

1. i. Show that the equation  $2 \sin x = \frac{4 \cos x - 1}{\tan x}$  can be expressed in the form  $6 \cos^2 x - \cos x - 2 = 0$ . [3]

- ii. Hence solve the equation  $2 \sin x = \frac{4 \cos x - 1}{\tan x}$ , giving all values of  $x$  between  $0^\circ$  and  $360^\circ$ . [4]

2. Solve each of the following equations, for  $0^\circ \leq x \leq 360^\circ$ .

i.  $\sin \frac{1}{2}x = 0.8$  [3]

ii.  $\sin x = 3 \cos x$  [3]

3. i. Show that the equation

$$\sin x - \cos x = \frac{6 \cos x}{\tan x}$$

can be expressed in the form

$$\tan^2 x - \tan x - 6 = 0.$$

[2]

- ii. Hence solve the equation  $\sin x - \cos x = \frac{6 \cos x}{\tan x}$  for  $0^\circ \leq x \leq 360^\circ$ .

[4]

4. In this question you must show detailed reasoning.

Solve the equation  $2\cos^2 x = 2 - \sin x$  for  $0^\circ \leq x \leq 180^\circ$ . [5]

5. In this question you must show detailed reasoning.

Solve the equation  $3 \sin^2 \theta - 2 \cos \theta - 2 = 0$  for  $0^\circ \leq \theta \leq 360^\circ$ . [5]

6. In this question you must show detailed reasoning.

Given that  $5\sin 2x = 3\cos x$ , where  $0^\circ < x < 90^\circ$ , find the exact value of  $\sin x$ . [4]

7. In this question you must show detailed reasoning.

Solve the equation  $\tan 2x = -\sqrt{3}$  for  $0^\circ \leq x < 360^\circ$ . [5]

8. The cubic polynomial  $f(x)$  is defined by  $f(x) = 4x^3 + 9x - 5$ .

(a) Show that  $(2x - 1)$  is a factor of  $f(x)$  and hence express  $f(x)$  as the product of a linear factor and a quadratic factor. [4]

(b) (a) Show that the equation

$$4 \sin 2\theta \cos 2\theta + \frac{5}{\cos 2\theta} = 13 \tan 2\theta$$

can be expressed in the form

$$4 \sin^3 2\theta + 9 \sin 2\theta - 5 = 0. [4]$$

(b) Hence solve the equation

$$4 \sin 2\theta \cos 2\theta + \frac{5}{\cos 2\theta} = 13 \tan 2\theta$$

for  $0 \leq \theta \leq 2\pi$ . Give each answer in an exact form. [4]

9. (a) Show that the equation

$$2 \sin x \tan x = \cos x + 5$$

can be expressed in the form

$$3 \cos^2 x + 5 \cos x - 2 = 0. \quad [3]$$

(b) Hence solve the equation

$$2 \sin 2\theta \tan 2\theta = \cos 2\theta + 5,$$

giving all values of  $\theta$  between  $0^\circ$  and  $180^\circ$ , correct to 1 decimal place. [5]

10. (a) Solve the equation  $\sin^2 \theta = 0.25$  for  $0^\circ \leq \theta < 360^\circ$ . [3]

(b) In this question you must show detailed reasoning.

Solve the equation  $\tan 3\phi = \sqrt{3}$  for  $0^\circ \leq \phi < 90^\circ$ . [3]

11. (a) Given that  $\sqrt{2 \sin^2 \theta + \cos \theta} = 2 \cos \theta$ , show that  $6 \cos^2 \theta - \cos \theta - 2 = 0$ . [2]

(b) In this question you must show detailed reasoning. [3]

Solve the equation

$$6 \cos^2 \theta - \cos \theta - 2 = 0,$$

giving all values of  $\theta$  between  $0^\circ$  and  $360^\circ$  correct to 1 decimal place. [4]

(c) Explain why not all the solutions from part (b) are solutions of the equation [1]

$$\sqrt{2 \sin^2 \theta + \cos \theta} = 2 \cos \theta.$$

END OF QUESTION paper

# Mark scheme

Question	Answer/Indicative content	Marks	Part marks and guidance	
1	$2\sin x \frac{\sin x}{\cos x} = 4\cos x - 1$ $2\sin^2 x = 4\cos^2 x - \cos x$ $2 - 2\cos^2 x = 4\cos^2 x - \cos x$ $6\cos^2 x - \cos x - 2 = 0 \quad \mathbf{AG}$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Use <math>\tan x = \frac{\sin x}{\cos x}</math> and rearrange to a form not involving fractions</p> <p>Use <math>\sin^2 x = 1 - \cos^2 x</math></p> <p>Obtain <math>6\cos^2 x - \cos x - 2 = 0</math> with no errors seen</p>	<p>Must be used and not just stated</p> <p>Must multiply all terms by <math>\cos x</math> so <math>4\cos^2 x - 1</math> is M0, but allow M1 for <math>\cos x(4\cos x - 1)</math> even if subsequent errors</p> <p>Must be used and not just stated</p> <p>Must be used correctly, so M0 for <math>1 - 2\cos^2 x</math></p> <p>Not dependent on previous M mark, so M0 M1 possible</p> <p>Must be attempting quadratic in <math>\cos x</math> so M0 for <math>\cos^2 x = 1 - \sin^2 x</math></p> <p>Must be equation ie = 0</p> <p>Allow poor notation (eg <math>\cos</math> not <math>\cos x</math>, or <math>\tan x = \frac{\sin}{\cos}(x)</math>) as long as final answer is correct</p> <p><b>Examiner's Comments</b></p> <p>Most candidates could quote both of the required identities and then attempt to use them. Whilst <math>\sin^2 x = 1 - \cos^2 x</math> was usually used correctly, the use of <math>\tan x</math> caused more problems as candidates were expected to also deal with the fraction to gain the method mark and a number struggled to do so. Some candidates used poor notation, such as omitting the <math>x</math> from their trigonometric ratios, and others spoiled an otherwise</p>

					correct solution by failing to give an equation as their final answer.
	ii	$(3\cos x - 2)(2\cos x + 1) = 0$ $\cos x = 2/3, \cos x = -1/2$ $x = 48.2^\circ, 312^\circ, 120^\circ, 240^\circ$	M1	Attempt to solve quadratic in $\cos x$	<p>This M mark is just for solving a 3 term quadratic (see guidance sheet for acceptable methods)</p> <p>Condone any substitution used, including <math>x = \cos x</math></p> <p>Attempt <math>\cos^{-1}</math> of at least one of their roots</p> <p>Allow for just stating <math>\cos^{-1}(\text{their root})</math> inc if <math> \cos x  &gt; 1</math></p> <p>Not dependent so M0 M1 possible</p> <p>If going straight from <math>\cos x = k</math> to <math>x = \dots</math> then award M1 only if their angle is consistent with their <math>k</math></p> <p>Allow 3sf or better</p> <p>Must come from correct solution of quadratic – ie correct factorisation or correct substitution into formula so A0 if two correct roots from eg <math>(3\cos x + 2)(2\cos x + 1) = 0</math></p> <p>Allow radian equivs – 0.841, 5.44, <math>2\pi/3</math> or 2.09, <math>4\pi/3</math> or 4.19</p> <p>Must now be in degrees</p> <p><b>SR</b> If no working shown then allow <b>B1</b> for 2 correct angles (poss in rads) or <b>B2</b> for 4 correct angles, no extras</p> <p><b>Examiner's Comments</b></p> <p>This part of the question was done very well by the majority of the candidates, who were able to identify the fact that</p>
	ii		M1	Attempt to find $x$ from root(s) of quadratic	
	ii		A1	Obtain at least 2 correct angles	
	ii		A1	Obtain all 4 correct angles, with no extra in given range	

					the given equation was a quadratic in $\cos x$ and attempt an appropriate method to solve it. The four required roots then usually followed, though some candidates struggled to find the secondary angles with $318.2^\circ$ being a common wrong answer. Others lost marks by discarding the negative root to the quadratic, failing to realise that this would also lead to valid solutions.
<b>Total</b>			<b>7</b>		
2	i	$\frac{1}{2}x = 53.1^\circ, 126.9^\circ$	B1	Obtain $106^\circ$ , or better	Allow answers in the range [106.2, 106.3] Ignore any other solutions for this mark Must be in degrees, so 1.85 rad is B0
	i	$x = 106^\circ, 254^\circ$	M1	Attempt correct solution method to find second angle  Obtain $254^\circ$ , or better	Could be $2(180^\circ - \text{their } 53.1^\circ)$ or $(360^\circ - \text{their } 106^\circ)$ Allow valid method in radians, but M0 for eg $(360 - 1.85)$
	i		A1	<b>Examiner's Comments</b>  Most candidates were able to correctly find the first angle though a few halved rather than doubled the result of $\sin^{-1} 0.8$ . Finding the second angle proved more challenging with the most common error being to simply subtract their first answer from $180^\circ$ . The fact that this resulted in a second angle that was smaller than the first did not seem to deter them. Whilst some candidates were able to use the symmetry of the $\sin \frac{1}{2}x$ graph to find the second angle, the more successful method was to find the possible solutions for $\frac{1}{2}x$ from the $\sin x$ graph and then double all the solutions.	Allow answers in the range [253.7°, 254°] A0 if in radians (4.43) A0 if extra incorrect solutions in range  <b>SR</b> If no working shown then allow B1 for $106^\circ$ and B2 for $254^\circ$ (max B2 if additional incorrect angles)

		ii	$\tan x = 3$	B1	State $\tan x = 3$	Allow B1 for correct equation even if no, or an incorrect, attempt to solve Give BOD on notation eg $\sin/\cos(x)$ as long as correct equation is seen or implied at some stage
		ii	$x = 71.6^\circ, 252^\circ$	M1	Attempt to solve $\tan x = k$	Not dep on B1, so could gain M1 for solving eg $\tan x = 1/3$ Could be implied by a correct solution  A0 if extra incorrect solutions in range
		ii		A1	Obtain $71.6^\circ$ and $252^\circ$ , or better  <b>Examiner's Comments</b>  This part of the question was better attempted, and most candidates scored full marks with ease. A few struggled to find the second angle, or lost the final mark through a lack of precision when rounding. Some candidates made life difficult for themselves by attempting to square both sides and use $\sin^2 x + \cos^2 x \equiv 1$ , but this was very rarely done correctly. Potential pitfalls included forgetting to square the coefficient of 3, omitting to use both the positive and negative square roots and finally realising that only two of the four solutions were valid. At least this was a valid method, which could not be said for those candidates who started with $\sin x + \cos x = 1$ .	<b>Alt method:</b> <b>B1</b> Obtain $10\sin^2 x = 9$ or $10\cos^2 x = 1$ <b>M1</b> Attempt to solve $\sin^2 x = k$ or $\cos^2 x = k$ (allow M1 if just the positive square root used) <b>A1</b> Obtain $71.6^\circ$ and $252^\circ$ , with no extra incorrect solutions in range  <b>SR</b> If no working shown at all then allow B1 for each correct angle (max B1 if additional incorrect angles), but allow full credit if $\tan x = 3$ seen first
		<b>Total</b>		<b>6</b>		
3		i	$\tan x(\sin x - \cos x) = 6 \cos x$ $\tan x(\frac{\sin x}{\cos x} - 1) = 6$ $\tan x(\tan x - 1) = 6$	M1	Use $\tan x = \frac{\sin x}{\cos x}$ correctly once	Must be used clearly at least once – either explicitly or by writing eg 'divide by $\cos x$ ' at side of solution Allow M1 for any equiv eg $\sin x = \cos x \tan x$ Allow poor notation eg writing just $\tan$ rather than $\tan x$
		i	$\tan^2 x - \tan x = 6$ $\tan^2 x - \tan x - 6 = 0$ <b>AG</b>	A1	Obtain $\tan^2 x - \tan x - 6 = 0$	Correct equation in given form, including $= 0$

				<p><b>Examiner's Comments</b></p> <p>A variety of methods were seen for this proof, some more efficient than others. Most candidates did get there in the end, but full credit was only given if the correct notation had been used throughout. Candidates must also ensure that each step is clearly and convincingly detailed when a proof has been requested.</p>	<p>Correct notation throughout so A0 if eg <math>\tan</math> rather than <math>\tan x</math> seen in solution</p>
	ii	$(\tan x - 3)(\tan x + 2) = 0$ $\tan x = 3, \tan x = -2$	M1	Attempt to solve quadratic in $\tan x$	<p>This M mark is just for solving a 3 term quadratic (see guidance sheet for acceptable methods)</p> <p>Condone any substitution used, inc <math>x = \tan x</math></p> <p>Attempt <math>\tan^{-1}k</math> at least once</p> <p>Not dependent on previous mark so M0M1 possible</p> <p>If going straight from <math>\tan x = k</math> to <math>x = \dots</math>, then award M1 only if their angle is consistent with their <math>k</math></p> <p>Allow 3sf or better</p> <p>Must come from a correct method to solve the quadratic (as far as correct factorisation or substitution into formula)</p> <p>Allow radian equivs ie 1.25 / 4.39 / 2.03 / 5.18</p>
	ii	$x = \tan^{-1}(3), x = \tan^{-1}(-2)$	M1	Attempt to solve $\tan x = k$ at least once	<p>Allow 3sf or better</p> <p>Must now all be in degrees</p> <p>Allow 3sf or better</p> <p>A0 if other incorrect solutions in range <math>0^\circ - 360^\circ</math> (but ignore any outside this range)</p>
	ii	$x = 71.6^\circ, 252^\circ, 117^\circ, 297^\circ$	A1	Obtain two correct solutions	<p>Allow 3sf or better</p> <p>Must now all be in degrees</p> <p>Allow 3sf or better</p> <p>A0 if other incorrect solutions in range <math>0^\circ - 360^\circ</math> (but ignore any outside this range)</p>
	ii		A1	<p>Obtain all 4 correct solutions, and no others in range</p> <p><b>Examiner's Comments</b></p> <p>This question was generally very well done, and many candidates gained full marks on this question. The most common error was to completely discount the solution resulting from <math>\tan^{-1}(-2)</math> as it resulted in a negative angle rather than appreciating it would still generate other angles within the given range. It was also disappointing to see candidates with a correct method failing to gain full marks due to rounding errors. As in previous questions involving</p>	<p><b>SR</b> If no working shown then allow <b>B1</b> for each correct solution (max of <b>B3</b> if in radians, or if extra solns in range).</p>



				trigonometry, some candidates did not ensure their calculator was in the correct mode before proceeding. Angles given in radians could gain some credit, but candidates did not actually consider which measure they were using so the typical error was $\tan^{-1}(3) = 1.25$ and hence 189.25.	
		<b>Total</b>	<b>6</b>		
4		<p>DR</p> $2(1 - \sin^2 x) = 2 - \sin x$ $2\sin^2 x - \sin x = 0$ $\sin x(2\sin x - 1) = 0$ $\sin x = \frac{1}{2} \text{ so } x = 30 \text{ or } x = 150$ $\sin x = 0 \text{ so } x = 0 \text{ or } x = 180$	<p>M1(AO3.1a)</p> <p>A1(AO1.1)</p> <p>M1(AO1.1a)</p> <p>A1(AO1.1)</p> <p>A1(AO1.1)</p> <p>[5]</p>	<p>Use <math>\cos^2 x = 1 - \sin^2 x</math> and simplify</p> <p>Obtain <math>2\sin^2 x - 1\sin x = 0</math></p> <p>Attempt to solve a 2 term quadratic in <math>\sin x</math> and use correct order of operations to obtain <math>x</math></p> <p>Both values are required</p> <p>Both values are required</p>	<p>One step of simplification must be seen</p> <p>Use any valid method Must be seen</p>
		<b>Total</b>	<b>5</b>		
5		<p>DR</p> $3(1 - \cos^2 \theta) - 2\cos \theta - 2 = 0$ $3\cos^2 \theta + 2\cos \theta - 1 = 0$	<p>M1(AO3.1a)</p> <p>A1(AO1.1)</p> <p>M1(AO1.1a)</p>	<p>Attempt to use <math>\sin^2 \theta = 1 - \cos^2 \theta</math></p>	

		$(3\cos\theta - 1)(\cos\theta + 1) = 0$ $\cos\theta = \frac{1}{3} \quad \cos\theta = -1$ $\theta = 70.5^\circ, 289^\circ, 180^\circ$	A1(AO2.2a) A1(AO1.2) [5]	Obtain correct equation  Attempt to solve quadratic  Obtain at least two correct angles  Obtain all 3 angles, and no others	Factorise or BC	
		<b>Total</b>	<b>5</b>			
6		<b>DR</b> $5\sin 2x = 3\cos x \Rightarrow 10\sin x \cos x = 3\cos x$ $\cos x(10\sin x - 3) = 0$ $\cos x \neq 0 \text{ for } 0^\circ < x < 90^\circ$ so $\sin x = \frac{3}{10}$	B1(AO 1.1)  M1(AO1.1a) E1(AO2.1) A1(AO1.1) [4]	Use $\sin 2x = 2\sin x \cos x$ to obtain correct identity  Attempt to factorise	<b>SC2</b> For use of identity followed by cancelling $\cos x$ , leading to $\sin x = \frac{3}{10}$ .	
		<b>Total</b>	<b>4</b>			
7		<b>DR</b> $2x = -60$	M1(AO1.1) M1(AO2.1)			

		$2x = 180 - 60$ or $360 - 60$  $x = 60$ or $150$  $2x = 120 + 360$ or $300 + 360$  $x = 60$ or $150$ or $240$ or $330$	OR $2x = 120$ OR $2x = 300$  $2x = 120$ or $300$  $x = 60$ or $150$  $x = 60 + 180$ or $150 + 180$  $x = 60$ or $150$ or $240$ or $330$	A1(AO1.1) M1(AO2.1)  A1(AO1.1)  [5]	$2x = -60$ or $x = -30^\circ$ $x = -30 + 90$ or $-30 + 2 \times 90$  $x = 60$ or $150$  $x = -30 + 3 \times 90$ or $-30 + 4 \times 90$	one value enough for M1  both  both  both  all four
		Total		5		
8	i	$f(1/2) = 1/2 + 9/2 - 5 = 0$ $f(x) = (2x - 1)(2x^2 + x + 5)$		B1   M1	Confirm $f(1/2) = 0$ , with detail shown  Attempt complete division or equiv	$4(1/2)^3 + 9(1/2) - 5 = 0$ is sufficient B0 for just $f(1/2) = 0$ No conclusion needed If using division to justify then must draw attention to the zero remainder  Must be dividing by $(2x - 1)$ Must be complete method - ie all 3 terms attempted Long division - must subtract lower line (allow one slip) Inspection - expansion must give at least three correct terms of the cubic Coefficient matching - must be valid attempt at all

coeffs of quadratic, considering all relevant terms each time  
 Synthetic division - must be using 0.5 (not  $-0.5$ ) and adding within each column (allow one slip); expect to see

$$\begin{array}{r|rrrr}
 0.5 & 4 & 0 & 9 & -5 \\
 & & 2 & 1 & \\
 \hline
 & 4 & 2 & 10 & 
 \end{array}$$

Allow  $4x^2 + 2x + 10$  from dividing by  $x - \frac{1}{2}$

Must be written as a product  
 Allow  $(x - \frac{1}{2})$   
 ( $4x^2 + 2x + 10$ )

ISW any attempt to write as 3 linear factors, or to find roots

A1

A1

[4]

Obtain correct quotient

Obtain  
 $(2x - 1)(2x^2 + x + 5)$

**Examiner's Comments**

This part of the question was very well answered, with the majority of the candidates able to produce a correct product. Algebraic long division was the most common approach and candidates coped well with the lack of a quadratic term. Coefficient matching was less common, but was equally successful. Most candidates made explicit use of the factor theorem to show that  $(2x - 1)$  was a factor, and this invariably had sufficient detail to be convincing. If using algebraic long division it was not sufficient to simply have a 0 on the bottom line; attention had to be drawn to the lack of remainder for the mark to be awarded.

$$4 \sin 2\theta \cos 2\theta + \frac{5}{\cos 2\theta} = \frac{13 \sin 2\theta}{\cos 2\theta}$$

ii (a)  $4 \sin 2\theta \cos^2 2\theta + 5 = 13 \sin 2\theta$

$4 \sin 2\theta (1 - \sin^2 2\theta) + 5 = 13 \sin 2\theta$

$4 \sin 2\theta - 4 \sin^3 2\theta + 5 = 13 \sin 2\theta$

B1

Use  
 $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$   
 or  $\tan 2\theta \cos 2\theta = \sin 2\theta$

B1

Correct method to remove fraction(s)

B1

Use  $\cos^2 2\theta = 1 - \sin^2 2\theta$

Must be explicit, and correct notation when used  
 Allow even if errors elsewhere in equation

Any correct equation seen no longer containing fractions (allow recovery from a slip in notation)

Must be explicit, and correct notation when used Allow even if errors elsewhere in equation

		$4\sin^3 2\theta + 9\sin 2\theta - 5 = 0$	<p>B1 [4]</p>	<p>Obtain correct equation, from correct working</p>	<p>Must be correct notation throughout Dependent on B1 B1 B1 awarded</p> <p><b>NB - must annotate answer space</b></p>	
<p>iii (b)</p> $(2 \sin 2\theta - 1)(2\sin^2 2\theta + \sin 2\theta + 5) = 0$ $\sin 2\theta = \frac{1}{2}$ $2\theta = \frac{1}{6}\pi, \frac{5}{6}\pi, \frac{13}{6}\pi, \frac{17}{6}\pi$ $\theta = \frac{1}{12}\pi, \frac{5}{12}\pi, \frac{13}{12}\pi, \frac{17}{12}\pi$			<p>B1 M1 A1</p>	<p>State that <math>\sin 2\theta = \frac{1}{2}</math> oe</p> <p>Attempt to solve <math>\sin 2\theta = \pm \frac{1}{2}</math> to find at least one root</p> <p>Obtain at least 2 correct roots</p>	<p>Could just be stated, or implied by later method</p> <p>Correct order of operations ie <math>\frac{1}{2} (\sin^{-1} \frac{1}{2})</math> Allow M1 if angle(s) found in degrees (<math>15^\circ</math>, <math>75^\circ</math> etc)</p> <p>Must be in radians, and given in an exact form Allow recurring</p>	


			A1 [4]	<table border="1"> <tr> <td>Obtain 4 correct roots</td> <td>           decimals, or mixed numbers             Must be in radians, and given in an exact form            Allow recurring decimals, or mixed numbers            ISW any angles that come from an incorrect quadratic quotient, or an incorrect attempt to find the roots of the quadratic quotient            A0 if any extras in range <math>[0, 2\pi]</math> that are not clearly from their quadratic roots         </td> </tr> </table> <p><b>Examiner's Comments</b></p> <p>Most candidates recognised the link with the previous part of the question and appreciated that they had to solve the cubic in <math>\sin 2\theta</math>. However a significant minority did not realise that this was related to the cubic that they had already factorised in part (i) and made a fresh attempt to solve, sometimes even attempting the use of the quadratic formula on the cubic. Many did recognise the link and attempted to use the root of <math>\frac{1}{2}</math>, but this was sometimes equated to <math>\sin \theta</math> rather than <math>\sin 2\theta</math>. Candidates who correctly stated that <math>\sin 2\theta = \frac{1}{2}</math> were usually able to solve the equation to find at least two roots, with only the most astute candidates giving all 4 roots as their answer. Candidates seem to be comfortable working with angles in radians in an exact form, and there were only a few instances of angles being given in degrees instead.</p>	Obtain 4 correct roots	decimals, or mixed numbers  Must be in radians, and given in an exact form Allow recurring decimals, or mixed numbers ISW any angles that come from an incorrect quadratic quotient, or an incorrect attempt to find the roots of the quadratic quotient A0 if any extras in range $[0, 2\pi]$ that are not clearly from their quadratic roots	
Obtain 4 correct roots	decimals, or mixed numbers  Must be in radians, and given in an exact form Allow recurring decimals, or mixed numbers ISW any angles that come from an incorrect quadratic quotient, or an incorrect attempt to find the roots of the quadratic quotient A0 if any extras in range $[0, 2\pi]$ that are not clearly from their quadratic roots						
		<b>Total</b>	12				

9	a	$2 \sin x \left( \frac{\sin x}{\cos x} \right) = \cos x + 5$ $2 \sin^2 x = \cos^2 x + 5 \cos x$ $2(1 - \cos^2 x) = \cos^2 x + 5 \cos x$ $2 - 2 \cos^2 x = \cos^2 x + 5 \cos x$ $3 \cos^2 x + 5 \cos x - 2 = 0$	<p>M1 (AO 3.1a)</p> <p>M1 (AO 3.1a)</p> <p>A1 (AO 2.1)</p> <p>[3]</p>	<p>Uses <math>\tan x = \sin x / \cos x</math></p> <p>Uses <math>\sin^2 x = 1 - \cos^2 x</math></p> <p><b>AG</b> – correct working throughout</p> <p><u>Examiner's Comments</u></p> <p>This question was done well by many. Because the answer aimed for was given, examiners paid close attention to the written detail and expected fully accurate working for the answer mark. Removing the denominator sometimes led to error, with <math>\cos^2 x + 5</math> appearing on the RHS, and less regularly <math>2 \sin x \cos x \sin x</math> on the LHS.</p>	<p>Must show sufficient working to justify the given answer</p>	
	b	$(3 \cos 2\theta - 1)(\cos 2\theta + 2) = 0$ $\cos 2\theta = \frac{1}{3} \text{ (and } \cos 2\theta = -2)$ $\theta = \frac{1}{2} \arccos \left( \frac{1}{3} \right)$	<p>M1 (AO 1.1a)</p> <p>A1 (AO 1.1)</p> <p>M1 (AO 1.1)</p>	<p>Attempt to solve 3-term quadratic</p> <p>Condone <math>\cos x = \frac{1}{3}</math></p> <p>Correct order of operation to find one value of <math>\theta</math> (or both)</p>	<p><math>(2\theta =) 70.52877\dots,</math></p>	



		$\theta = 35.3^\circ$  $\theta = 144.7^\circ$	<p>A1 (AO 1.1)</p> <p>A1 (AO 1.1)</p> <p>[5]</p>	<div style="border: 1px solid black; padding: 5px;"> <p>values of <math>2\theta</math> correct)</p> <p>One correct value to the nearest integer or better</p> <p>Cao (35.3 and 144.7)</p> </div> <p>289.471...</p> <p>Any additional values in the range loses final A mark if earned</p>	
		<b>Total</b>	<b>8</b>		
10	a	$\sin \theta = 0.5$ and $-0.5$ or $\sin \theta = \pm\sqrt{0.25}$ both  $\theta = 30^\circ$ and $150^\circ$  $\theta = 210^\circ$ and $330^\circ$	<p>B1 (AO1.1a)</p> <p>B1 (AO1.1)</p> <p>B1 (AO1.1)</p>	<div style="border: 1px solid black; padding: 5px;"> <p>"-0.5" may be implied by <b>all 4 answers</b></p> <p>Ignore other answers for this B1</p> <p>NB Correct ans with no wking: B1B1B1</p> </div>	<p><math>\sin \theta = 0.5</math>, <math>\theta = 30</math> and 210</p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px auto;">B0B0B0</div> <p><math>\sin \theta = \pm 0.5</math>, <math>\theta = 30</math> and 210</p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px auto;">B1B0B0</div>

			[3]	<p><u>Examiner's Comments</u></p> <p>Many candidates omitted <math>\sin \theta = -0.5</math>, usually obtaining <math>30^\circ</math> but not always <math>150^\circ</math>. Some of those who included <math>\sin \theta = -0.5</math> only gave one of the two other answers. Some candidates found <math>\sin^{-1}(0.25) = 14.5^\circ</math>. Some of these then found <math>14.5^2</math> or <math>\sqrt{14.5}</math>.</p>																			
	b	<p><b>DR</b>  <math>60^\circ</math> and <math>240^\circ</math> seen or implied</p> <p><math>20^\circ</math> seen</p> <p><math>\phi = 20^\circ, 80^\circ</math>      With no other sol'ns</p>	<p>B1 (AO1.1a)</p> <p>B1 (AO1.1)</p> <p>B1 (AO1.1)</p> <p>[3]</p>	<table border="1" data-bbox="1032 421 1713 847"> <tr> <td data-bbox="1032 421 1373 847">Both needed, but ignore other values</td> <td data-bbox="1373 421 1713 847"></td> </tr> <tr> <td data-bbox="1032 751 1373 847">SC: correct ans with no wking: B0B1B0</td> <td data-bbox="1373 751 1713 847"></td> </tr> </table> <p><u>Examiner's Comments</u></p> <table border="1" data-bbox="1032 991 1713 1433"> <tr> <td colspan="2" data-bbox="1032 991 1713 1043">Most candidates obtained <math>3\phi = 60^\circ</math> but not</td> </tr> <tr> <td colspan="2" data-bbox="1032 1043 1713 1096">all included <math>3\phi = 240^\circ</math>. A few started with</td> </tr> <tr> <td data-bbox="1032 1096 1240 1214"><math>\tan \phi = \frac{\sqrt{3}}{3}</math></td> <td data-bbox="1240 1096 1713 1214">and hence <math>\phi = 30^\circ</math>. Some</td> </tr> <tr> <td data-bbox="1032 1214 1240 1287">thought that</td> <td data-bbox="1240 1214 1713 1287"><math>\tan^{-1}(\sqrt{3}) = 30^\circ</math>. Some</td> </tr> <tr> <td colspan="2" data-bbox="1032 1287 1713 1340">gave extraneous answers obtained by,</td> </tr> <tr> <td colspan="2" data-bbox="1032 1340 1713 1393">for example, <math>(180^\circ - 60^\circ) \div 3</math>.</td> </tr> <tr> <td colspan="2" data-bbox="1032 1393 1713 1433">Others gave answers outside the given</td> </tr> </table>	Both needed, but ignore other values		SC: correct ans with no wking: B0B1B0		Most candidates obtained $3\phi = 60^\circ$ but not		all included $3\phi = 240^\circ$ . A few started with		$\tan \phi = \frac{\sqrt{3}}{3}$	and hence $\phi = 30^\circ$ . Some	thought that	$\tan^{-1}(\sqrt{3}) = 30^\circ$ . Some	gave extraneous answers obtained by,		for example, $(180^\circ - 60^\circ) \div 3$ .		Others gave answers outside the given		
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gave extraneous answers obtained by,																							
for example, $(180^\circ - 60^\circ) \div 3$ .																							
Others gave answers outside the given																							

				domain. Some gave answers in radians.	
		<b>Total</b>	<b>6</b>		
11	a	$2(1 - \cos^2\theta) + \cos\theta = 4 \cos^2\theta$  $2 - 2 \cos^2\theta + \cos\theta = 4 \cos^2\theta$  $6 \cos^2\theta - \cos\theta - 2 = 0$	<b>M1</b> <b>(AO3.1a)</b>  <b>A1</b> <b>(AO2.2a)</b> <b>[2]</b>	<p>Correctly removing square root and use of <math>\sin^2\theta = 1 - \cos^2\theta</math> to obtain an equation in cos only</p> <p><b>AG</b> – sufficient working must be shown to establish given result</p>	
	b	<b>DR</b> $(2 \cos\theta + 1)(3 \cos\theta - 2) = 0$  $\cos\theta = -\frac{1}{2}$ and $\cos\theta = \frac{2}{3}$  $\cos\theta = \frac{2}{3} \Rightarrow \theta = 48.2, 311.8$  $\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120, 240$	<b>M1 (AO1.1)</b> <b>A1 (AO1.1)</b>  <b>A1 (AO1.1)</b>  <b>M1 (AO2.2a)</b> <b>[4]</b>	<p>Correct method for solving quadratic</p> <p>Any two correct values</p> <p>All four correct values</p>	<p>May use formula or completing the square</p> <p>48.189..., 311.810...</p> <p>And no others</p>
	c	<p>E.g. since <math>\cos\theta \neq -\frac{1}{2}</math> n the RHS of the equation</p> $\sqrt{2 \sin^2\theta + \cos\theta} = 2 \cos\theta$	<b>E1 (AO 2.3)</b> <b>[1]</b>		

			Total	7	
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