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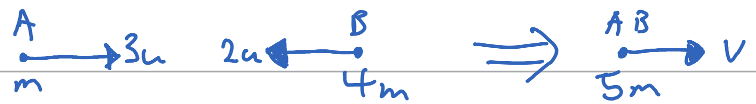
1. A railway truck A of mass m and a second railway truck B of mass $4m$ are moving in opposite directions on a smooth straight horizontal track when they collide directly. Immediately before the collision the speed of truck A is $3u$ and the speed of truck B is $2u$. In the collision the trucks join together. Modelling the trucks as particles, find

(a) the speed of A immediately after the collision, **(3)**

(b) the direction of motion of A immediately after the collision, **(1)**

(c) the magnitude of the impulse exerted by A on B in the collision. **(3)**



a) Cons. of momentum:

$\rightarrow \quad 3mu - 2u(4m) = 5mv$

$v = -u$

$\therefore$ Speed of A is u

b) A moves in the opposite direction to its initial motion

c) $\rightarrow \quad I = mv - mu$

$= 4m(-u - (-2u))$

$= 4mu$

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2.

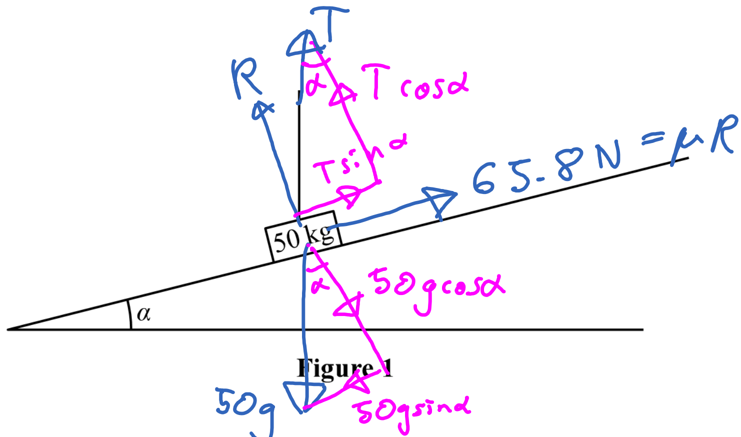
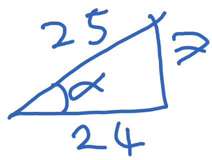


Figure 1

A block of mass 50 kg lies on a rough plane which is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{7}{24}$. The block is held at rest by a vertical rope, as shown in Figure 1, and is on the point of sliding down the plane. The block is modelled as a particle and the rope is modelled as a light inextensible string. Given that the friction force acting on the block has magnitude 65.8 N, find

(a) the tension in the rope,



(4)

(b) the coefficient of friction between the block and the plane.

(4)

∴ equilibrium

a) $\nearrow \quad 65.8 + T\sin\alpha - 50g\sin\alpha = 0$

$$T = \left[50g\left(\frac{7}{25}\right) - 65.8\right]\frac{25}{7}$$

$$= 255 \text{ N}$$

b) $\nwarrow \quad R + T\cos\alpha - 50g\cos\alpha = 0$

$$R = 50g\left(\frac{24}{25}\right) - 255\left(\frac{24}{25}\right)$$

$$= 225.6 \text{ N}$$

$F = \mu R$

$$\mu = \frac{F}{R} = \frac{65.8}{225.6} = 0.292$$

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3. [*In this question* **i** *and* **j** *are unit vectors directed due east and due north respectively*.]

A particle P is moving with constant velocity $(-6\mathbf{i} + 2\mathbf{j})$ m s^{-1}. At time $t = 0$, P passes through the point with position vector $(21\mathbf{i} + 5\mathbf{j})$ m, relative to a fixed origin O.

(a) Find the direction of motion of P, giving your answer as a bearing to the nearest degree.

(3)

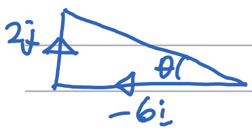
(b) Write down the position vector of P at time t seconds.

(1)

(c) Find the time at which P is north-west of O.

(3)

a)



$\tan\theta = \frac{2}{6}$

$\theta = 18.4°$

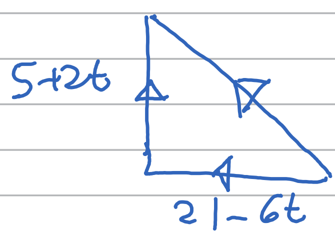
∴ Bearing is $270 + 18 = 288$

b) $\underline{r} = \underline{r}_0 + \underline{v}t$

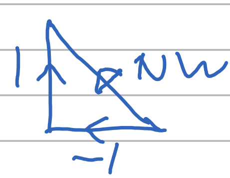
$= \begin{pmatrix} 21 \\ 5 \end{pmatrix} + t\begin{pmatrix} -6 \\ 2 \end{pmatrix}$

$= (21 - 6t)\underline{i} + (5 + 2t)\underline{j}$

c)



$\equiv$



$5 + 2t = -(21 - 6t)$

$4t = 26$

$t = 6.5$

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4. The points P and Q are at the same height h metres above horizontal ground. A small stone is dropped from rest from P. Half a second later a second small stone is thrown vertically downwards from Q with speed 7.35 m s^{-1}. Given that the stones hit the ground at the same time, find the value of h.

(7)

$P: s = h$ $\qquad$ $Q: s = h$

$u = 0$ $\qquad$ $u = 7.35 \text{ ms}^{-1}$

$v = x$ $\qquad$ $v = x$

$a = g$ $\qquad$ $a = g$

$t = T$ $\qquad$ $t = T - 0.5$

$[s = ut + \frac{1}{2}at^2]$ $\qquad$ $[s = ut + \frac{1}{2}at^2]$

$h = \frac{g}{2}T^2 \quad (1)$ $\qquad$ $h = 7.35(T-0.5) + \frac{g}{2}(T-0.5)^2$

$\Rightarrow \frac{g}{2}T^2 = 7.35T - \frac{7.35}{2} + \frac{gT^2}{2} - \frac{gT}{2} + \frac{g}{8}$

$2.45 = 2.45T$

$T = 1$

In (1): $h = \frac{g}{2}(1)^2 = 4.9\text{ m}$

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5.

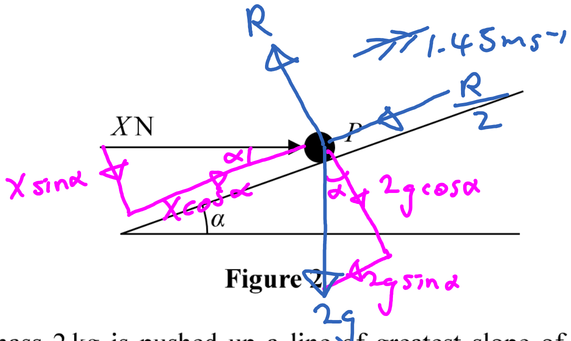
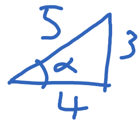


Figure 2

A particle P of mass 2 kg is pushed up a line of greatest slope of a rough plane by a horizontal force of magnitude X newtons, as shown in Figure 2. The force acts in the vertical plane which contains P and a line of greatest slope of the plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

The coefficient of friction between P and the plane is 0.5



Given that the acceleration of P is 1.45 m s^{-2}, find the value of X.

(10)

$\nwarrow$ equilibrium: $R - X\sin\alpha - 2g\cos\alpha = 0$

$$R = \frac{3X}{5} + 2g\left(\frac{4}{5}\right)$$

$\nearrow$ $[F = ma]$

$$X\cos\alpha - \frac{R}{2} - 2g\sin\alpha = 2(1.45)$$

$$\frac{4X}{5} - \frac{3X}{10} - \frac{4g}{5} - \frac{6g}{5} = 2.9$$

$$X = 45\text{ N}$$

Leave blank

6. A uniform rod AC, of weight W and length $3l$, rests horizontally on two supports, one at A and one at B, where $AB = 2l$. A particle of weight $2W$ is placed on the rod at a distance x from A. The rod remains horizontal and in equilibrium.

(a) Find the greatest possible value of x.

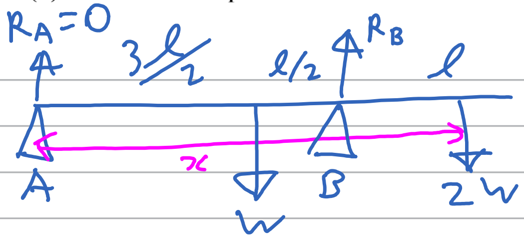
(5)

The magnitude of the reaction of the support at A is R. Due to a weakness in the support at A, the greatest possible value of R is $2W$,

(b) find the least possible value of x.

(5)

a)



At max x, rod is on limit of tilting about B

$\Rightarrow R_A = 0$

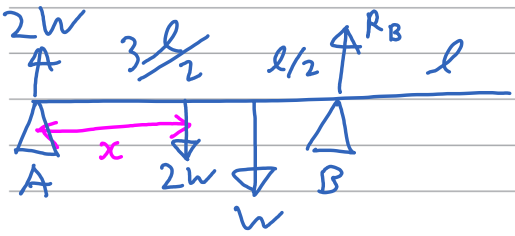
$$A\circlearrowleft \quad 2Wx + \frac{3l}{2}W - 2lR_B = 0$$

$$\uparrow \quad R_B = W + 2W$$

$$\Rightarrow 2Wx + \frac{3l}{2}W - 6Wl = 0$$

$$x = \frac{9l}{4}$$

b)



At min x, $R = 2W$

$$B\circlearrowright \quad \frac{lW}{2} + (2l - x)2W = 2l \cdot 2W$$

$$\frac{l}{2} + 4l - 2x = 4l$$

$$x = \frac{l}{4}$$

Leave blank

7. A train travels along a straight horizontal track between two stations A and B. The train starts from rest at A and moves with constant acceleration until it reaches its maximum speed of 108 km h^{-1}. The train then travels at this speed before it moves with constant deceleration coming to rest at B. The journey from A to B takes 8 minutes.

(a) Change 108 km h^{-1} into m s^{-1}.

(2)

(b) Sketch a speed-time graph for the motion of the train between the two stations A and B.

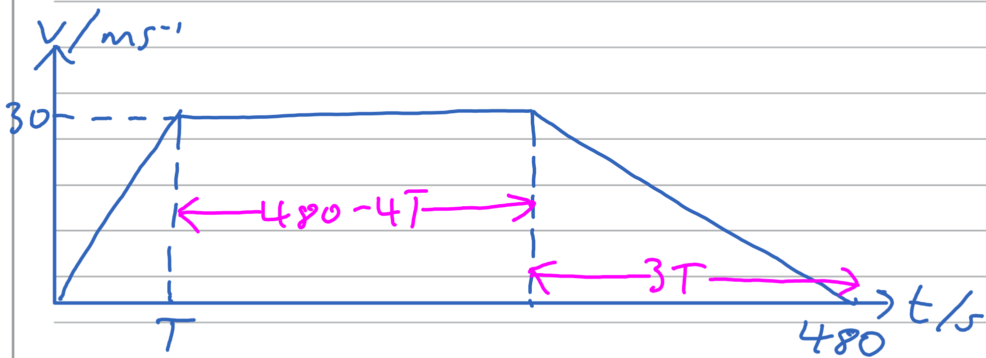
(2)

Given that the distance between the two stations is 12 km and that the time spent decelerating is three times the time spent accelerating,

(c) find the acceleration, in m s^{-2}, of the train.

(6)

a) $108 \text{ km h}^{-1} = \frac{108 \times 1000}{60 \times 60} \text{ ms}^{-1} = 30 \text{ ms}^{-1}$

b) 

c) $12000 = \frac{1}{2}(480 - 4T + 480)\,30$

$800 = 960 - 4T$

$T = 40\text{s}$

$a = \frac{\Delta v}{\Delta t} = \frac{30}{40} = 0.75 \text{ ms}^{-2}$

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8.

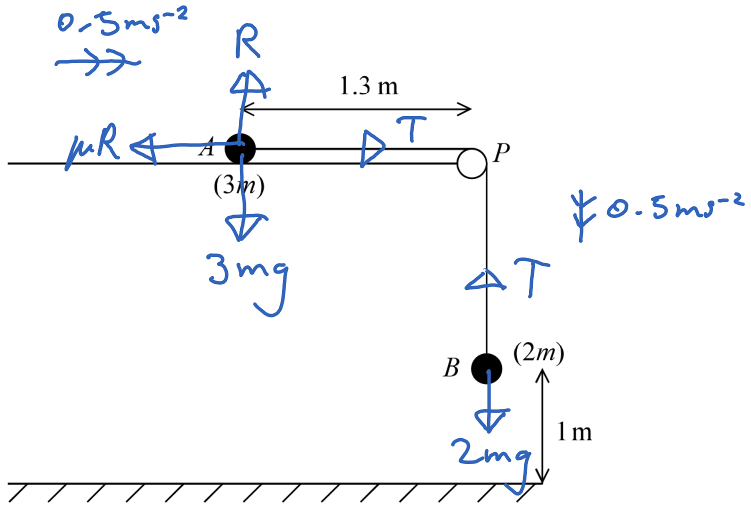

Figure 3

A particle A of mass $3m$ is held at rest on a rough horizontal table. The particle is attached to one end of a light inextensible string. The string passes over a small smooth pulley P which is fixed at the edge of the table. The other end of the string is attached to a particle B of mass $2m$, which hangs freely, vertically below P. The system is released from rest, with the string taut, when A is 1.3 m from P and B is 1 m above the horizontal floor, as shown in Figure 3.

Given that B hits the floor 2 s after release and does not rebound,

(a) find the acceleration of A during the first two seconds,
(2)

(b) find the coefficient of friction between A and the table,
(8)

(c) determine whether A reaches the pulley.
(6)

a) $\downarrow$ $s = 1\text{m}$ $\quad [s = ut + \frac{1}{2}at^2]$

$u = 0$

$v = x$ $\quad 1 = \frac{1}{2}a(2)^2$

$a = ?$

$t = 2\text{s}$ $\quad a = 0.5\text{ms}^{-2}$

b) $B: [F = ma] \downarrow$ $\quad$ A: $\uparrow$ of equilibrium

$2mg - T = 2m(0.5)$ $\quad R = 3mg$

$T = m(2g - 1)$

Leave blank

Question 8 continued

A: $\rightarrow [F = ma]$

$T - \mu R = 3m(0.5)$

$m(2g-1) - \mu(3mg) = 1.5m$

$\mu = \dfrac{2g - 2.5}{3g}$

$= 0.582$

c) When A is 0.3m from the pulley, the string becomes slack $\Rightarrow T = 0$

$\uparrow R = 3mg \qquad \rightarrow [F = ma]$

$-\mu R = 3ma$

$a = \dfrac{-0.582(3mg)}{3m}$

$= -5.70\,ms^{-2}$

Find speed of A when string becomes slack:

$s = 1m,\ u = 0,\ v = ?,\ a = 0.5ms^{-2},\ t = 2s$

$v = u + at = 0.5 \times 2 = 1ms^{-1}$

Find displacement after string becomes slack:

$[v^2 = u^2 + 2as] \rightarrow$

$s = \dfrac{0 - 1^2}{2(-5.7)} = 0.088m \qquad \therefore$ A does not reach the pulley