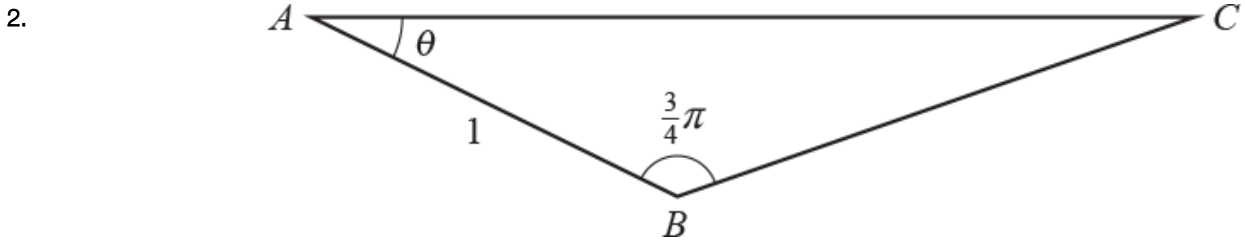


1. For a small angle θ , where θ is in radians, show that $1 + \cos \theta - 3 \cos^2 \theta \approx -1 + \frac{5}{2} \theta^2$.

[4]



The diagram shows triangle ABC , in which angle $A = \theta$ radians, angle $B = \frac{3}{4}\pi$ radians and $AB = 1$ unit.

- (a) Use the sine rule to show that $AC = \frac{1}{\cos \theta - \sin \theta}$.

[3]

- (b) Given that θ is a small angle, use the result in part (a) to show that

$$AC \approx 1 + p\theta + q\theta^2,$$

where p and q are constants to be determined.

[4]

3. Use small angle approximations to estimate the solution of the equation $\frac{\cos \frac{1}{2}\theta}{1 + \sin \theta} = 0.825$, if θ is small enough to neglect terms in θ^3 or above.

[4]

END OF QUESTION paper

Mark scheme

Question	Answer/Indicative content	Marks	Guidance	
1	<p>When θ is small $1 + \cos\theta - 3\cos^2\theta$ $\approx 1 + \left(1 - \frac{1}{2}\theta^2\right) - 3\left(1 - \frac{1}{2}\theta^2\right)^2$</p> $= 1 + \left(1 - \frac{1}{2}\theta^2\right) - 3\left(1 - \theta^2 + \frac{1}{4}\theta^4\right)$ $= 1 + 1 - \frac{1}{2}\theta^2 - 3 + 3\theta^2 - \frac{3}{4}\theta^4$ <p>Since θ is small, we can neglect the higher order terms</p> <p>so $1 + \cos\theta - 3\cos^2\theta \approx -1 + \frac{5}{2}\theta^2$ as required</p>	<p>M1(AO 1.1a)</p> <p>M1(AO1.1)</p> <p>E1(AO2.5)</p> <p>E1(AO2.1)</p> <p>[4]</p>	<p>Attempt to use cos $\approx 1 - \frac{1}{2}\theta^2$ or $= 1 + \left(1 - \frac{1}{2}\theta^2 + \dots\right)$ $- 3\left(1 - \frac{1}{2}\theta^2 + \dots\right)^2$ Multiply out</p> <p>For explanation of loss of θ^4 term and consistent use of notation throughout (Working need not be fully correct) AG Clearly obtained www Condone θ^4 term missing without explanation and inconsistent notation</p>	<p>OR</p> <p>M1 Attempt to use $\cos\theta \approx 1 - \frac{1}{2}\theta^2$</p> <p>M1 use trigonometric identity $1 + \cos\theta - 3\cos^2\theta$ $= 1 + \cos\theta - \frac{3}{2} - \frac{3}{2}\cos 2\theta$</p> <p>E1 For showing clearly which identity has been used and consistent use of notation throughout E1 AG Clearly obtained www Condone inconsistent notation</p>
	Total	4		
2	a $\frac{AC}{\sin \frac{3}{4}\pi} = \frac{1}{\sin\left(\pi - \frac{3}{4}\pi - \theta\right)}$	M1(AO2.1)	Attempt sine rule	

		$AC = \frac{\sin \frac{3}{4}\pi}{\sin \frac{1}{4}\pi \cos \theta - \cos \frac{1}{4}\pi \sin \theta}$ $\sin \frac{3}{4}\pi = \sin \frac{1}{4}\pi = \cos \frac{1}{4}\pi \text{ so } AC = \frac{1}{\cos \theta - \sin \theta}$	M1(AO2.1) E1(AO2.2a) [3]	<p>For expanding $\sin\left(\frac{1}{4}\pi - \theta\right)$</p> <p>AG, so must show sufficient working; e.g. stating $\sin \frac{3}{4}\pi = \sin \frac{1}{4}\pi = \cos \frac{1}{4}\pi$</p> <p>or using $\frac{1}{\sqrt{2}}$ oe for each</p>	
	b	$AC = \left(1 + \left(-\theta - \frac{1}{2}\theta^2\right)\right)^{-1}$ $AC = 1 + (-1)\left(-\theta - \frac{1}{2}\theta^2\right) + \frac{(-1)(-2)}{2}\left(-\theta - \frac{1}{2}\theta^2\right)^2 + \dots$ $AC \approx 1 + \theta + \frac{3}{2}\theta^2$	B1(AO1.1) M1(AO3.1a) A1(AO1.1) A1(AO1.1) [4]	<p>Using both small angle approximations</p> <p>Attempt binomial expansion of AC, with at least the first two terms present</p> <p>$p = 1$</p> <p>$q = \frac{3}{2}$</p>	
		Total	7		

3		$\frac{1 - \frac{1}{8}\theta^2}{1 + \theta} = 0.825$ $0.125\theta^2 + 0.825\theta - 0.175 = 0$ $\theta = 0.206 \text{ or } -6.81 \text{ (3 sf)}$ <p>Discard -6.81 as not small. $\theta = 0.206$ (3 sf)</p>	M1 (AO1.1a) A1 (AO1.1) A1 (AO1.1) A1 (AO2.3) [4]	BC Statement needed and $\theta = 0.206$ alone	
		Total	4		