



# Admissions Testing Service

## STEP Examiner's Report 2015

Mathematics

STEP 9465/9470/9475

October 2015



Test

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### **STEP Mathematics (9465, 9470, 9475)**

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# SI 2015 Report

## General Comments

The aim of this report is to account for how well, or how poorly, the candidates performed this year while at the same time attempting to indicate their corresponding areas of strength, or weakness. If I should concentrate marginally more in the direction of the candidates' weaknesses, the reader should understand that it is with the hope that both future STEP candidates, and the teachers preparing them for this examination in years to come, will have the opportunity to focus on those areas of common weakness in an effort to ensure a better preparedness for what is unquestionably the most demanding of examinations in the UK for students of pre-university years.

For the record, the scripts are marked by a team of postgraduate mathematical students – many working towards a doctorate in this, or another, closely-related subject – who spend days poring over the scripts, working in small teams under the supervision of the Principal Examiner and carefully appointed “question captains”. Their powers of concentration are truly phenomenal and they not only appreciate the need for mathematical rigour but (due to having once been in exactly this position themselves) are also deeply sympathetic towards the candidates; making every effort first to understand what has been presented to them by the candidates and then to reward genuinely good mathematics when it appears, no matter how hastily and/or messily it has been set down onto paper. Thus it is that the comments produced by the Principal Examiner within this Report are merely summaries of what these markers have passed on to him (or her) at the end of the marking period. Moreover, since the candidates' backgrounds are entirely unknown to the markers, any comments – critical or otherwise – cannot possibly be taken to have been directed towards specifically chosen targets.

More than 2000 candidates sat SI this year, which represents another increase of around 10% over last year's entry numbers. Once again, however, it is sadly the case that many of these candidates have simply not prepared sufficiently well to be in a position to emerge from the experience with any amount of positive feelings of success at the results of their efforts. In the first instance, many candidates (this year, more than half of the entry) attempt more than the recommended six questions. This automatically penalises them for the time that they have spent on extra questions whose marks will not count towards their final total – remember that only the highest-scoring six questions count towards a candidate's final total; note also that a grade 1 can usually be obtained from four questions which have been completed reasonably successfully, or from two questions done completely correctly plus four “halves”, or from any in-between combination of question-scores. Thus, it is strongly advised that candidates spend a few minutes at some stage of the examination reading the questions carefully with a view to deciding which of them they would best attempt.

Overall, this year's paper worked out in very much the same way as had the 2014 paper, with a mean score of around 43-44% and with approximately 45% of candidates failing to exceed a total of 40 marks; though totals in excess of 100 marks were slightly down on 2014. Part of the reason for this is that the five applied maths questions were very much non-standard this year, and this prevented a lot of very easy marks (for routine beginnings) being picked up by candidates attempting these questions. The mean score for Qs.9-13 thus fell from  $6\frac{1}{2}$  in 2014 to  $2\frac{1}{2}$  in 2015. Another trend of recent years is the widespread dislike for the vectors questions, which have become both unpopular and very low-scoring for candidates.

Points of general application regarding candidates' attempts this year are little different to usual – far too many candidates produce only fragmentary attempts (often, as mentioned above, to almost every question they attempt) at solutions, with little apparent intent to persevere beyond the first obstacle. Presentation was also particularly poor this year, with most candidates making life hard both for themselves and for the markers who genuinely wish to find credit-worthy mathematics in order to award the marks available. In long questions such as these, with the barest minimum of structure provided, candidates need a lot of prior practice at past STEP questions in learning how to supply their own.

Curve-sketching skills continue to be a weakness, as candidates tend to veer away from justifying what they have drawn; algebra and calculus skills are very mixed, and it was especially clear this year how little candidates like being required to formulate their own solution-strategies – no doubt being the result of an over-reliance on being told exactly what to do, as is customarily the case in AS- and A-level papers.

On the other side of the coin, there was a very pleasing number of candidates who produced exceptional pieces of work on 5 or 6 questions (or more), and thus scored very highly indeed on the paper overall. Around 80 of them scored 90+ marks of the 120 available, and they should be very proud of their performance – it is a significant and noteworthy achievement.

### Comments on individual questions

[Examiner's note: in order to extract the maximum amount of profit from this report, I would firmly recommend that the reader studies this report alongside the *Hints and Solutions* or *Marking Scheme* supplied separately.]

**Q1** Traditionally, question 1 is intended to be the most generous and/or helpful question on the paper and, most years, it is attempted by almost everyone. This year – despite the fact that it is obviously the question most similar to one that might appear on an A-level paper – the lack of given structure, and the requirement for sketching, clearly put many candidates off, and only three-quarters of candidates even started it. Once started, however, it proved to be the highest-scoring question for candidates (as was the intention), eliciting a mean score of over  $13\frac{1}{2}$  out of 20. Most serious attempts were thus highly successful, and it was generally only in part (ii) that marks were commonly lost in bulk. Sadly, many who did move into (ii) with some measure of success, overlooked the fact that it was a fairly straightforward follow-on from all the information gained or used in part (i), and many attempts were thus successful but unnecessarily long-winded.

**Q2** This was the most popular question of all, attempted by around 85% of candidates, and producing a mean success rate of just over 10 out of 20. Part (i) was almost always concluded successfully, and by the anticipated method of the use of the addition-formulae for sine and cosine; a method then used sensibly, along with the well-known double-angle formulae, in (ii) to establish that  $\cos\alpha$  was indeed a root of the given cubic equation. For many candidates, though, the other roots of this cubic were often “found” by guesswork, and many candidates thought it appropriate to “cancel”  $x$  with  $\cos\alpha$  in some strange way, rather than resort to the use of the quadratic formula. Part (iii) was frequently not attempted at all, and many who did boldly venture forth therein did so by **not** using the results of parts (i) and (ii) as instructed. A few rather presumptuously assumed that  $\cos 15^\circ$  was a solution, when a little care would have revealed that it is, in fact,  $2\cos 15^\circ$  that fits the bill; the key being that  $2\cos 45^\circ$  is the  $\sqrt{2}$  at the end of the given equation.

**Q3** This question was attempted by just over half of the candidature, but produced a mean score of only 4 marks ... largely due (I suspect) to the twin difficulty of a lack of supplied structure and the poor ability of candidates to do their own modelling. In many instances, the two cases  $b < 3a$  and  $b > 3a$ , given in the question, proved to be unhelpful as many candidates chose specific values of  $b$  in each of these ranges as “exemplar” values of  $b$  and then supposed that this sufficed in establishing the boundary cases; when, in fact, the given information was intended to guide where they were to end up rather than from where they should begin. It was also especially disappointing to see that so many candidates struggled to explain what they were doing, thinking that some poorly labelled diagram would ‘do the trick’. The poor thinking behind the diagrams usually meant, for instance, that one of the two possible scenarios for when the guard stood at the midpoint of a side was completely unconsidered. Rather strangely – and disastrously in terms of scoring any marks at all – it was very common indeed for candidates to have considered the **area** of the courtyard that was visible to the guard, despite the very clear reference to the “length of the perimeter” in the question.

**Q4** Amongst the pure maths questions, this one was least popular, with less than a third of candidates making an attempt at it, and producing a mean score of  $4\frac{1}{2}$  marks. Unlike Q3, there was a lot of helpful information given in the question, and key intermediate results also. Those candidates who realised that the gradient of the rod was  $\tan\theta$  answered the first part quite acceptably, although it was relatively common to see the gradient of the curve,  $\frac{1}{2}x$ , being used as the “ $m$ ” in the formula  $y = mx + c$  for a straight line.

It is, unfortunately, often the case that when an answer is given with the view that it will prove helpful to candidates, that they then miraculously manage to obtain it through any means possible, and there were many wayward attempts to justify the given final answer without any clear supporting evidence: the biggest difficulty arose from the need to use  $A_x = 0$  in order to eliminate  $b$ .

**Q5** More than 60% of candidates attempted this question, and scores were relatively healthy, with a mean of almost  $7\frac{1}{2}$ . Those candidates who realised that  $x$  was being treated as a constant within the integrals generally found it fairly straightforward to make good progress; those who didn’t were doomed to failure from the outset. The sketches were generally completed fairly successfully, though few candidates managed to be entirely convincing, especially in (ii), where the modulus function needed to be employed (although some candidates thought the matter through sufficiently carefully without it by considering the various intervals of the domain of  $g$ ).

**Q6** This vectors question proved both unpopular and low-scoring, eliciting a mean of only 2.3 out of 20. In many cases, this was because candidates started their “solution” with a diagram before abandoning it altogether and moving on elsewhere. Most of the remaining attempts assumed that the quadrilateral was a square, rectangle or parallelogram to begin with – whether through a misreading of the question or through an inability to deal with a general scenario it is hard to say. Moreover, the usual convention of underlining vector quantities (thereby distinguishing them from scalar ones) was almost universally avoided, and this made it extremely difficult to give serious consideration to much of what was written, as candidates moved from scalar to vector and back again.

**Q7** This was another popular question, since most candidates were able to make some progress with the ideas involved, though few seemed to have a particularly thorough grasp. The change in the variable being considered – from  $x$  to  $f(x)$  and then to  $a$  – was evidently the source of much of the confusion, though I am sure that candidates would have made better progress with a more carefully laid out plan for working through the different possibilities. In the end, it all boiled down to the fact that a (continuous) function takes its maximum value *on a finite interval* at either an endpoint or at a maximum turning-point. Thereafter, it was important to do some sensible comparisons using inequalities. Thus, there were easy enough marks to be had and the mean score of 7.7 out of 20 was the third highest on any question, after questions 1 & 2.

**Q8** This was another very popular question, with three-quarters of the candidature making a start at it. However, the mean score of 5.4 almost certainly arose from the acquisition of the 3 marks allocated to the bit of introductory bookwork plus 2 or 3 marks gained by considering a suitable pairing of terms for considering  $S$  in (ii). Inductive proofs were unnecessary in (i), and almost invariably went wrong in (ii); this was a shame when the appearance of the term  $(N - m)^k$  in the given expression really indicated for the use of the binomial theorem. It should have been relatively straightforward to apply the given initial result of (ii) in the two cases that followed, but each required the addition of an extra  $0^k$  term ... perversely, to make the odd number of terms into an even one, and then v.v. (since it is now the isolated middle term that is crucial), and this extra leap of intuition was clearly where the difficulty lay. The final arguments relied on a sound grasp of what had gone before, and most candidates had given up by this point.

**Qs.9-11** These mechanics questions were – as mentioned earlier – very non-standard, and hence found very difficult. Of the relatively few attempts appearing from candidates, almost none of them got further than a hesitant start. There were two or three easy marks to be had in Q9 in finding the general time for any one bullet to land, but very few candidates were able to cope with replacing a specific launch angle with the variable angle given

In Q10, the real key to successful progress was to avoid worrying about any constants of proportionality (such as that introduced by the unstated width of the bus), so very few candidates managed to produce the early given result in a satisfactorily justifiable way. Introducing an extra proportionality relationship for the journey time was then a leap too far, even for those who had started well. Moreover, there were some candidates who only seem to be able to maximise or minimise a function by using calculus, and this provided an extra layer of unnecessary clutter here. Fewer than 200 candidates started this question, and most of these attempts had little more than a sketchy diagram for the markers to consider.

Q11 attracted double the number of attempts of Q10, but had a marginally lower mean score, and this was slightly surprising. A few years ago, statics questions such as this would have been gobbled up with glee by many candidates, happy to collect some very easy mechanics marks. The great hurdle for the weaker candidates remains the widespread inability to draw a good diagram with all relevant forces marked on it in appropriate directions. Sadly, here, almost all diagrams failed to include all of the relevant forces, and decent progress beyond that point was, therefore, essentially impossible. Remarkably few candidates managed even to explain satisfactorily that the two frictional forces were equal (by taking moments about the central axis of each of the two cylinders).

**Qs12 & 13** These questions were also less routine than has usually been the case in recent years, although they were nowhere near as demanding as the 2-3 mark mean score might suggest. In Q12, despite the reference to the Poisson Distribution in the introduction, part (i) required a simple statement of a Binomial term. Part (ii) then proved difficult as it became clear that few candidates could manipulate a summation of terms in order to establish a result they might have anticipated being allowed to quote in an ordinary A-level examination. A few candidates managed part (iii) perfectly adequately without having gone very far with (ii), and this represented a shrewd use of “examination technique” on their part.

Most attempts at Q13 got little further on in the question than writing the simple, general term for  $P(A)$ ; namely  $\left(\frac{5}{6}\right)^{n-1}\left(\frac{1}{6}\right)$ . Even though the Geometric Distribution is not expected here, the probability of the run of independent events “ $n - 1$  failures followed by a success” should be within the scope of any STEP candidate (who has studied any small amount of probability and/or statistics). Many candidates decided that (ii) and (iii) also required a general expression for each of  $P(B)$  and  $P(B \cap C)$ , whereas these were clearly intended to be numerical, and a brief ‘*symmetry*’ argument quickly reveals their respective probabilities to be  $\frac{1}{2}$  and  $\frac{1}{3}$ . Parts (iv) and (v) were tougher, but required the candidates to see that each was the sum of an infinite number of terms, and the helpfully given series expansion at the end of the question helped wrap these up. One third of all candidates made an attempt at this question, but almost none of them got around to either of these final two parts.

## STEP 2 2015 Report

As in previous years the Pure questions were the most popular of the paper with questions 1, 2 and 6 the most popular. The least popular questions on the paper were questions 8, 11 and 13 with fewer than 250 attempts for each of them. There were many examples of solutions in this paper that were insufficiently well explained given that the answer to be reached had been provided in the question.

### Question 1

This was a popular question, but a number of common errors resulted in a relatively low average score for the attempts made. A number of candidates did not appreciate that it is necessary in the first part to show both that the gradient is positive for all relevant values of  $x$  and to check the value when  $x=0$ . Additionally, many candidates failed to note that the next part of this question instructed them to use the result shown in the second section of part (i) and instead used a graphical method. Other common errors included an incorrect use of the chain rule in the second part leading to a sign error and incorrect statements of formulae for the sums.

### Question 2

The average mark for question 2 was the highest on the paper with a large number of good solutions produced using a wide variety of different methods. However, many solutions did not explain clearly the method being used – it is advisable to make every step of the solution clear, especially in the case of questions where the answer that is to be reached has been given. In many cases the diagrams drawn were not sufficiently large to allow candidates to work easily on the question and on a number of occasions the sizes of two angles were reversed in the diagram leading to other points being in the wrong position on the diagram.

### Question 3

On the whole attempts at this question were good with a significant number of candidates obtaining full marks. In the first part of the question a number of candidates did not interpret the difference between successive terms of the sequence as triangles which included a particular length edge and chose to enumerate all possible cases – if this was carried out correctly it was still possible to achieve full marks on this section. Even when unsuccessful in this part of the question many candidates were able to write down correct expressions for the general cases. The proof by induction was generally well done, although a number of candidates failed to justify the first case fully (which can easily be done by enumerating all of the cases). The final part of the question (the corresponding result for an odd number of rods) was not attempted by all candidates. Of those that did, those who attempted to use induction rather than applying the result from earlier in the question struggled to reach the correct answer.

### Question 4

This was a generally well attempted question, although it was a common error to draw graphs that were not continuous, even in some cases with statements that they were continuous. Marks were also lost through mislabelling of points on the graphs or through incorrect attempts to use arguments based on graphical transformations to deduce the shape of the graph. A number of candidates when trying to find the stationary points stated that they were going to differentiate a function, but then integrated it.



### Question 5

This question had one of the lower average marks for the Pure maths questions on the paper. Most candidates were able to produce a proof by induction for the first part, but the vast majority failed to realise that there was more that needed to be done to prove the result stated in terms of arctan. As is the case for a number of other questions, candidates need to give a clear explanation of each step of the solution. Where candidates identified the relationship between the two parts of the question the second part was generally well attempted.

### Question 6

This was the most popular question on the paper with over 1000 attempts made. The first section did not present significant difficulties to candidates and the integration was generally well completed, although occasionally with an error in the factor. The second part proved difficult for a number of candidates who failed to change the variable in the integral correctly, or in some cases did not change the variable in every position that it occurred. Other candidates did not apply a correct result for dealing with the trigonometric functions involved or did not clearly show how the required result was reached as the solution jumped through several steps to a statement of the result asked for in the question. There were very few successful attempts at the final part of the question, but they did include a variety of methods for evaluating the integral once the substitution had been made.

### Question 7

The first part of this question was generally well answered, although a significant number of answers did not give the equation of the new circle. The case in part (i) where the two circles have the same radius was often not considered and the explanations for there not being such a circle in some cases were often not sufficiently clear. A significant number of candidates made the incorrect assumption in the second part that the centres of the three circles must lie on a straight line or attempted this part of the question with incorrect methods, such as equating the equations of the two given circles. In the final part of the question not all candidates realised that  $y^2$  must be positive and were unable to obtain the required inequality by any other means.

### Question 8

This was one of the least attempted questions on the paper and the average score for the question was quite low. However, there were a number of very good answers to the question. Part (i) was answered correctly by the majority of candidates, but part (ii) was approached in a much more complicated manner than necessary by many candidates, attempting to work out the equation of the line rather than comparing vectors in its direction. Where the vectors were considered, solutions could have been made clearer by better grouping of the terms. A number of solutions referred to division of vectors rather than comparing coefficients. In the final part some candidates did not identify the simplest relationship between the vectors to ensure that Q lies halfway between P and R. Generally, more complicated relationships did not lead to correct solutions to this part of the question.

### Question 9

This was the most popular of the Mechanics questions, but many candidates struggled to achieve good marks. In the first section many candidates had difficulties in finding the correct angles to work with – a clear diagram is very helpful in tackling this problem. Candidates often introduced new notation to help with the steps toward the solution, but this was sometimes poorly chosen and made solution of the problem more difficult. Explanations of the methods being used were also often poor – in particular the triangles being used at different stages were not clearly identified. There were also a number of errors when taking moments or when recalling exact values of the sine and cosine functions. There were a number of good attempts at the second part of the question, but a large number of candidates calculated the kinetic energy incorrectly.

### Question 10

This question received generally very poor attempts, including a large number of partial attempts. The majority of attempts failed to get the correct expression of the velocity in the first part and this limited the number of marks that could be awarded for the remainder of the question. A very small number of attempts were awarded full marks and there were a considerable number of attempts in which correct methods were attempted following an incorrect solution to the first part of the question.

### Question 11

This was the least popular question on the paper. Many answers to the first part did not give good explanations of the method for obtaining the velocity of A. Similarly, in the second part there were a number of statements such as “conservation of velocity” or “conservation of the modulus of momentum” used to support the answer without sufficient explanation to show that a valid method was being applied. Those candidates who attempted to use the equations of motion under uniform acceleration were unable to reach the solution. Part (iii) was very poorly answered with almost no correct solutions offered. In the final part of the question very few candidates were able to identify the part of the reasoning that led to  $v$  not being equal to zero in all of the cases identified.

### Question 12

Many solutions to this question did not include sufficient explanation to gain full credit. In the first part, marks were not awarded simply for stating that the value of  $\frac{1}{4}$  could be achieved by multiplying  $\frac{1}{2}$  by  $\frac{1}{2}$  (often with an additional multiplication by 1) – an explanation of where this calculation comes from was also required. In the second part a number of candidates stated that it was symmetric and so the answer must be  $\frac{1}{4}$  but with insufficient explanation why. In part (iii), some candidates obtained a geometric series which was then summed to get the probability of C winning if the first two tosses are TT. In the final part some correct answers were offered, but without explanation of the method. A number of candidates made incorrect assumptions such as that  $p+q=1$ , or  $p+q+r=1$ . When finding the probability that C wins a lot of candidates were able to achieve some of the marks by working out the probability in terms of  $q$ .

### Question 13

This was not a popular question and those solutions that were offered generally showed a limited understanding of continuous probability distributions. The integration that was required was also generally quite poorly carried out. Often these mistakes made it difficult to answer the final section of part (i). Part (ii) was only attempted by a small proportion of candidates.

A very similar number of candidates to 2014 once again ensured that all questions received a decent number of attempts, with seven questions being very popular rather than five being so in 2014, but the most popular questions were attempted by percentages in the 80s rather than 90s. All but one question was answered perfectly at least once, the one exception receiving a number of very close to perfect solutions. About 70% attempted at least six questions, and in those cases where more than six were attempted, the extra attempts were usually fairly superficial

1. This was the most popular question, being attempted by 85% of candidates, it was however only moderately successful although a number achieved full marks. Quite often, candidates ignored the helpful approach suggested by the LHS of the first required result, though, of course, it was possible to start from the first defined integral and achieve the same result. Many needlessly lost marks through omitting fairly straightforward steps such as the final evaluation in the last part of the question and failing to substantiate the simplified form of the result of part (i). Some got very carried away with  $\tan$  or  $\sinh$  substitutions in part (i), usually unsuccessfully and leading to monstrous amounts of algebraic working. A few failed to change the limits of integration in [part (ii)].

2. Nearly three quarters attempted this, though again with moderate success as the main feature of the question was proof, and this was frequently handled cavalierly. Whilst it was not a crucial aspect of the question, ignoring the fact that the question deals with sequences of positive numbers was careless. Answers to the first part suffered at times from lack of argument or backwards logic. Part (ii) was generally well answered, although there were some silly counter-examples. This part suffered from those who completely missed the point of what the question was all about, forgetting the initial definition. Whilst most appreciated that part (iv) was true, there were many different methods used to attempt to prove it, and often unsuccessfully. Whilst induction using algebra is fairly straightforward, differentiation with or without logarithms and graphical methods frequently came to grief.

3. Under 20% attempted this, making it the least popular Pure question on the paper, and it was the least successfully attempted of all questions on the paper. Candidates seemed to find it intimidating, and many gave up before part (ii). They often got confused when dealing with separate cases and did not seem to understand what was required to show  $\sec \theta > 0$  in part (i). Those that did make a stab at (ii) usually omitted a factor of two and most failed to find the correct limit to use.

4. Along with questions 5 and 7, attempted by just over three quarters, this was the third most popular question, though a little less successful than the most popular question 1. The first part was frequently not well attempted, but the second part was usually mastered. Attempts at the third part suffered from arguments with poor logical structure, though many did not get a start on this part.

5. Marginally less successful than question 2, a lot of candidates earned about half of the marks. Unfortunately, many candidates approached this on the basis of their knowledge of the standard irrationality proof for root two employing rational numbers expressed in lowest terms rather than observing the specified argument. In part (i), proving step 5 was frequently beset with omissions, and simple steps like  $0 < \sqrt{2} - 1 < 1$  were not acknowledged let alone justified. The first result of part (ii) caused few problems except to those that did not appreciate 'if and only if', but defining a suitable set in order to construct a similar argument to prove the irrationality of the cube roots of 2 and 2 squared was beyond most leading to mostly spurious logic.

6. About three fifths of the candidates attempted this question but without great success. The first part tripped up many through needing to prove 'if and only if'. The first part of (ii) yielded good

scoring opportunities for those that did make progress on this question, though some fell by the wayside when it came to the situation that would not generate these possible values. Some attempts at the last result failed as the counter-example was not always shown to be a counter-example.

7. This was as successful as question 2 and so was third equal most popular and second equal most successful. Usually the very first result was comfortably answered, but there were many flaws in part (i) as many could not carry out a proper formal induction. In part (ii), which saw a lot fall by the wayside, some candidates thought that  $x$  commutes with  $\frac{d}{dx}$ , and often, candidates invented random formulae for  $D^n(1-x)^m$  from looking at the first few cases. Not surprisingly, working towards a given result, many came up with the correct result, but through spurious working such as substituting  $x = 1$  in  $(1-x)^m$  before using the differential operator.

8. This was a little less popular than question 1, but still was attempted by more than 80% of candidates, and was the question with highest scores. Many managed part (i) although several candidates did not realise that  $r$  was a function of  $\theta$ . Part (ii) was generally fine as far as the transformed differential equation but then the correct use of partial fractions to integrate having separated variables was less frequent than it should have been. A surprising number made no attempt to sketch any solutions despite doing the rest of the question either well or perfectly. Nobody realised that the constant  $A$  was truly arbitrary in part (ii) because of the modulus signs appearing in the log terms from the integral. The sketches tested all but the best.

9. Just over 20% attempted this, making it the most popular non-Pure question, and attempts at it were slightly less successful than question 1. Quite a few found the first required equation from applying Newton's 2<sup>nd</sup> Law, when some made sign errors through not being careful with directions, and then integrating rather than from conserving energy.  $x_0$  was found easily by the majority, and the expression for the acceleration was commonly still by Newton, though a lot of marks were lost by not substituting in  $x = x_0$ . The last part was poorly done in general with few getting more than an opening line, and if they made a sensible substitution, very few expanded it correctly. Not many even attempted the last part of the question.

10. Whilst this was the least popular question with just over 8% attempting it, it was only slightly less successfully attempted than question 5, though those making substantial attempts at it invariably scored half to two thirds of the marks comfortably. Most successfully wrote the position vector of one of the particles and then differentiated with respect to time to obtain the velocity correctly, though a few succeeded by adding velocities. The second displayed equation was almost always correctly derived, though many did far too much work obtaining the corresponding equations for the other particle when it could just be written down. Deducing  $\ddot{x}$  and  $\dot{y}$  was fine, but  $\ddot{\theta}$  frequently wasn't. At this point, finding initial values for  $\dot{y}$  and  $\dot{\theta}$  caused some issues, if it was realised that these were needed, and although many wrote the uniform acceleration equation for the displacement of the midpoint of the rod, the final result eluded many.

11. Marginally more popular than the Probability and Statistics questions, this was attempted by just over 10% with, on average, slightly less success than question 4, though students either got almost full marks or virtually none. Most did part (i) correctly except the final part, identifying the force from the hinge. The most common mistake was in part (ii) by those that assumed that there were no perpendicular forces acting on  $m_1$  and  $m_2$ . Students that correctly considered the total moment for part (ii) obtained the answer. Some students got the direction of centripetal force wrong.

12. The two Probability and Statistics questions were equally popular being attempted by about 10% of the candidates with, overall, this one achieving the same sort of scores as question 6. About a fifth of the candidates attempting it got right through the question. Most however did not seem to know what a probability generating function was, and it was often confused with the probability density function. Equally there was confusion between the labels of the random variables and of the PGFs. However most were happy working with the arithmetic congruent to moduli.

13. The large majority of attempts got almost no marks, and as a consequence this was the second worst scoring question. A lot failed to draw the right sort of graph to attempt the first part of (i) or, if they did, frequently miscalculated the area to find  $P(X + Y < t)$  in the case  $1 < t \leq 2$ . The next major problem was an inability to see how to find the cumulative distribution function of  $(X + Y)^{-1}$ . A surprisingly large number failed to multiply by  $t$  before integrating to find the expectation. A handful of candidates got most of the question right although only one made it clear with a symmetry argument why they could write down  $E\left(\frac{X}{X+Y}\right) = \frac{1}{2}$ .

## Explanation of Results STEP 2015

All STEP questions are marked out of 20. The mark scheme for each question is designed to reward candidates who make good progress towards a solution. A candidate reaching the correct answer will receive full marks, regardless of the method used to answer the question.

All the questions that are attempted by a student are marked. However, only the 6 best answers are used in the calculation of the final grade for the paper.

There are five grades for STEP Mathematics which are:

- S – Outstanding
- 1 – Very Good
- 2 – Good
- 3 – Satisfactory
- U – Unclassified

The rest of this document presents, for each paper, the grade boundaries (minimum scores required to achieve each grade), cumulative percentage of candidates achieving each grade, and a graph showing the score distribution (percentage of candidates on each mark).

### STEP Mathematics I (9465)

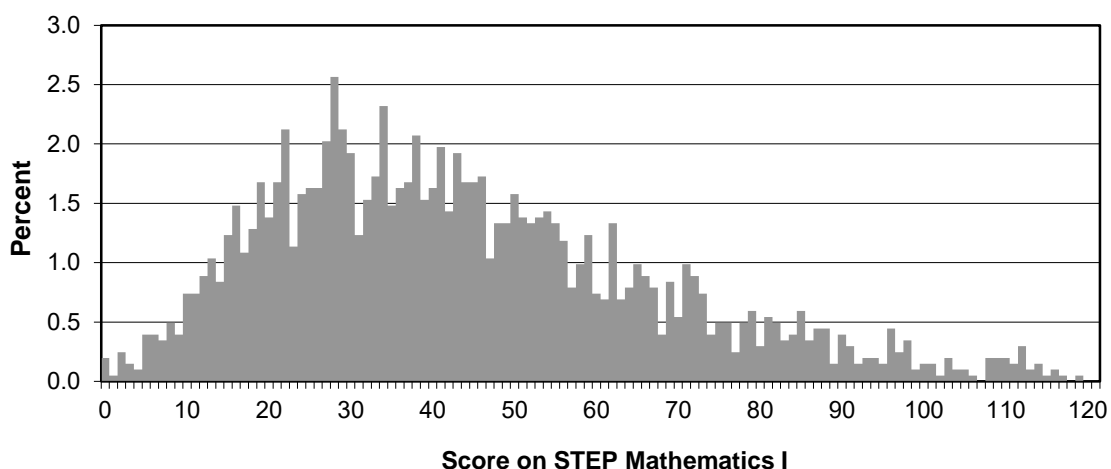
#### Grade boundaries

| Maximum Mark | S  | 1  | 2  | 3  | U |
|--------------|----|----|----|----|---|
| 120          | 96 | 65 | 45 | 28 | 0 |

#### Cumulative percentage achieving each grade

| Maximum Mark | S   | 1    | 2    | 3    | U     |
|--------------|-----|------|------|------|-------|
| 120          | 3.5 | 18.6 | 42.6 | 73.1 | 100.0 |

#### Distribution of scores



### STEP Mathematics II (9470)

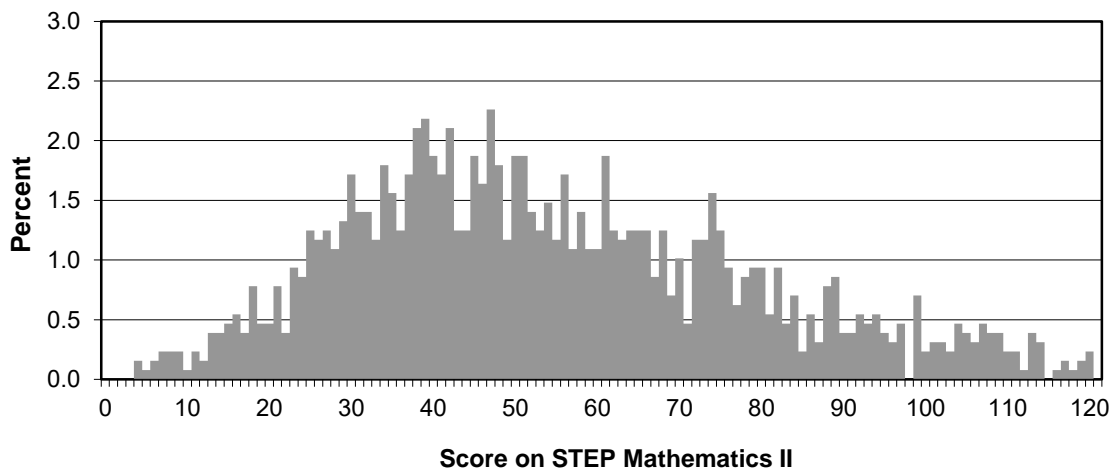
*Grade boundaries*

|              |    |    |    |    |   |
|--------------|----|----|----|----|---|
| Maximum Mark | S  | 1  | 2  | 3  | U |
| 120          | 94 | 68 | 60 | 30 | 0 |

*Cumulative percentage achieving each grade*

|              |     |      |      |      |       |
|--------------|-----|------|------|------|-------|
| Maximum Mark | S   | 1    | 2    | 3    | U     |
| 120          | 7.9 | 27.9 | 37.9 | 85.5 | 100.0 |

*Distribution of scores*



### STEP Mathematics III (9475)

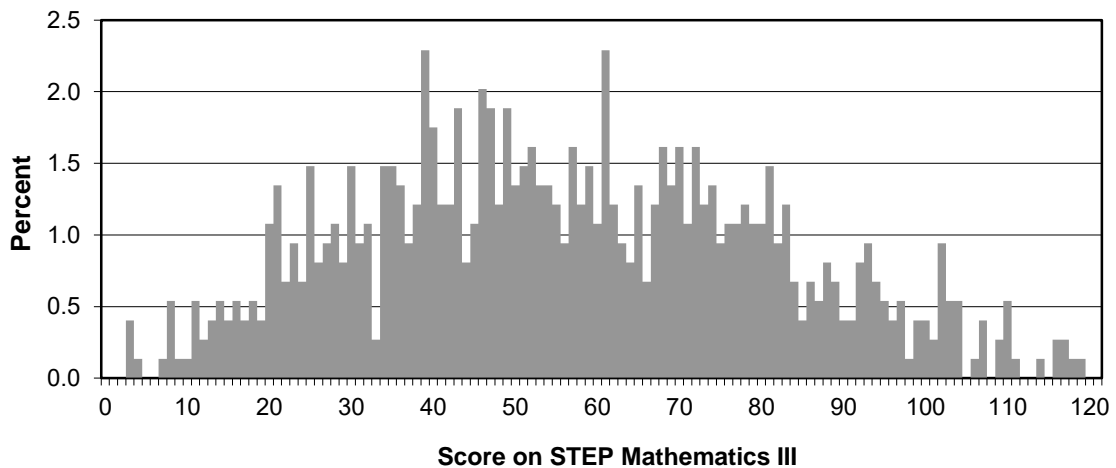
*Grade boundaries*

|              |    |    |    |    |   |
|--------------|----|----|----|----|---|
| Maximum Mark | S  | 1  | 2  | 3  | U |
| 120          | 88 | 65 | 54 | 29 | 0 |

*Cumulative percentage achieving each grade*

|              |      |      |      |      |       |
|--------------|------|------|------|------|-------|
| Maximum Mark | S    | 1    | 2    | 3    | U     |
| 120          | 11.8 | 37.3 | 51.4 | 85.5 | 100.0 |

*Distribution of scores*





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