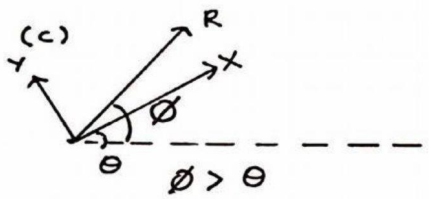
$Y = 4mg\cos\theta - F$

$= 4mg\cos\theta - \frac{5amg}{b}\cos\theta$

$= \frac{4bmg\cos\theta - 5amg\cos\theta}{b}$

$= mg\cos\theta\left(\frac{4b-5a}{b}\right)$

(c)



$\phi > \theta$

$\Rightarrow (\phi - \theta) > 0$

$\tan(\phi - \theta) > \tan 0$

$\tan(\phi - \theta) > 0$

$\frac{Y}{X} > 0$

$$\frac{\cancel{mg\cos\theta}\left(\frac{4b-5a}{\cancel{b}}\right)}{\frac{5a\cancel{mg}}{\cancel{b}}\cancel{\cos\theta}} > 0$$

$4b - 5a > 0$

$4b > 5a$

$b > \frac{5a}{4}$

$b \leq 2a$ [∵ b cannot be larger than the rod]

$\therefore \frac{5a}{4} < b \leq 2a$

Q6

(Total 11 marks)

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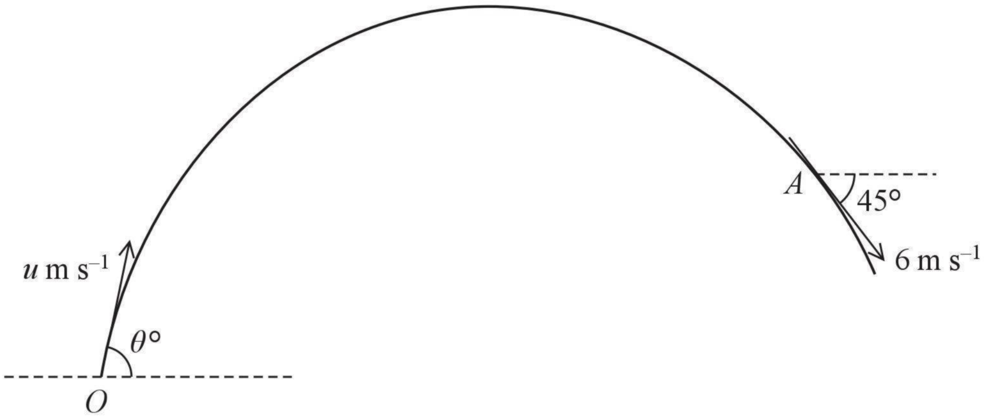

Figure 4

At time $t = 0$, a particle P of mass 0.7 kg is projected with speed u m s^{-1} from a fixed point O at an angle $\theta°$ to the horizontal. The particle moves freely under gravity. At time $t = 2$ seconds, P passes through the point A with speed 6 m s^{-1} and is moving downwards at 45° to the horizontal, as shown in Figure 4.

Find

(a) the value of θ, **(6)**

(b) the kinetic energy of P as it reaches the highest point of its path. **(3)**

For an interval of T seconds, the speed, v m s^{-1}, of P is such that $v \leqslant 6$

(c) Find the value of T. **(5)**

(a) Horizontal velocity is constant $\Leftrightarrow$ $u\cos\theta = 6\cos 45°$

$$u = \frac{6\cos 45°}{\cos\theta}$$

$R(\uparrow)$: $s =$
$u = u\sin\theta$
$v = -6\sin 45°$
$a = -g$
$t = 2$

$v = u + at$

$-6\sin 45° = u\sin\theta - g(2)$

$$-6\sin 45° = \left(\frac{6\cos 45°}{\cos\theta} \times \sin\theta\right) - 2g$$

$2g - 6\sin 45° = 6\cos 45° \tan\theta$

$$\tan\theta = \frac{2g - 6\sin 45°}{6\cos 45°}$$

$$\theta = \arctan\left(\frac{2g - 6\sin 45°}{6\cos 45°}\right)$$

$\theta = 74.556595...°$

$\therefore \theta \approx 74.6°$ (3 SF)

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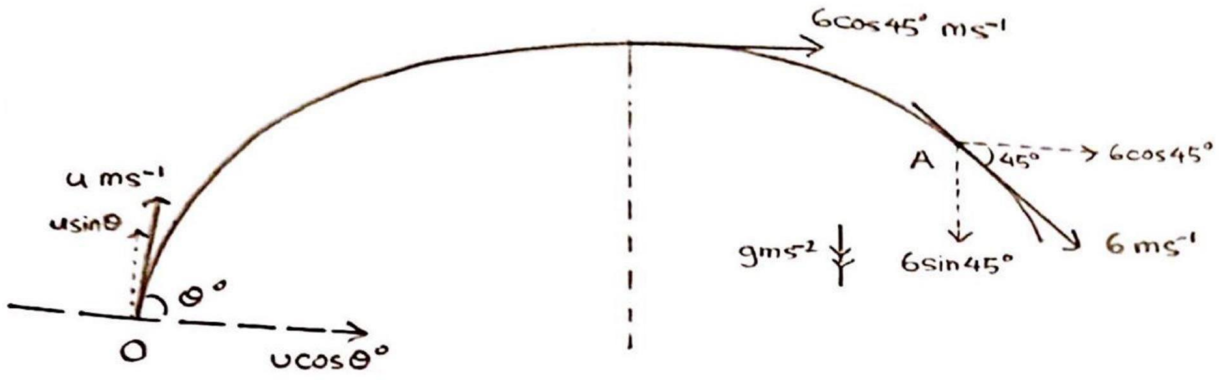
Question 7 continued

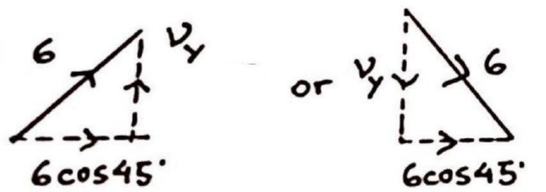


(b) At the highest point, the only component of velocity acting is the horizontal component.

KE at highest point = $\frac{1}{2}mv^2 = \frac{1}{2}(0.7)(6\cos 45°)^2$

$= 0.35\,(18)$

$= 6.3$ J

(c) 

$6^2 = v_y^2 + (6\cos 45°)^2$

$36 = v_y^2 + 18$

$18 = v_y^2$

$v_y = \pm\sqrt{18} = \pm 3\sqrt{2}$ ms⁻¹

When the ball moves at 6 ms⁻¹ for the first time,

$R(\uparrow)$: $s =$

$u = u\sin\theta = 15.35735931\ldots$

$v = 3\sqrt{2}$

$a = -g$ ms⁻²

$t = ?$

$v = u + at$

$3\sqrt{2} = (15.357\ldots) - gt$

$t = 1.134154962\ldots$ s

P 4 6 9 6 0 R A 0 2 7 2 8

Turn over

Question 7 continued

When the ball moves at $6\,ms^{-1}$ for the second time,

$R(\uparrow)$: $s=$

$u = 15.357359...$

$v = -3\sqrt{2}$

$a = -g$

$t = ?$

$v = u + at$

$-3\sqrt{2} = (15.357...) - gt$

$t = 2\,s$

$T = 2 - 1.134...$

$= 0.865845...$

$\therefore T \approx 0.866\,s$ (3 SF)

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Q7

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TOTAL FOR PAPER: 75 MARKS

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