1. (a) Given that $5\sin\theta = 2\cos\theta$, find the value of $\tan\theta$.

(1)

(b) Solve, for $0 \le x < 360°$,

$$5\sin 2x = 2\cos 2x,$$

giving your answers to 1 decimal place.

(5)
(Total 6 marks)

2. (a) Show that the equation

$$5\sin x = 1 + 2\cos^2 x$$

can be written in the form

$$2\sin^2 x + 5\sin x - 3 = 0$$

(2)

(b) Solve, for $0 \le x < 360°$,

$$2\sin^2 x + 5\sin x - 3 = 0$$

(4)
(Total 6 marks)

3. (i) Solve, for $-180° \le \theta < 180°$,

$$(1 + \tan\theta)(5\sin\theta - 2) = 0.$$

(4)

(ii) Solve, for $0 \le x < 360°$,

$$4\sin x = 3\tan x.$$

(6)
(Total 10 marks)

4. (a) Show that the equation

$$4 \sin^2 x + 9 \cos x - 6 = 0$$

can be written as

$$4 \cos^2 x - 9 \cos x + 2 = 0.$$

(2)

(b) Hence solve, for $0 \le x < 720°$,

$$4 \sin^2 x + 9 \cos x - 6 = 0,$$

giving your answers to 1 decimal place.

(6)
(Total 8 marks)

5. Solve, for $0 \le x < 360°$,

(a) $\sin(x - 20^o) = \frac{1}{\sqrt{2}}$

(4)

(b) $\cos 3x = -\frac{1}{2}$

(6)
(Total 10 marks)

6. (a) Show that the equation

$$3 \sin^2\theta - 2 \cos^2\theta = 1$$

can be written as

$$5 \sin^2\theta = 3.$$

(2)

(b) Hence solve, for $0° \leq \theta < 360°$, the equation

$$3 \sin^2\theta - 2 \cos^2\theta = 1,$$

giving your answers to 1 decimal place.

(7)
(Total 9 marks)

7. Find all the solutions, in the interval $0 \leq x < 2\pi$, of the equation

$$2 \cos^2 x + 1 = 5 \sin x,$$

giving each solution in terms of π.

(Total 6 marks)

8. (a) Given that $\sin \theta = 5\cos \theta$, find the value of $\tan \theta$.

(1)

(b) Hence, or otherwise, find the values of θ in the interval $0 \leq \theta < 360°$ for which

$$\sin \theta = 5\cos \theta,$$

giving your answers to 1 decimal place.

(3)
(Total 4 marks)

9. Solve, for $0 < \theta < 360°$, giving your answers to 1 decimal place where appropriate,

(a) $2 \sin \theta = 3 \cos \theta$,

(3)

(b) $2 - \cos \theta = 2 \sin^2 \theta$.

(6)
(Total 9 marks)

10. (a) Find all the values of θ, to 1 decimal place, in the interval $0° \le \theta < 360°$ for which

$$5 \sin(\theta + 30°) = 3.$$

(4)

(b) Find all the values of θ, to 1 decimal place, in the interval $0° \le \theta, < 360°$ for which

$$\tan^2\theta = 4.$$

(5)
(Total 9 marks)

11. Solve, for $-90° < x < 90°$, giving answers to 1 decimal place,

(a) $\tan (3x + 20°) = \dfrac{3}{2}$,

(6)

(b) $2 \sin^2 x + \cos^2 x = \dfrac{10}{9}$.

(4)
(Total 10 marks)

12. Solve, for $0 \le x \le 180°$, the equation

(a) $\sin(x + 10°) = \dfrac{\sqrt{3}}{2}$,

(4)

(b) $\cos 2x = -0.9$, giving your answers to 1 decimal place.

(4)
(Total 8 marks)

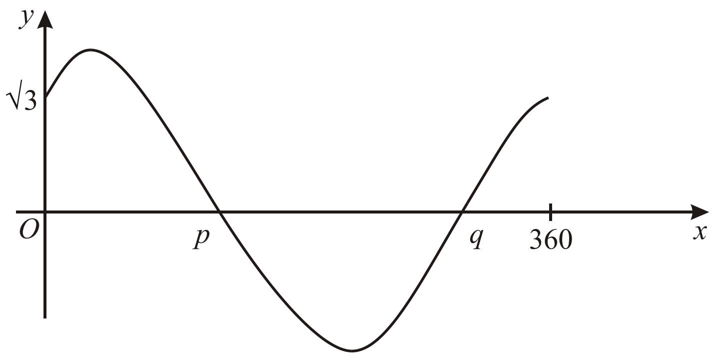
13. Solve, for $0 \leq \theta < 2\pi$, the equation

$$\sin^2 \theta = 1 + \cos \theta,$$

giving your answers in terms of π.

(Total 5 marks)

14.



The diagram above shows the curve with equation $y = k \sin (x + 60)°$, $0 \leq x \leq 360$, where k is a constant.

The curve meets the y-axis at $(0, \sqrt{3})$ and passes through the points $(p, 0)$ and $(q, 0)$.

(a) Show that $k = 2$.

(1)

(b) Write down the value of p and the value of q.

(2)

The line $y = -1.6$ meets the curve at the points A and B.

(c) Find the x-coordinates of A and B, giving your answers to 1 decimal place.

(5)
(Total 8 marks)

15. (a) Show that the equation

$$5\cos^2 x = 3(1 + \sin x)$$

can be written as

$$5\sin^2 x + 3\sin x - 2 = 0.$$

(2)

(b) Hence solve, for $0 \leq x < 360°$, the equation

$$5\cos^2 x = 3(1 + \sin x),$$

giving your answers to 1 decimal place where appropriate.

(5)
(Total 7 marks)

16. (i) Prove that $\tan\theta + \cot\theta \equiv 2\operatorname{cosec} 2\theta$, $\left(\theta \neq \frac{n\pi}{2},\ n \in \mathbb{Z}\right)$.

(5)

(ii) Given that $\sin\alpha = \frac{5}{13}$, $0 < \alpha < \frac{\pi}{2}$, find the exact value of

(a) $\cos\alpha$,

(b) $\cos 2\alpha$.

(4)

Given also that $13\cos(x + \alpha) + 5\sin x = 6$, and $0 < \alpha < \frac{\pi}{2}$,

(c) find the value of x.

(5)
(Total 14 marks)

17. (a) Given that $3 \sin x = 8 \cos x$, find the value of $\tan x$.

(1)

(b) Find, to 1 decimal place, all the solutions of

$$3 \sin x - 8 \cos x = 0$$

in the interval $0 \leq x < 360°$.

(3)

(c) Find, to 1 decimal place, all the solutions of

$$3 \sin^2 y - 8 \cos y = 0$$

in the interval $0 \leq y < 360°$.

(6)
(Total 10 marks)

18. Find all the values of θ in the interval $0 \leq \theta < 360°$ for which

(a) $\cos(\theta - 10°) = \cos 15°$,

(3)

(b) $\tan 2\theta = 0.4$,

(5)

(c) $2 \sin \theta \tan \theta = 3$.

(6)
(Total 14 marks)

19. The curve C has equation $y = \cos\left(x + \frac{\pi}{4}\right)$, $0 \leq x \leq 2\pi$.

(a) Sketch C.

(2)

(b) Write down the exact coordinates of the points at which C meets the coordinate axes.

(3)

(c) Solve, for x in the interval $0 \leq x \leq 2\pi$,

$$\cos\left(x + \frac{\pi}{4}\right) = 0.5,$$

giving your answers in terms of p.

(4)
(Total 9 marks)

20. Find, in degrees, the value of θ in the interval $0 \leq \theta < 360°$ for which

$$2\cos^2\theta - \cos\theta - 1 = \sin^2\theta.$$

Give your answers to 1 decimal place where appropriate.

(Total 8 marks)

21. (a) Sketch, for $0 \leq x \leq 360°$, the graph of $y = \sin(x + 30°)$.

(2)

(b) Write down the coordinates of the points at which the graph meets the axes.

(3)

(c) Solve, for $0 \leq x < 360°$, the equation

$$\sin(x + 30°) = -\tfrac{1}{2}.$$

(3)
(Total 8 marks)

22. Find, in degrees to the nearest tenth of a degree, the values of x for which $\sin x \tan x = 4$, $0 \leq x < 360°$.

(Total 8 marks)

23. (a) Solve, for $0 \leq x < 360°$, the equation $\cos(x - 20°) = -0.437$, giving your answers to the nearest degree.

(4)

(b) Find the exact values of θ in the interval $0 \leq \theta < 360°$ for which $3\tan\theta = 2\cos\theta$.

(6)

(Total 10 marks)

1. (a) $\tan\theta = \frac{2}{5}$ (or 0.4) (i.s.w. if a value of θ is subsequently found) B1 1

Requires the correct value with no incorrect working seen.

(b) awrt 21.8 (α) B1

(Also allow awrt 68.2, ft from $\tan\theta = \frac{5}{2}$ in (a), but no other ft)

(This value must be seen in part (b). It may be implied by a correct solution, e.g. 10.9)

$180 + \alpha$ $(= 201.8)$, or $90 + (\alpha/2)$ (if division by 2 has already occurred)

(α found from $\tan 2x = \ldots$ or $\tan x = \ldots$ or $\sin 2x = \pm\ldots$ or $\cos 2x = \pm\ldots$)

$360 + \alpha$ $(= 381.8)$, or $180 + (\alpha/2)$

(α found from $\tan 2x = \ldots$ or $\sin 2x = \ldots$ or $\cos 2x = \ldots$)

OR $540 + \alpha$ $(= 561.8)$, or $270 + (\alpha/2)$

(α found from $\tan 2x = \ldots$)

Dividing at least one of the angles by 2

(α found from $\tan 2x = \ldots$ or $\sin 2x = \ldots$ or $\cos 2x = \ldots$)

$x = 10.9,\ 100.9,\ 190.9,\ 280.9$ (Allow awrt) A1 5

<u>**Note**</u>

Extra solution(s) in range: Loses the final A mark.
Extra solutions outside range: Ignore (whether correct or not).

<u>Common answers:</u>
10.9 and 100.9 would score B1 M0 A0 (Ensure that <u>these</u> M marks are awarded)
10.9 and 190.9 would score B1 M0 A0 (Ensure that <u>these</u> M marks are awarded)
<u>Alternatives:</u>
(i) $2\cos 2x - 5\sin 2x = 0$ $R\cos(2x + \lambda) = 0$ $\lambda = 68.2 \Rightarrow 2x + 68.2 = 90$ B1
$2x + \lambda = 270$
$2x + \lambda = 450$ or $2x + \lambda = 630$
Subtracting λ and dividing by 2 (at least once)
(ii) $25\sin^2 2x = 4\cos^2 2x = 4(1 - \sin^2 2x)$
$29\sin^2 2x = 4$ $2x = 21.8$ B1

The M marks are scored as in the main scheme, but extra solutions will be likely, losing the A mark.

Using radians:
B1: Can be given for awrt 0.38 (β)
For $\pi + \beta$ or $180 + \beta$
For $2\pi + \beta$ or $3\pi + \beta$ (Must now be consistently radians)
For dividing at least one of the angles by 2
A1: For this mark, the answers must be in degrees.
(Correct) answers only (or by graphical methods):
B and M marks can be awarded by implication, e.g.
10.9 scores B1 M0 M0 A0
10.9, 100.9 scores B1 M0 A0
10.9, 100.9, 190.9, 280.9 scores full marks.
Using 11, etc. instead of 10.9 can still score the M marks by implication.

[6]

2. (a) $5\sin x = 1 + 2\left(1 - \sin^2 x\right)$

$2\sin^2 x + 5\sin x - 3 = 0$ (*) A1cso 2

Note

for a correct method to change $\cos^2 x$ into $\sin^2 x$
(must use $\cos^2 x = 1 - \sin^2 x$)
A1 need 3 term quadratic printed in any order with = 0 included

(b) $(2s-1)(s+3)=0$ **giving** $s =$

[$\sin x = -3$ has no solution] **so** $\sin x = \frac{1}{2}$ A1

$\therefore\ x = 30, 150$ B1 B1ft 4

Note

for attempt to solve given quadratic (usual rules for solving quadratics) (can use any variable here, s, y, x, or $\sin x$)
A1 requires no incorrect work seen and is for $\sin x = \frac{1}{2}$
or $x = \sin^{-1}\frac{1}{2}$ $y = \frac{1}{2}$ is A0 (unless followed by $x = 30$)

B1 for 30 (α) not dependent on method
2nd B1 for $180 - \alpha$ provided in required range (otherwise $540 - \alpha$)

Extra solutions outside required range: Ignore

Extra solutions inside required range: Lose final B1

Answers in radians: Lose final B1

S.C. Merely writes down two correct answers is M0A0B1B1

Or $\sin x = \frac{1}{2}$ $\therefore$ $x = 30, 150$ **is M1A1B1B1**

Just gives one answer : 30 only is M0A0B1B0 or 150 only is M0A0B0B1

NB Common error is to factorise wrongly giving $(2\sin x + 1)(\sin x - 3) = 0$

[sin x = 3 gives no solution] sin $x = -\frac{1}{2}$ $\Rightarrow$ x = 210, 330

This earns A0 B0 B1ft

Another common error is to factorise correctly (2sin x –1)

(sin x + 3) = 0 and follow this sin $x = \frac{1}{2}$, sin x = 3 then x = 30°, 150°

This would be A0 B1 B1

[6]

3. (i) $\tan\theta = -1$ $\Rightarrow$ $\theta = -45$, 135 B1, B1ft

$\sin\theta = \frac{2}{5}$ $\Rightarrow$ $\theta = 23.6$, 156.4 (AWRT: 24, 156) B1, B1ft 4

Note

1st B1 for – 45 seen (α, where $|\alpha| < 90$)

2nd B1 for 135 seen, <u>or ft</u> (180 + α) if α is negative, or (α – 180) if α is positive.
If $\tan\theta = k$ is obtained from <u>wrong working</u>, 2nd B1ft is still available.

3rd B1 for awrt 24 (β, where $|\beta| < 90$)

4th B1 for awrt 156, <u>or ft</u> (180 – β) if β is positive, or – (180 + β) if β is negative.
If $\sin\theta = k$ is obtained from <u>wrong working</u>, 4th B1ft is still available.

(ii) $4\sin x = \frac{3\sin x}{\cos x}$

$4\sin x\cos x = 3\sin x$ $\Rightarrow$ $\sin x(4\cos x - 3) = 0$

Other possibilities (after squaring): $\sin^2 x(16\sin^2 x - 7) = 0$,
$(16\cos^2 x - 9)(\cos^2 x - 1) = 0$

x = 0, 180 <u>seen</u> B1, B1

x = 41.4, 318.6 (AWRT: 41, 319) B1, B1ft 6

Note

1st for use of tan $x = \frac{\sin x}{\cos x}$. Condone $\frac{3\sin x}{3\cos x}$.

2nd for correct work leading to 2 factors (may be implied).

1st B1 for 0, 2nd B1 for 180.

3rd B1 for awrt 41 (γ, where $|\gamma| < 180$)

4th B1 for awrt 319, <u>or ft</u> (360 – γ).
If $\cos\theta = k$ is obtained from <u>wrong working</u>, 4th B1ft is still available.

N.B. Losing sin x = 0 usually gives a maximum of 3 marks M1M0B0B0B1B1

<u>Alternative:</u> (squaring both sides)

1st for squaring both sides and using a ‘quadratic’ identity.

e.g. $16\sin^2\theta = 9(\sec^2\theta - 1)$

2nd for reaching a factorised form.

e.g. $(16\cos^2\theta - 9)(\cos^2\theta - 1) = 0$

Then marks are equivalent to the main scheme. Extra solutions, if not rejected, are penalised as in the main scheme.

<u>For both parts of the question:</u>

<u>Extra solutions outside required range: Ignore</u>

<u>Extra solutions inside required range:</u>
For each <u>pair</u> of B marks, the 2nd B mark is lost if there are two correct values and one or more extra solution(s), e.g. $\tan\theta = -1$
$\Rightarrow \theta = 45, -45, 135$ is B1 B0

<u>Answers in radians:</u>
Loses a maximum of 2 B marks in the whole question (to be deducted at the first and second occurrence).

[10]

4. (a) $4(1 - \cos^2 x) + 9\cos x - 6 = 0$ $\quad 4\cos^2 x - 9\cos x + 2 = 0$ (*) A1 2

<u>Note</u>

Uses $\sin^2 x = 1 - \cos^2 x$ (may omit bracket) **not** $\sin^2 x = \cos^2 x - 1$

A1: Obtains the printed answer without error – **must have = 0**

(b) $(4\cos x - 1)(\cos x - 2) = 0$ $\cos x = \ldots, \quad \frac{1}{4}$ A1

$x = 75.5 \quad (\alpha)$ B1

$360 - \alpha$, $360 + \alpha$ or $720 - \alpha$
284.5, 435.5, 644.5 A1 6

<u>Notes</u>

Solves the quadratic with usual conventions
A1: Obtains ¼ accurately– ignore extra answer 2 but penalise e.g. –2.
B1: allow answers which round to 75.5
$360 - \alpha$ ft their value, $360 + \alpha$ ft their value or $720 - \alpha$ ft
A1: Three **and only three** correct exact answers in the range achieves the mark

Special cases

In part (b) Error in solving quadratic $(4\cos x - 1)(\cos x + 2)$
Could yield, **M1A0B1M1M1A1** losing one mark for the error

Works in radians:
Complete work in radians :Obtains 1.3 **B0**. Then allow
for $2\pi - \alpha$, $2\pi + \alpha$ or $4\pi - \alpha$ Then gets 5.0, 7.6, 11.3 **A0 so 2/4**
Mixed answer 1.3, 360 – 1.3, 360 + 1.3, 720 – 1.3 still gets **B0M1M1A0**

[8]

5. (a) 45 (α) (This mark can be implied by an answer 65) B1
$180 - \alpha$, Add 20 (for at least one angle)
65 155 A1 4

Extra solution(s) in range: Loses the A mark.
Extra solutions outside range: Ignore (whether correct or not).
Common solutions:
65 (only correct solution) will score B1 M0 A0 (2 marks)
65 and 115 will score B1 M0 A0 (2 marks)

44.99 (or similar) for ***a*** is B0, and 64.99, 155.01 (or similar) is A0.

(b) 120 or 240 (β): (This mark can be implied by an answer 40 or 80) B1
(Could be achieved by working with 60, using 180 – 60 and/or 180 + 60)
$360 - \beta$, $360 + \beta$ (or 120 + an angle that has been divided by 3)
Dividing by 3 (for at least one angle)
40 80 160 200 280 320 First A1: at least 3 correct A1A1 6

Extra solution(s) in range: Loses the final A mark.
Extra solutions outside range: Ignore (whether correct or not).
Common solutions:
40 (only correct solution) will score B1 M0 M0 A0 A0 (2 marks)
40 and 80 (only correct solutions) B1 M0 A0 A0 (3 marks)
40 and 320 (only correct solutions) B1 M0 M0 A0 A0 (2 marks)

<u>Answers without working:</u>
Full marks can be given (in both parts), B and M marks by implication.

<u>Answers given in radians</u>:
Deduct a maximum of 2 marks (misread) from B and A marks.
(Deduct these at first and second occurrence.)

<u>Answers that begin</u> with statements such as $\sin(x - 20) = \sin x - \sin 20$ or $\cos x = -\frac{1}{6}$, then go on to find a value of 'α' or 'β', however badly, <u>can</u> continue to earn the first M mark in either part, but will score <u>no further marks</u>.

<u>Possible misread</u>: $\cos 3x = \frac{1}{2}$, giving 20, 100, 140, 220, 260, 340

Could score up to 4 marks B0 A0 A1 for the above answers.

[10]

6. (a) $3 \sin^2 \theta - 2 \cos^2 \theta = 1$
$3 \sin^2 \theta - 2(1 - \sin^2 \theta) = 1$ (Use of $\sin^2 \theta + \cos^2 \theta = 1$)
$3 \sin^2 \theta - 2 + 2 \sin^2 \theta = 1$
$5 \sin^2 \theta = 3$ cso **AG** A1 2

N.B: **AG**; need to see at least one line of working after substituting $\cos^2 \theta$.

(b) $\sin^2 \theta = \frac{3}{5}$, so $\sin \theta = (\pm)\sqrt{0.6}$

Attempt to solve both $\sin\theta = +..$ and $\sin\theta = - \ldots$
(may be implied by later work)

$\theta = 50.7685°$ awrt $\theta = 50.8°$ (dependent on first only) A1

θ (= 180° – 50.7685$_c$°); = 129.23…° awrt 129.2° A1ft

[f.t. dependent on first M and 3rd M]

$\sin\theta = -\sqrt{0.6}$

$\theta = 230.785°$ and 309.23152° awrt 230.8°, 309.2° (both) M1A1 7

First Using $5\sin^2\theta = 3$ to find value for $\sin\theta$ or θ

[Allow such results as $\sin\theta = \frac{3}{5}$, $\sin\theta = \frac{\sqrt{3}}{5}$ ….. for

Second Considering the – value for $\sin\theta$. (usually later)

First A1: Given for awrt 50.8°. **Not** dependent on **second** M.

Third For (180 – **candidate's** 50.8)°, need not see written down

Final **Dependent** on **second** M (but **may be implied by answers)**

For (180 + candidate's 50.8)° **or** (360 – candidate's 50.8)° **or equiv**.

Final A1: Requires both values. (**no follow through**)

[Finds $\cos^2\theta = k$ (k = 2/5) and so $\cos\theta = (\pm)$... then mark equivalently]

NB Candidates who **only consider positive value for sin θ**
can score max of 4 marks: M1M0A1M1A1M0A0 – Very common.
Candidates **who score first but have wrong sin θ** can score
maximum M1M1A0M1AftM1A0

SC Candidates who obtain one value from each set, e.g 50.8 and 309.2
M1M1(bod)A1M0A0M1(bod)A0

Extra values out of range – no penalty

[9]

7. $2(1 - \sin^2 x) + 1 = 5\sin x$

$2\sin^2 x + 5\sin x - 3 = 0$

$(2\sin x - 1)(\sin x + 3) = 0$

$\sin x = \frac{1}{2}$ A1

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ A1cso 6

Use of $\cos^2 x = 1 - \sin^2 x$.
Condone invisible brackets in first line if $2 - 2\sin^2 x$ is present
(or implied) in a subsequent line.
Must be using $\cos^2 x = 1 - \sin^2 x$. Using $\cos^2 x = 1 + \sin^2 x$ is M0.

Attempt to solve a 2 or 3 term quadratic in $\sin x$ up to $\sin x = \ldots$
Usual rules for solving quadratics. Method may be factorising, formula
or completing the square

Correct factorising for correct quadratic and $\sin x = \frac{1}{2}$.
So, e.g. $(\sin x + 3)$ as a factors $\rightarrow \sin x = 3$ can be ignored. A1

Method for finding any angle in any range consistent with (either of) their trig. equation(s) in degrees or radians (even if x not exact).
[Generous M mark]
Generous mark. Solving any trig. equation that comes from minimal working (however bad).
So $x = \sin^{-1}/\cos^{-1}/\tan^{-1}$ (number) $\rightarrow$ answer in degrees or radians correct for their equation (in any range)

Method for finding second angle consistent with (either of) their trig. equation(s) in radians.
Must be in range $0 \le x < 2\pi$. Must involve using π(e.g. $\pi \pm \ldots, 2\pi - \ldots$) but … can be inexact.
Must be using the same equation as they used to attempt the 3rd M mark.
Use of π must be consistent with the trig. equation they are using (e.g. if using $\cos^{-1}$ then must be using $2\pi - \ldots$)
If finding both angles in degrees: method for finding 2nd angle equivalent to method above in degrees and an attempt to change both angles to radians.

$\frac{\pi}{6}, \frac{5\pi}{6}$ c.s.o. Recurring decimals are okay (instead of $\frac{1}{6}$ and $\frac{5}{6}$).
Correct decimal values (corrected or truncated) before the final answer of $\frac{\pi}{6}, \frac{5\pi}{6}$ is acceptable. A1 cso

Ignore extra solutions outside range; deduct final A mark for extra solutions in range.

Special case

Answer only $\frac{\pi}{6}, \frac{5\pi}{6}$ M0, M0, A0, A1

Answer only $\frac{\pi}{6}$ M0, M0, A0, M0 A0

Finding answers by trying different values (e.g. trying multiples of π) in $2\cos^2 x + 1 = 5\sin x$: as for answer only.

[6]

8. (a) $\tan\theta = 5$ B1 1

Must be seen explicitly, e.g. $\tan\theta = \tan^{-1} 5 = 78.7$ or equiv. is B0, unless $\tan\theta = 5$ is also seen.

(b) $\tan\theta = k$ $\left(\theta = \tan^{-1} k\right)$

$\theta = 78.7$, 258.7 (Accept awrt) A1, A1ft 3

The M mark may be implied by working in (a).

A1ft for $180 + \alpha$. ($a \neq k$).

Answers in radians would lose both the A marks.
Extra answers between 0 and 360: Deduct the final mark.
Alternative:
Using $\cos^2 \theta = 1 - \sin^2 \theta$ (or equiv.) and proceeding to $\sin \theta = k$ (or equiv.): then A marks as in main scheme.

[4]

9. (a) $\tan \theta = \frac{3}{2}$ Use of $\tan \theta = \frac{\sin\theta}{\cos\theta}$

$\theta = 56.3°$ cao A1

$= 236.3°$ ft 180° + their principle value A1ft 3

Maximum of one mark is lost if answers not to 1 decimal place

(b) $2 - \cos \theta = 2(1 - \cos^2 \theta)$ Use of $\sin^2 \theta + \cos^2 \theta = 1$

$2\cos^2 \theta - \cos \theta$ A1

Allow this A1 if both $\cos \theta = 0$ and $\cos \theta = \frac{1}{2}$ are given

$\cos \theta = 0 \Rightarrow \theta = 90°, 270°$ one solution A1

$\cos \theta = \frac{1}{2} \Rightarrow \theta = 60°, 300°$ one solution A1 6

[9]

10. (a) $\sin(\theta + 30) = \frac{3}{5}$ ($\frac{3}{5}$ on RHS) B1

$\theta + 30 = 36.9$ (α = AWRT 37) B1

or $= 143.1$ (180 – α)

$\underline{\theta = 6.9, \quad 113.1}$ A1cao 4

(b) $\tan \theta = \pm 2)$ or $\sin \theta = \pm \frac{2}{\sqrt{5}}$ or $\cos \theta = \pm \frac{1}{\sqrt{5}}$ B1

($\tan \theta = 2 \Rightarrow$) $\theta = 63.4$ (β = AWRT 63.4)

or $\underline{243.4}$ ($180 + \beta$)

($\tan \theta = -2 \Rightarrow$) $\theta = \underline{116}.6$ ($180 - \beta$)

or $\underline{296.6}$ (180 + their 116.6) 5

[9]

(a) for 180 – their first solution. Must be at the correct stage i.e. for θ + 30

(b) ALL M marks in (b) must be for $\theta = ...$

1st for 180 + their first solution
2nd for 180 – their first solution
3rd for 180+ their 116.6 or 360 – their first solution

<u>Answers Only</u> can score full marks in both parts

Not 1 d.p.: loses A1 in part (a). In (b) all answers are AWRT.

Ignore extra solutions outside range

Radians Allow M marks for consistent work with radians only, but all A and B marks for angles must be in degrees. Mixing degrees and radians is M0.

MISREADS

$5x^2$ misread as $5x^3$

$\frac{5x^3+2}{x^{\frac{1}{2}}} = 5x^{\frac{5}{2}} + 2x^{-\frac{1}{2}}$ A0

$f(x) = 3x + \frac{5x^{\frac{7}{2}}}{\left(\frac{7}{2}\right)} + \frac{2x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ $(+C)$ A1ft

$6 = 3 + \frac{10}{7} + 4 + C$

$C = -\frac{17}{7}$, $f(x) = 3x + \frac{10}{7}x^{\frac{7}{2}} + 4x^{\frac{1}{2}} - \frac{17}{7}$ A0, A1

11. (a) $\arctan\frac{3}{2} = 56.3°$ $(= \alpha)$ (seen anywhere) B1

$\alpha - 20°$, $(\alpha - 20)° \div 3$ M1M1

$\alpha + 180°$ $(= 236.3°)$, $\alpha - 180°$ $(= -123.7°)$ (One of these)

$x = -47.9°, 12.1°, 72.1°$ A1A1 6

First Subtracting (allow adding) 20° from α

Second Dividing that result by 3 (order vital !)

[So 12.1° gains B1M1M1]

Third Giving a third quadrant result

First A1 is for 2 correct solutions,

Second A1 for third correct solution.

B1: Allow 0.983 (rads) or 62.6 (grad), and possible Ms but A0A0]

EXTRA

Using expansion of $\tan(3x + 20°) = \frac{3}{2}$

Getting as far as $\tan 3x$ = number (0.7348..) First

$\tan 3x = 36.3°, 216.3°, -143.7°$ 36.3° B1

$x = 12.1°, 72.1°, -47.9°$ Third quad result Third

Divide by 3 Second

Answers as scheme A1A1

(b) $2\sin^2 x + (1 - \sin^2 x) = \frac{10}{9}$ or $2(1 - \cos^2 x) + \cos^2 x = \frac{10}{9}$

$\sin^2 x = \frac{1}{9}$ or $\cos^2 x = \frac{8}{9}$ or $\tan^2 x = \frac{1}{8}$ or $\sec^2 x = \frac{9}{8}$ or $\cos 2x = \frac{7}{9}$ A1

$x = 19.5°, –19.5°$ A1A1ft 4

for use of $\sin^2 x + \cos^2 x = 1$ or $\sin^2 x$ and $\cos^2 x$ in terms of cos2x

Note: Max. deduction of 1 for not correcting to 1 dec. place.

Record as 0 first time occurs but then treat as f.t.

Answers outside given interval, ignore
Extra answers in range, max. deduction of 1 in each part [Final mark]
(i.e. 4 or more answers within interval in (a), –1 from any gained A marks;
3 or more answers within interval in (b), –1 from any gained A marks

[10]

12. (a) $(x + 10 =)$ 60 α B1
120

(M: $180 - \alpha$ or $\pi - \alpha$)

$x = 50$ $x = 110$ (or 50.0 and 110.0) A1 4

(M: subtract 10)

First M: Must be subtracting from 180 <u>before</u> subtracting 10.

(b) $(2x =)$ 154.2 β B1

Allow a.w.r.t. 154 or a.w.r.t. 2.69 (radians)

205.8

(M: $360 - \beta$ or $2\pi - \beta$)

$x = 77.1$ $x = 102.9$ A1 4

M: Divide by 2

First M: Must be subtracting from 360 <u>before</u> dividing by 2, <u>or</u> dividing by 2 then subtracting from 180.

[8]

In each part:
Extra solutions outside 0 to 180: Ignore.
extra solutions between 0 and 180 : A0.

Alternative for (b): (Double angle formula)

$1 - 2\sin^2 x = -0.9$ $2\sin^2 x = 1.9$ B1

$\sin x = \sqrt{0.95}$

$x = 77.1$

$x = 180 - 77.1 = 102.9$ A1

13. Using $\sin^2\theta + \cos^2\theta = 1$ to give a quadratic in $\cos\theta$.
Attempt to solve $\cos^2\theta + \cos\theta = 0$

$(\cos\theta = 0) \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$ B1, B1

$(\cos\theta = -1) \Rightarrow \theta = \pi$ B1 5

(Candidate who writes down 3 answers only with no working scores a maximum of 3)

[5]

14. (a) Substitute $x = 0$, $y = \sqrt{3}$ to give $\sqrt{3} = k\frac{\sqrt{3}}{2} \Rightarrow k = 2$

(or verify result) must see $\frac{\sqrt{3}}{2}$ B1 1

(b) $p = 120$, $q = 300$ (f.t. on $p + 180$) B1, B1ft 2

(c) *arc* sin(–0.8) = –53.1 or *arc* sin(0.8) = 53.1 B1
(x + 60) = 180 – arc sin(–0.8) or equivalent 180 + arc sin 0.8
First value of x = 233.1 – 60, i.e. x = 173.1 A1
OR (x + 60) = 360 + arc sin(–0.8) or equivalent 360 – arc sin 0.8,
i.e. x = 246.9 A1 5

[8]

15. (a) $5(1 - \sin^2 x) = 3(1 + \sin x)$
$5 - 5\sin^2 x = 3 + 3\sin x$
<u>$0 = 5\sin^2 x + 3\sin x - 2$</u> (*) A1 cso 2

(b) $0 = (5\sin x - 2)(\sin x + 1)$

$\sin x = \frac{2}{5}, -1$ (both) A1 cso

$\sin x = \frac{2}{5} \Rightarrow x = \underline{23.6}$ (α = 23.6 or 156.4) B1

, <u>156.4</u> (180 – α)

$\sin x = -1 \Rightarrow x = = \underline{270}$ (ignore extra solutions <u>outside</u> the range) B1 5

[7]

16. (a) (i) Using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$ *or* $\cot\theta = \frac{1}{\tan\theta}$

Forming a single fraction

LHS = $\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ *or* LHS = $\frac{1 + \tan^2\theta}{\tan\theta}$

Reaching the expression $\frac{1}{\sin\theta\cos\theta}$ A1

Using $\sin 2\theta = 2\sin\theta\cos\theta$

LHS = $\frac{2}{2\sin\theta\cos\theta} = \frac{2}{\sin 2\theta} = 2\operatorname{cosec}2\theta$ RHS (*) cso A1 5

(ii) $\cos\alpha = \frac{12}{13}$ Use of $\sin^2\alpha + \cos^2\alpha = 1$ or right angled triangle but accept stated A1

(b) Use of $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ (*or* $1 - 2\sin^2 \alpha$ *or* $2\cos^2 \alpha - 1$)

$\cos 2\alpha = \frac{119}{169}$ A1 4

(c) Use of $\cos(x + \alpha) = \cos x \cos \alpha - \sin x \sin \alpha$

Substituting for $\sin \alpha$ and $\cos\alpha$

$12 \cos x - 5 \sin x + 5\sin x = 6$ ($12 \cos x = 6$) A1

$x = \frac{\pi}{3}$ awrt 1.05 A1 5

[14]

17. (a) $\tan x = \frac{8}{3}$ (or exact equivalent, or 3 s.f. or better) B1 1

(b) $\tan x = \frac{8}{3}$ $x = 69.4°$ (α), $x = 249.4°$ (180 + α) A1, A1ft 3

(c) $3(1 - \cos^2 y) - 8 \cos y = 0$ $3 \cos^2 y + 8 \cos y - 3 = 0$ A1

$(3 \cos y - 1)(\cos y + 3) = 0$, $\cos y = ...,$ $\frac{1}{3}$ (or –3) A1

$y = 70.5°$ (β), $x = 289.5°$ ($360 - \beta$) A1 A1ft 6

[10]

18. (a) $\theta - 10 = 15$ $\theta = 25$ ($\cos(\theta - 10) = \cos\theta - \cos 10$, etc, is B0) B1

$\theta - 10 = 345$ $\theta = 355$ M: Using 360 – "15" (can be implied) A1 3

Stating $\theta = 345$ scores A0

(Other methods: for <u>complete</u> method, A1 for 25 and A1 for 355)

(b) $2\theta = 21.8\ldots$ (α) (At least 1 d.p.) (Could be implied by a correct θ). B1

$2\theta = \alpha + 180$ or $2\theta = \alpha + 360$ or $2\theta = \alpha + 540$ (One more solution)

$\theta = 10.9, 100.9, 190.9, 280.9$ (divide by 2) A1ft A1 5

(A1ft: 2 correct, ft their α) (A1: all 4 correct cao, at least 1 d .p.)

(c) $2\sin\theta \left(\frac{\sin\theta}{\cos\theta}\right) = 3$, $2\sin^2\theta = 3\cos\theta$ A1

$2(1 - \cos^2\theta) = 3\cos\theta$

$2\cos^2\theta + 3\cos\theta - 2 = 0$

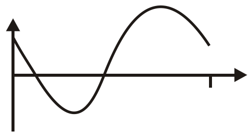
$(2\cos\theta - 1)(\cos\theta + 2) = 0$ $\cos\theta = \frac{1}{2}$ (M: solve 3 term quadratic

up to $\cos \theta = \ldots$ or $x = \ldots$) A1

$\theta = 60$, $\theta = 300$ A1 6

[14]

19. (a)



Cosine/Sine shape, period 2π B1

All "features" correct. B1 2

For first B1, if full domain is not shown, at least 0 to π must be shown, and in this case there must be indication of scale on the x-axis.
For second B1, ignore anything outside the 0 to 2π domain.

(b) $\left(0, \frac{1}{\sqrt{2}}\right), \left(\frac{\pi}{4}, 0\right), \left(\frac{5\pi}{4}, 0\right)$ or, e.g. $x = 0, y = \frac{1}{\sqrt{2}}$. B1 B1 B1 3

Allow 45° and 225°, allow any exact form.

Penalties: Non-exact, lose 1 mark (maximum).
Missing zeros, or coordinates wrong way round, lose 1 mark (maximum).

(c) $\left(x + \frac{\pi}{4} =\right) \frac{\pi}{3}$ or 60° or 1.05 (2 d.p. at least) B1

Other value $\left(2\pi - \frac{\pi}{3} =\right) \frac{5\pi}{3}$

Subtract $\frac{\pi}{4}$

(For the M marks, allow working in degrees or radians to 2 d.p. at least, but not degrees and radians mixed unless corrected later.)

$x = \frac{\pi}{12}, x = \frac{17\pi}{12}$ A1 4

Allow 0.08π and 1.42π (2 d.p. or better)
Ignore extras outside 0 to 2π.

n.b. There are other correct approaches for the first using the symmetry of the graph.

[9]

20. $2\cos^2\theta - \cos\theta - 1 = 1 - \cos^2\theta$

$3\cos^2\theta - \cos\theta - 2 = 0$ A1

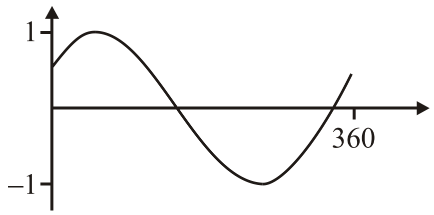
$(3\cos\theta + 2)(\cos\theta - 1) = 0$ $\quad \cos\theta = -\frac{2}{3}$ or 1 A1

$\theta = 0, \quad \theta = 131.8°$ B1 A1

$\theta = (360 - \text{"131.8"})° = 228.2°$ A1 ft

[8]

21. (a)



Scales (–1, 1 and 360) B1

Shape, position B1

(b) (0, 0.5) (150, 0) (330, 0) B1 B1 B1

(c) (x + 30=) 210° or 330° B1

One of these

x = 180° , 300°

M: Subtract 30, A: Both A1

[8]

22. $\sin^2 x = 4 \cos x$

$1 - \cos^2 x = 4 \cos x$

$\cos^2 x + 4 \cos x - 1 = 0$ A1

$$\cos x = \frac{-4 \pm \sqrt{16+4}}{2}$$

$$= \frac{\sqrt{20} - 4}{2}$$, second root has no real solution for x A1, B1

$x = 76.3°$ *or* $283.7°$ A1 A1 ft 8

[8]

23. (a) $x - 20° = 115.9°\ldots$
Or 244.08°...

Any solution (awrt 116° or 244°) B1
360° – candidate's first answer
+ 20° at correct stage

x = 136°, 264° A1 4

(b) $$3\frac{\sin\theta}{\cos\theta} = 2\cos\theta$$

Use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$3 \sin\theta = 2\cos^2\theta = 2(1 - \sin^2\theta)$

Use of $\cos^2\theta = 1 - \sin^2\theta$

$2\sin^2\theta + 3\sin\theta - 2 = 0$

3 term quadratic in sin = 0 A1

$(2\sin\theta - 1)(\sin\theta + 2) = 0$

Attempt to solve

$\sin\theta = -2$(No solution)

$\sin\theta = \frac{1}{2}$

At least $\frac{1}{2}$ A1

So $\theta = 30°, 150°$ Both A1 6

[10]

1. While many struggled with this question, strong candidates often produced clear, concise, well-structured responses.

Finding the value of $\tan\theta$ in part (a) proved surprisingly difficult. The most common wrong answer was $\frac{5}{2}$ instead of $\frac{2}{5}$, but many candidates failed to obtain *any* explicit value of $\tan\theta$. Some, not recognising the link between the two parts of the question, failed in part (a) but went on to find a value of $\tan 2x$ in part (b) before solving the equation. Most candidates achieved an acute value for $2x$ and then used the correct method to find the second solution. At this stage some omitted to halve their angles and some did not continue to find the other two solutions in the given range. Alternative methods using double angle formulae were occasionally seen, but were rarely successful. Some candidates resorted to using interesting 'identities' such as $\cos 2x = 1 - \sin 2x$.

2. (a) Most candidates correctly substituted $1 - \sin^2 x$ for $\cos^2 x$, but some lost the accuracy mark through incorrect manipulation of their equation or failure to put "equals zero".

(b) Most factorised or used the formula correctly and earned the first two marks. The most common errors again involved wrong signs. Most candidates correctly obtained the two answers 30 and 150 degrees. Some however gave the second angle as 210, others as 330 and another significant group gave three answers. Those who had made sign errors were able to get a follow through mark for giving a second angle consistent with their first.

3. The style of this question was unfamiliar to many candidates and this produced a generally poor performance, with weaker candidates often scoring no marks at all and many good candidates struggling to achieve more than half marks overall. Much time was wasted on multiple solutions, especially for part (i).

In part (i), where values of $\tan\theta$ and $\sin\theta$ could have been written down directly from the given equation, the most common strategy was to multiply out the brackets. This often led to protracted manipulations involving trigonometric identities and, more often than not, no answers.

Most candidates did a little better in part (ii), starting off correctly by expressing $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$ and often proceeding to divide by $\sin\theta$ and find a value for $\cos\theta$. What very few realised, however, was that $\sin\theta = 0$ was a possibility, giving further solutions 0 and 180°. Those who tried squaring both sides of the equation were often let down by poor algebraic skills. Just a few candidates resorted to graphical methods, which were rarely successful.

In both parts of the questions, candidates who were able to obtain one solution often showed competence in being able to find a corresponding second solution in the required range. Familiarity with trigonometric identities varied and it was disappointing to see $\sin\theta = 1 - \cos\theta$ so often.

4. In part (a) most candidates correctly substituted for (sinx)^2 but some lost the A mark through incorrect signs or a failure to put their expression equal to zero.

For part (b) most factorised or used the formula correctly and earned the B1. Unfortunately some who failed to achieve the given answer in a) carried on with their own version of the equation. There were many completely accurate solutions, but others stopped after 360 – 75.5 or did just 360 + 75.5 and some candidates tried combinations of 180 +/– 75.5 or 270 +/– 75.5. A few candidates mixed radians and degrees.

This question was answered well by a majority of candidates.

5. The most able candidates completed this question with little difficulty, sometimes using sketches of
the functions to identify the possible solutions. Generally speaking, however, graphical approaches
were not particularly successful.

In part (a), most candidates were able to obtain 45° and went onto get $x = 65°$ but it was disappointing to see 115° so frequently as the second answer (180 – 65). A surprising number of candidates subtracted 20 rather than adding, giving the answers 25° and 115°.
A number of candidates gave their first angle in radians and then proceeded to get further solutions by mixing degrees with radians. It was encouraging that few candidates thought $\sin(x - 20)$ was equivalent to $\sin x - 20 \sin$.

In part (b), the majority of candidates were able to obtain one or two correct solutions, but sometimes 'correct' answers followed incorrect working. Those with a good understanding of trigonometric functions produced very concise solutions, adding 360° and 720° to their values of $3x$, then dividing all values by 3. Weaker candidates often gave solutions with no clear indication of method, which were very difficult for examiners to follow.

As in part (a), disastrous initial steps such as $\cos 3x = -\frac{1}{2} \Rightarrow \cos x = -\frac{1}{6}$ were rare.

6. For the majority of candidates part (a) produced 2 marks, but part (b) was variable. Good candidates could gain full marks in (b) in a few lines but the most common solution, scoring a maximum of 4 marks, did not consider the negative value of $\sin\theta$. There were many poorly set out solutions and in some cases it was difficult to be sure that candidates deserved the marks given; a statement such as $5\sin^2\theta = 3 \Rightarrow \sin\theta = \frac{\sqrt{3}}{\sqrt{5}}$, so $\theta = 50.8°, 309.2°$, could be incorrect thinking, despite having two of the four correct answers.

7. Most candidates used the appropriate trigonometrical identity and many continued to find a correct quadratic equation insin x. However, poor algebraic manipulation (mainly sign errors and careless use of brackets) led to a number of candidates obtaining quadratic equations that were harder to solve than the correct one. Other errors seen included sign errors in factorising, more than 2 solutions given to the equationsin $x = \frac{1}{2}$ for $0 \le x < 2\pi$ and solutions given as decimals rather than multiples of π. A number of candidates gave solutions in degrees first. Some of these candidates then showed a misunderstanding of the required method for converting their solutions to radians. Weaker candidates incorrectly substituted $1 - \cos x$ for $\sin x$ or made no progress at all with this question.

8. Finding the value of $\tan\theta$ in part (a) proved surprisingly difficult for some candidates. Some, not recognising the link between the two parts of the question, failed in part (a) but went on to find a correct value in part (b) before solving the equation. Those who had a value for $\tan \theta$ were usually able to attempt a solution to the equation. Most candidates achieved an acute value for θ but then some omitted or used wrong methods to find the second solution. Another, almost invariably unsuccessful, method seen in part (b) was the use of $\sin^2\theta + \cos^2 \theta = 1$, or a false variation such as $\sin \theta + \cos\theta = 1$.

9. The work on trigonometry was of a high quality. There are 6 answers to be found in this question and the majority of candidates did find all of them. In part (a), the error $\tan\theta = \frac{2}{3}$ was sometimes seen and a few missed the second solution. In part (b), one or both solutions associated with $\cos\theta = 0$ were sometimes lost. Almost all candidates gave the answers to the degree of accuracy specified.

10. This was probably the least well answered question on the paper, with well under 50% achieving full marks in either part. Most candidates started part (a) quite well by dividing by 5 and finding 36.9 as their first solution to $\theta + 30$. It was disappointing though to see how many went on to say $\theta = 6.9$ or 173.1 . There were still some who thought that $\sin(\theta + 30) = \sin \theta + \sin 30$ and they made no progress.

In part (b) under 40% remembered to include the ± when taking the square root. Most knew how to find a second solution when solving a tan equation but there were some candidates who simply kept adding 90 to their first solution and others who took their value away from 270.

There were only a few candidates who had their calculators still in radian mode and gave answers that were a mixture of radians and degrees. Candidates should be encouraged to get a feel for the size of angles in both degrees and radians, an answer of 1.1 degrees for $\tan \theta = 2$ should raise suspicion.

11. Although tan $(3x + 20^{\circ}) = \tan 3x + \tan 20^{\circ}$ was seen from weaker candidates, the first three marks were gained by a large number of candidates. Consideration of the third quadrant results was generally only seen from the better candidates and so two correct solutions, and more particularly three correct solutions, were not so common. Candidates who used the correct expansion of tan $(3x + 20^{\circ})$ often made subsequent errors and it was rare to see all marks gained from this approach.

In part (b) many candidates gained the first method mark, although "division errors" such as "$2\sin^2 x + \cos^2 x = \frac{10}{9} \Rightarrow 2\tan^2 x + 1 = \frac{10}{9}$" were common.

It was surprising to see the number of candidates who, having reached a correct result such as $\sin^2 x = \frac{1}{9}$, lost at least one of the final two marks, and it was only the best candidates who scored all four marks in this part. The most common, and not unexpected error, was to solve only $\sin x = + \frac{1}{3}$, but often the answer was not corrected to 1 decimal place and so this mark was lost.

12. Well-prepared candidates often scored full marks on this question. In part (a), most recognised $\frac{\sqrt{3}}{2}$ as sin 60° (or were able to find 60° otherwise) and went on to subtract 10° to find one correct solution. The second solution (110°) was often missing or incorrect, a common mistake here being to subtract 50° from 180°. Just a few candidates started with $\sin(x + 10) = \sin x + \sin 10$, making little progress.

Performance in part (b) was similar, in that again the second solution was often missing or incorrect. Most candidates rounded their answers to 1 decimal place as required. Only a few were tempted to proceed from $\cos 2x = -0.9$ to $\cos x = -0.45$.

In both parts, it was rare to see extra solutions either within or outside the given range.

13. Some candidates didn't attempt this at all. Those who did, generally got the first A lot managed to factorise correctly or solve using the formula to get 0 and -1. Some however divided through by cos θ to lose the cos θ=0 solutions. The solution $\frac{3\pi}{2}$ was frequently omitted or written as $\frac{3\pi}{4}$. Most candidates who had the correct answers did give them in the correct form ; 1.57, 3.14, 4.71 or 90, 180, 270 were seen only a few times. Extra solutions of 0 and 2π unfortunately were seen quite frequently.

14. Not enough steps of working were shown by many candidates here. Frequently k=√3/sin60=2 was stated without reference to the fact that sin60=√3/2

(b) This was well answered and many candidates had both parts correct. Of those who didn't quite a few gained a B1 follow through for p+180.

(c) Most candidates gained a few marks here, but not many had full marks. The most common mistakes were to find -53.1, then subtract the 60 to get -113.1 resulting in 180-113.1 and 360-113.1. Some didn't give their answers to 1dp, and others mistakenly stated that 0.8=sinx+sin60 and proceeded to geta range of erroneous solutions!

15. Most candidates knew the appropriate trigonometric identity to answer part (a) and full marks were usually score here. The candidates usually went on to solve the equation correctly but some errors occurred after this. Some students realized that $x = -90$ was a solution to $\sin x = -1$, but could not find its equivalent value in the required range, sometimes listing 90 instead or indeed as well. Many students found the other two solutions in the 1st and 2nd quadrants, but some false solutions occasionally appeared based on $180+\alpha$ or $360-\alpha$.

16. Trigonometric identities are not popular with many candidates but the answers to the first part of the question showed that work in this area is improving and more than three-quarters of the candidates were able to make a correct start to the proof. It is also pleasing to note that there were many complete and formally elegant solutions. Most of these started from the left hand side of the identity, using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$, putting the expressions over a common denominator and using a double angle formula to complete the proof. Part (ii)(a) was well done but in part (ii)(b) many did not realise a double angle formula was needed and some just doubled their answer to part (ii)(a). The candidate was expected to produce appropriate working leading to an exact answer to obtain credit in part (ii)(b). In part (ii)(c), many tried to use a $R\cos(x-\alpha)$ method of solution and produced much work often leading to a fallacious answer. Those who expanded, substituted, and obtained $12\cos x = 6$ sometimes lost the final mark by giving $x = 60°$, not recognising that the question implies the angles are in radians.

17. While most candidates appeared to know that $\tan x = \dfrac{\sin x}{\cos x}$, many were unable to perform correctly the required steps to find the value of $\tan x$ in part (a). Common wrong answers here were $\tan x = \dfrac{3}{8}$ and, for example, $0.375\tan x = 0$. The link between parts (a) and (b) was often not recognised, so candidates tended to start again (and were sometimes more successful!). Those who were able to find a suitable value for $\tan x$ usually knew how to find the third quadrant solution for x. There was often more success in part (c), where most candidates realised that they needed to use $\sin^2 y + \cos^2 y = 1$ to form an equation in $\cos y$. Those who did not use the identity made no progress and those who misquoted it as, for example, $\sin y + \cos y = 1$, fared little better. There were however, many completely correct solutions to this part and it was pleasing that candidates were usually able to find both y values in the required interval. Weaker candidates frequently abandoned this question.

18. Performance in the three parts of this question was rather varied, with part (b) proving the best source of marks for many candidates. In part (a), while most scored the first mark for finding $\theta = 25°$, a significant number made the mistake of giving 335° as the second solution (360° – 25° rather than (360° – 15°) + 10°). Occasionally the value of cos 15° was used as a value for q, or cos (θ – 10°) was expressed as cos θ – cos 10°.

The main problem in part (b) was that of incomplete solution sets, with candidates dividing by 2 before generalising, leading only to θ = 10.9° and θ = 190.9°.

While some candidates made no progress at all with part (c), the majority at least attempted to use the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$. Some found it difficult to simplify and proceed from there, but those who noticed the $\sin^2\theta$ were often able to continue, using $\sin^2\theta + \cos^2\theta = 1$ to attempt a full solution. Those who managed to reach the correct quadratic equation in cos θ almost always completed their solution successfully, giving both θ = 60° and θ = 300°.

19. In part (a), there were many good attempts to sketch the cosine curve, with the translation “to the left” well understood. One of the two available marks was often lost, however, sometimes because curves were incomplete for the required domain and sometimes because the position of the maximum was incorrect.

Exact coordinates were demanded in part (b), and few candidates scored all 3 marks available here, even though the mark scheme generously allowed x values in degrees as an alternative to radians. The point $\left(0, \frac{1}{\sqrt{2}}\right)$ was usually omitted or given as a rounded decimal, zero coordinates were often omitted and coordinates were sometimes reversed.

While a few candidates used their sketch to help them to solve the equation in part (c), most tackled this independently, with variable success. The first correct solution $\frac{\pi}{12}$ was often seen, the other $\left(\frac{17\pi}{12}\right)$ much less frequently. A common mistake was to calculate $2\pi - \left(\frac{\pi}{4} - \frac{\pi}{3}\right)$ instead of $\left(2\pi - \frac{\pi}{3}\right) - \frac{\pi}{4}$. Working in degrees and changing to radians at the end was a successful strategy for many candidates, but sometimes degrees and radians were combined to give confused values.

20. Candidates who did not recognise this “standard” type of trigonometric equation were unable to get started, but the vast majority realised the need to use $\sin^2\theta + \cos^2\theta = 1$, and many were able to simplify the equation to find the correct three-term quadratic in $\cos\theta$. From there onwards, many completely correct solutions were seen, although some candidates, having found 131.8°, were unable to obtain the corresponding third quadrant solution. The solution θ = 0 was sometimes found and then rejected as invalid, and occasionally extra solutions (wrong rather than outside the required interval) were included.

21. Some good graph sketches were seen in part (a), where the usual method was either direct "plotting" or translation of the graph of $y = \sin x$. Many candidates, however, did lose a mark here because their graph was incomplete, stopping at the x-axis where $x = 330°$, or because indication of scale on the axes was inadequate. A few translated the graph of $y = \sin x$ in the wrong direction, but were given follow-through credit in the remainder of the question where possible.

Those who produced good sketches were usually able to give the correct coordinates for the intersection points with the x-axis, but the intersection (0, 0.5) with the y-axis was often carelessly omitted.

While a few candidates were able to make efficient use of their sketches to solve the equation in part (c), the majority followed the usual equation-solving procedure and often had difficulty in reaching the required solutions. Typically, $(x + 30) = -30$ was seen, leading to $x = -60$, but complete methods showing clearly $(x + 30) = 210$ and $(x + 30) = 330$ were disappointingly rare.

22. No Report available for this question.

23. No Report available for this question.