CAMBRIDGE ASSESSMENT

STEP Examiners' Report 2011

Mathematics
STEP 9465/9470/9475

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TEST

Contents

STEP Mathematics (9465, 9470, 9475)

STEP 2011 Paper I: Principal Examiner's Report

Introductory comments

There were again significantly more candidates attempting this paper than last year (just over 1100), but the scores were significantly lower than last year: fewer than 2% of candidates scored above 100 marks, and the median mark was only 44, compared to 61 last year. It is not clear why this was the case. One possibility is that Questions 2 and 3, which superficially looked straightforward, turned out to be both popular and far harder than candidates anticipated. The only popular and well-answered questions were 1 and 4.

The pure questions were the most popular as usual, though there was noticeable variation: questions 1–4 were the most popular, while question 7 (on differential equations) was fairly unpopular. Just over half of all candidates attempted at least one mechanics question, which one-third attempted at least one probability question, an increase on last year. The marks were surprising, though: the two best-answered questions were the pure questions 1 and 4, but the next best were statistics question 12 and mechanics question 9. The remainder of the questions were fairly similar in their marks.

A number of candidates ignored the advice on the front cover and attempted more than six questions, with a fifth of candidates trying eight or more questions. A good number of those extra attempts were little more than failed starts, but still suggest that some candidates are not very effective at question-picking. This is an important skill to develop during STEP preparation. Nevertheless, the good marks and the paucity of candidates who attempted the questions in numerical order does suggest that the majority are being wise in their choices. Because of the abortive starts, I have generally restricted my attention to those attempts which counted as one of the six highest-scoring answers in the detailed comments.

On occasions, candidates spent far longer on some questions than was wise. Often, this was due to an algebraic slip early on, and they then used time which could have been far better spent tackling another question. It is important to balance the desire to finish a question with an appreciation of when it is better to stop and move on.

Many candidates realised that for some questions, it was possible to attempt a later part without a complete (or any) solution to an earlier part. An awareness of this could have helped some of the weaker students to gain vital marks when they were stuck; it is generally better to do more of one question than to start another question, in particular if one has already attempted six questions. It is also fine to write "continued later" at the end of a partial attempt and then to continue the answer later in the answer booklet.

As usual, though, some candidates ignored explicit instructions to use the previous work, such as "Hence", or "Deduce". They will get no credit if they do not do what they are asked to! (Of course, a question which has the phrase "or otherwise" gives them the freedom to use any method of their choosing; often the "hence" will be the easiest, though.)

It is wise to remember that STEP questions do require a greater facility with mathematics and algebraic manipulation than the A-level examinations, as well as a depth of understanding which goes beyond that expected in a typical sixth-form classroom. STEP candidates are therefore recommended to heed the sage advice on the STEP Mathematics website, `http://www.admissionstests.cambridgeassessment.org.uk/adt/step`:

> From the point of view of admissions to a university mathematics course, STEP has three purposes. ... Thirdly, it tests motivation. It is important to prepare for STEP (by working through old papers, for example), which can require considerable dedication. Those who are not willing to make the effort are unlikely to thrive on a difficult mathematics course.

Students will also benefit from reading the detailed STEP I solutions which I have written over the previous few years after attempting the papers; these are available from the "Test Preparation" section of the above website.

Common issues

There were a number of common errors and issues which appeared across the whole paper.

The first was a lack of fluency in algebraic manipulations. STEP questions often use more variables than A-level questions (which tend to be more numerical), and therefore require candidates to be comfortable engaging in extended sequences of algebraic manipulations with determination and, crucially, accuracy. This is a skill which requires plenty of practice to master.

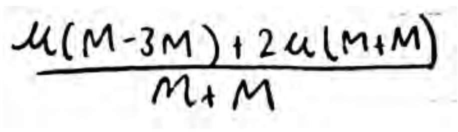
Another area of weakness is logic. The first section of the STEP Specification specifically lists: "Mathematical vocabulary and notation: including: 'equivalent to'; 'necessary and sufficient'; 'if and only if'; '$\Rightarrow$'; '$\iff$'; '$\equiv$'". A lack of confidence in this area showed up several times. In particular, a candidate cannot possibly gain full marks on a question which reads "Show that X if and only if Y" unless they provide an argument which shows that Y follows from X *and vice versa*.

Along with this comes the need for explanations in English: a sequence of formulæ or equations with no explicit connections between them can leave the reader (and writer) confused as to the meaning: Does one statement follow from the other? Are they equivalent statements? Or are they perhaps simultaneous equations? For example, writing $x = 2$ followed by $x^2 = 4$ is not the same as writing $x = 2$ followed by $2x = 4$, and both are different from writing $x = 2$ followed by $y = 3$. Brief connectives or explanations ("thus", "so", "$\therefore$", "$\Rightarrow$" or "$\iff$") would help, and sometimes longer sentences are necessary. The solutions booklet is more verbose than candidates' solutions need to be, but gives an idea of how English can be used.

In some cases, the need for explanations is even greater. Where a question instructs the candidate to prove a statement, writing down an equation without justification is likely to gain the candidate few (if any) marks. For example, it may well suffice to write "Taking moments around A" or even just "$\mathscr{M}(A)$" to indicate the source of the following equation. In a pure mathematics question, something like "Substituting (1) into (2):" may well be adequate.

Another related issue continues to be legibility. Many candidates at some point in the paper lost marks through misreading *their own writing*. Common confusions include muddling their symbols, the most common being: M and m; V and v; u and n; u and N;

x and n; α and 2; a and 9; s, S and 5; and occasionally z and 2. It is sad that, at this stage, candidates are still wasting marks because of bad writing habits. A particularly striking example is shown in this candidate's work:



This apparently reads

$$\frac{u(M-3m)+2u(M+m)}{M+m}$$

One frequent error was dividing by zero. On several occasions, an equation of the form $xy = xz$ appeared, and candidates blithely divided by x to reach the conclusion $y = z$. This may or may not be true, depending upon whether or not x could be zero. A better approach in general is to rearrange to get $x(y-z) = 0$ and to deduce that either $x = 0$ or $y = z$. Alternatively, if it is known with certainty that $x \neq 0$, it is fine to divide by x, but one must *explicitly* indicate that $x \neq 0$.

Again, I give a strong reminder that it is vital to draw appropriate, clear, accurate diagrams when attempting some questions, mechanics questions in particular: it was shocking how many candidates attempted to solve a collision question without a diagram or a moments question with a tiny, rough sketch!

Question 1

This was an easy and very popular first question, attempted by almost all of the candidates. It was also the most successfully answered, with a median mark of 14.

(i) Most candidates differentiated the equation implicitly, reaching the specified gradient with ease. Some decided to rearrange the equation into the form $y = \cdots$ before differentiating; this was frequently unsuccessful as the final manipulations required were relatively tricky.

The next step, showing that $p = \pm q$, was also generally answered well, even by candidates who had become stuck on the first part of the question. It was surprising to see many solutions using implicit differentiation to find the gradient of the straight line $ax + by = 1$, rather than rearranging the equation to get $y = mx + c$ and then reading off the answer.

The final step was also generally answered well, though a number of candidates could not see how to use the previous result ($p = \pm q$) to progress. Furthermore, there were a few candidates who managed to deduce one of the possibilities but not the other; this was a little strange, as the argument was essentially identical.

(ii) While the majority of candidates used the result that the products of normal gradients is -1, relatively few paid enough attention to notice that the equation of the curve was different to that of part (i). This led them to conclude that $a^2q^2 = -b^2p^2$, from which they were not able to make any progress. (Note here that the left hand side is always positive and the right hand side is always negative.)

Of those who overcame this hurdle and attempted the question posed, some succeeded in reaching the specified conclusion, while most deduced that $a^2q^2 = b^2p^2$ and then became stuck, unable to see how to progress. This is presumably because this equations were more complex than in the first part of the question. Also, some candidates who gave otherwise good answers did not consider both possible cases $aq = +bp$ and $aq = -bp$.

Question 2

This was a very popular question which was answered spectacularly poorly: almost half of candidates scored 0. Of the other half, the median score was 8, with them succeeding on the first part but getting no further.

For the first part, most candidates (presumably familiar with examples such as $\int x\mathrm{e}^x \,\mathrm{d}x$) attempted to integrate by parts, even though that was not the most appropriate method in this case. Those who did use parts frequently tried to integrate $x/(1+x)$ and tended to get nowhere. Very few understood that a second integration by parts was necessary to finish the job. Nevertheless, most of those who did attempt to use parts showed that they did have a reasonable working knowledge of the technique.

There were a number of shocking errors which recurred frequently. The first was to use the erroneous "rule" that $\int \mathrm{f}(x)\mathrm{g}(x)\,\mathrm{d}x = \int \mathrm{f}(x)\,\mathrm{d}x . \int \mathrm{g}(x)\,\mathrm{d}x$, so there were a number of candidates who wrote $\int \frac{\mathrm{e}^x}{1+x}\,\mathrm{d}x = \mathrm{e}^x \int \frac{1}{1+x}\,\mathrm{d}x$ or similar. Another common error was that E, a definite integral, would appear inside integrals or evaluations, giving statements or expressions such as $[xE]_0^1 = E(1) - E(0)$ or $\int E\,\mathrm{d}x$. There were also a number of candidates who applied parts and ended up with integrals inside integrals or integrals inside evaluations ($[\cdots]_0^1$).

A number of candidates also failed to distinguish between indefinite and definite integrals.

Parts (i) and (ii)

There were relatively few attempts at these later parts, though some who had been stumped by the first part of the question did succeed with this part.

Some candidates tried to apply the techniques of the first part to this integral, but many realised that a substitution was necessary and successfully executed it.

Most candidates who were successful with part (i) went on to complete the whole question.

Question 3

This was another very popular but poorly answered question. While most candidates were able to gain some marks, few proceeded beyond the initial part of the question, giving a median mark of 5 and an upper quartile of 7.

Candidates generally did well at the first part of this question, with the majority using the compound angle formulæ to expand the sines on the left hand side of the identity.

A number got stuck at this point, either because they did not know $\sin\frac{1}{3}\pi$ or $\cos\frac{1}{3}\pi$ or because they made algebraic errors. About a quarter of candidates were unable to complete the proof as they did not show that $\sin 3\theta$ is identical to $3\sin\theta - 4\sin^3\theta$ or equivalent; some simply stated the result with no justification. On the other hand, a number of candidates offered very nice arguments using de Moivre's theorem, which was very nice to see.

Some candidates used the factor formulæ, but these were in the minority.

(i) Many candidates who reached this point were able to differentiate the identity correctly, which was pleasing. A significant number failed at this hurdle, though. Of those who did differentiate correctly, though, the majority failed to realise that they could divide their new identity involving cosines by the original identity to reach the stated result. Since the rest of the question depended upon this idea, barely a quarter of candidates made any further progress. There was a strong hint though: the presence of $\cot\theta$ should suggest thinking about $\cos\theta/\sin\theta$.

(ii) Of those who had become stuck earlier, very few attempted this part, even though the first half was totally independent of part (i). Of those who did, a significant number became stuck after performing the given substitution, not realising that they could then use the identity $\sin(\frac{\pi}{2} - x) = \cos x$. There were also a number of candidates who successfully derived the result from scratch using the compound angle formula for tan.

Few candidates made it as far as the cosec equation. Of those who did, and realised that they needed to perform another division or equivalent, few were comfortable enough with the trigonometric manipulations involved to reach an expression involving $\operatorname{cosec} 2\theta$ and thence to reach the stated result.

Question 4

This was a popular question, attempted by two-thirds of candidates. It was also one of the most successfully answered, with a median mark of 11.

Candidates were very good at differentiating to find the coordinates of T, though there were some issues. Those who rearranged to find $y = \sqrt{4ax}$ generally did not handle the possibility that y could be negative. There were also a number of candidates who are still confused when trying to find the equation of a tangent: they used the *general expression* for $\mathrm{d}y/\mathrm{d}x$ rather than substituting in the values of x and y at the point of tangency. This gave them a "straight line" with equation $y - 2ap = \frac{2a}{y}(x - ap^2)$ which was then sometimes rearranged to give a quadratic.

The vast majority were fine with this step, though, and went on to successfully find the coordinates of T. Some used the symmetry of the situation to simply write down the equation of the second tangent, while others determined it from scratch.

There was one sticking point, though: at this level of work, candidates are expected to take care when dividing to ensure that they are not dividing by zero. A mark was therefore awarded for stating that $p - q \neq 0$ or $p \neq q$ when dividing by it, but very few candidates did so.

When it came to deducing the given formula for $\cos\phi$, most candidates made a good start, with the dot-product approach more popular than the cosine rule. However, there was a need for some fluent algebraic manipulations, in particular the ability to factorise. This should have been made a little easier by knowing the desired final result, but most candidates became bogged down at this point and were unable to deduce the given expression. The dot-product approach, with its slightly simpler algebra, was generally more successful.

The final part of the question, requiring candidates to deduce that the line FT bisects the angle PFQ, produced many spurious attempts. Few candidates appreciated the symmetry of the situation, and so went on to calculate $\cos(\angle TFQ)$ from scratch. Others attempted to find $\cos(\angle PFQ)$, presumably hoping to use a double angle formula or similar. These approaches were sometimes successful.

There were also candidates who attempted to answer this part by using right-angled trigonometry in one of the triangles, or by identifying similar or congruent triangles, even though none of these approaches made sense in this situation.

Question 5

This was a moderately popular question, attempted by half of the candidates. Most made a reasonably good start, but became stuck after deducing the value of I; only half of the candidates gained more than 7 marks.

The first part was answered fairly poorly. Most were able to correctly sketch the graph of $y = \sin x$ (though there were a few who could not) and $y = kx$, but very few made any attempt to justify from their sketch the required result. The most common mark for this part was 1 out of 4.

A handful of candidates decided to rewrite the equation as $\sin x/x = k$ and went on to draw a graph of $y = \sin x/x$, a far more challenging task which was successfully performed by some of the candidates.

The integration part (deducing the formula for I) was generally answered very well. There were some who did not understand how to split up the integral or the significance of $x = \alpha$ to this part, but the majority correctly handled the absolute value and the necessity to change the signs of the integrand in the integral from α to π.

Quite a number used geometrical arguments, considering a triangle from $x = 0$ to $x = \alpha$, a trapezium from $x = \alpha$ to $x = \pi$ and two regions under the curve; most of these also reached the given result.

Determining the stationary points of the function was found to be very challenging. While the differentiation was generally performed correctly, many struggled to factorise the resulting expression. Even among those who did, it was very common to divide by one of the factors, ignoring the possibility that it might be zero. Those who were lucky enough to divide by $\sin\alpha - \alpha\cos\alpha$ deduced the correct value for α at the minimum. Though they gained no immediate credit for this, they were able to continue attempting the rest of the question. Others, though, were less fortunate and asserted that the minimum occurred where $\sin\alpha = \alpha\cos\alpha$; they were unable to make any further progress.

It was very unusual to see any sort of decent justification that $\sin\alpha = \alpha\cos\alpha$ has no solutions in the required range.

Most of the students who reached this point correctly evaluated I at the stationary point to reach the given value.

Finally, students were required to show that the stationary point is a minimum, which few attempted. A significant amount of care was required for each of the approaches, and a small number did so successfully. A few only evaluated I or $\mathrm{d}I/\mathrm{d}\alpha$ on one side of the stationary point rather than on both. Students would do well to remember that there are at least three different general approaches to determining the nature of a stationary point, and that different methods might be more or less successful in different situations.

Question 6

This was another fairly popular question, but there were many very weak attempts; the median mark was 5.

The first part was answered very poorly. A significant number of candidates only worked out the first few terms and showed that they satisfied the given formula, without making any attempt to justify the formula in general. The majority attempted to write down the general term, with varying degrees of success: many forgot to take account of the minus sign, and so did not include $(-x)^r$. Another common error was to write expressions involving $(-n)!$. Few candidates gave any justification for removal of the minus signs, and solutions which correctly dealt with the case $r < 2$ were rare indeed.

(i) This part was answered very well by the majority of candidates. A number attempted to factorise the numerator as $1 - x + 2x^2 = (1 + x)(1 - 2x)$ or other incorrect ways. Very few answers were careful about the boundary cases, namely where $r = 0$ and $r = 1$, and so most candidates only achieved 4/5 on this part.

(ii) Only a minority of candidates made any progress on this part: most candidates were unable to correctly identify the rule $r^2/2^{r-1}$ for the terms of the sequence. Of those who did, the most common approach was to relate the sequence to that of part (i) as in the first approach described above. Some candidates were able to do the manipulations correctly, but there were a significant number who made slips along the way (for example, using $r^2/2^r$ or leaving out the initial term). Some used the second approach or a variant of it.

It was very pleasing to see some students use the third approach described, many of whom were successful.

Question 7

This was by far the least popular Pure Mathematics question, attempted by only one-third of candidates. The marks achieved were poor; the median mark was 3.

(i) For this first part of the question, very few candidates were able to justify the form of differential equation given. Many candidates talked about the water level decreasing until it reached $\alpha^2 H$, which is not what actually happens (it continually increases until it reaches that point). This lack of understanding had a knock-on effect later on, too, in that they were unable to generate the required differential equation in part (ii).

Nevertheless, from the given DE, a good number were able to separate the variables and at least make progress towards a solution, though only a minority were able to complete the task.

A number of candidates muddled H and h; this sloppiness prevented them from getting the right answer. It was also unfortunate that the symbols for proportionality and the Greek letter alpha are similar; students had to take extra care to keep those distinct as well.

Many candidates used the given result and approximation for $\ln(1+x)$ to deduce the stated approximation.

(ii) Few candidates were able to generate the correct differential equation for this part, with the offering of $\mathrm{d}h/\mathrm{d}t = c(\alpha^2 H - \sqrt{h})$ being far more common. (We were generous and only deducted a few marks if they made progress with this incorrect equation.) Of those who attempted to solve either the correct or this variant differential equation, very few could work out how to integrate the resulting expression. Even when they used the substitution $u = \sqrt{h}$, they were usually unable to integrate the fractional linear expression resulting from it. Some were more successful, especially when they used the substitution $v = \alpha\sqrt{H} - \sqrt{h}$.

A number of students wisely picked up a couple of marks by demonstrating the validity of the final approximation on the basis of the solution given in the question.

Question 8

This was a moderately popular pure mathematics question, attempted by about half of the candidates. There were a number of good solutions, though the median mark was only 6.

(i) (a) Most realised that $n^2 + 1 > 0$ and deduced that $m^3 > n^3$, though few gave any sort of explanation of how $m > n$ follows from this. A number of students split into four cases depending on the signs of m and n; this was frequently a laborious but correct approach.

The next part, requiring the expansion of $(n+1)^3$, was well answered, though few understood the significance of "if and only if": most only provided an argument for one of the two directions.

Most were able to combine the two conditions, and were also comfortable with solving quadratic inequalities.

(b) Few candidates understood the significance of the condition $n < m < n + 1$ where both n and m are supposed to be integers, leaving most unable to do this part. This also led them to struggle with part (ii).

(ii) Relatively few candidates understood what they were trying to do in this part. Many tried to replicate the approach given in part (i) with varying degrees of success.

There were a few concerning errors which occurred in a number of scripts. One was the arithmetic *faux pas* $\sqrt{\frac{1}{2}} = \frac{1}{4}$, the other was the belief that $\mathrm{f}(x) = \mathrm{f}(y)$ is equivalent to or implied by $\mathrm{f}'(x) = \mathrm{f}'(y)$.

Nonetheless, there were also a variety of other correct approaches, for example, some showed that $q - 2 \leqslant p \leqslant q + 2$ for all q, and then checked the five possible cases $p = q - 2$, etc., to ascertain all possible solutions.

Question 9

This was the most popular Mechanics question, attempted by about 40% of the candidates. It was well-answered overall; though the median mark was only 7, over a quarter of candidates achieved 14 or more.

In attempting to find $\tan\theta$, most candidates confidently drew a sketch of the situation and correctly wrote down the equations of motion. Some did not clearly indicate the meanings of their symbols, and this sometimes led to confusion later; some used x for time, which was bizarre.

The greatest stumbling block for the majority of candidates was the algebraic manipulations. Once they had reached the equation $gd_1^2/2v^2\cos^2\theta = d_1\tan\theta - d_2$, many seemed unsure how to proceed. And of those who did, a significant number were unfamiliar with the factorisation of $a^3 - b^3$, leaving them unable to complete this part despite being given the answer.

As a general rule, when using the "suvat" equations, it is worth indicating which equation is being used, and specifying the direction (horizontal or vertical) which is being considered.

A significant number of candidates did not even attempt the final part of the question (finding the range of the particle); it is unclear why this was the case.

Of those who did, many were successfully able to use their earlier work to determine the range. A number became stuck because of algebraic errors, but about 10% of attempts scored full marks.

It was also delightful to see the quadratic equation approach being successfully used at least once; there are often significantly different ways of approaching a problem in Mathematics, and this was a wonderful example.

Question 10

This Mechanics question was attempted by about one-quarter of candidates, but it was found to be fairly difficult with almost half of candidates scoring 5 or fewer marks.

The first part of this question led to many wordy solutions which did not reach the nub of the problem. Perhaps the wording "Explain why" rather than "Show that" or "Prove that" was part of the cause of this.

Many candidates realised that no energy is lost in the bounce, but few went on to make correct deductions from this. An easy and frequent mistake was to say that both A and B have the same kinetic energy when they collide; this is only true if they have the same mass.

The next part begins as a standard collision question, and many candidates were very comfortable with it. Many, however, had sign errors in their conservation of momentum or equation of restitution equation, preventing any significant further progress. This was particularly frequent among those candidates who did not draw a diagram with the velocities (and their directions) before and after the collision clearly marked. Of those who did, many were able to show that at least one of the particles moved upwards after the collision. Surprisingly few were able to show that both moved upward, in spite of this being a standard type of STEP question.

As mentioned in the overview, another frequent cause of difficulty in this question was a confusion between 'M' and 'm'; at this level, it is crucial that students have developed a mathematical writing style which is clear and avoids such errors.

A number of candidates made good attempts to find the maximum height of B. Many, however, simply wrote down a series of calculations with no indication of what they were attempting to calculate. As the answer was given, full marks could only be awarded if the argument was clearly correct. It also turned out to be possible to reach the given answer through a straightforward, yet incorrect, calculation; such solutions received very few marks.

Question 11

This question was attempted by about one-third of candidates, but was fairly poorly answered. About one-third of attempts scored 0 or 1, and the median mark was therefore only 4. Nevertheless, there were many good solutions, reaching about two-thirds of the way through, including a number of perfect scores.

An overall comment for this question is again that candidates need to explain their work. Particularly in questions where the answer is given, little credit will be given for simply writing down an equation which leads to the required answer in one step unless a justification can be seen.

For the first part of the question, most candidates drew decent diagrams, allowing them to proceed, but a fair number drew something which was either inaccurate (placing A below B or drawing the rod horizontally, for example), severely incomplete (no forces) or too small and scribbly to be useful. These led to subsequent difficulties when attempting to resolve forces or take moments.

About 20% of candidates stopped after drawing the diagram, gaining them either 0 or 1 (the modal score).

A number of candidates did not appreciate that the tensions in the two parts of the string were equal, and were therefore unable to proceed.

A common error seen when taking moments was something like $3d(mg) = 7d(T\sin\beta)$; this could be made to give the 'correct' answer, but received little credit. Another common error was to forget to include any forces in the moments equation.

Of those candidates who proceeded further than the diagram, most made good progress towards finding the length of the string.

A common assumption was that $\angle APB$ was a right angle, without giving any justification of this. A similar, though far less common, assertion was that $AP/PB = AG/GB$. Both of these happen to be true, but candidates are required to prove them to gain any credit for their argument.

Many different approaches were seen; the more common ones are described in the solutions above, and there were many variants of these.

When it came to finding the angle of inclination, a small number of candidates successfully did so. There were many attempts which fudged their working to reach the stated conclusion. A number stated that the angle APB is bisected by the line PG without giving any justification; such attempts gained few marks.

Question 12

Generally, Statistics questions are generally the least popular on STEP papers, and this year was no exception; this question was answered by fewer than one-quarter of candidates. In spite of this (or perhaps because of it), it was one of the most successfully answered on the paper, with a median mark of 11 and an upper quartile of 16.

(i) A few candidates failed to get started and scored no marks at all, or misread the question and gave the probability of failure instead, but the majority gained full marks on this part.

(ii) Most candidates were able to enumerate the possible cases of success or the cases of failure. However, a significant proportion did not provide any justification that they had considered *all* possible cases, and so lost at least 2 marks. In part (iii), this lack led a significant number to overlook one or more cases, and so get the wrong answer.

Several candidates failed to realise that the probability of the second person having a £2 coin depends upon the first person's coin.

(iii) Those who succeeded on part (ii) generally made good progress on this part as well; see the comments above.

Question 13

This question was attempted by around 20% of candidates, but was only counted as one of the best six questions for about three-quarters of them. Even among those, it was the most poorly answered question on the whole paper, with one-third of the attempts gaining only 0 or 1 mark, a median of 4 marks and an upper quartile of 8 marks.

(i) In this first part, many candidates did not even attempt to find k, and algebraic errors during the integration were common. A number of attempts to find k were obviously wrong, as they gave a negative area, but candidates did not notice this.

Most were unable to integrate $x\sqrt{4-x^2}$, with failed attempts to integrate by parts far outnumbering correct integrations of this expression.

A number of candidates attempted to calculate the median rather than the mean. Other bizarre interpretations of the term "mean" were also seen.

(ii) Few candidates made it this far. Of those who did, there were some very good solutions. One of the hardest parts was getting the logic correct: the phrasing of the question as "Show that X if Y" was often misinterpreted to mean "Show that if X then Y", though the majority of the marks were awarded in such cases.

(iii) The handful of students who made it this far were generally successful at this part, too.

Dr Julian Gilbey
Principal Examiner (Marking)

August 2011

STEP 2011 Paper 2 Report

General Remarks

There were just under 1000 entries for paper II this year, almost exactly the same number as last year. After the relatively easy time candidates experienced on last year's paper, this year's questions had been toughened up significantly, with particular attention made to ensure that candidates had to be prepared to invest more thought at the start of each question – last year saw far too many attempts from the weaker brethren at little more than the first part of up to ten questions, when the idea is that they should devote 25-40 minutes on four to six *complete* questions in order to present work of a substantial nature. It was also the intention to toughen up the final "quarter" of questions, so that a complete, or nearly-complete, conclusion to any question represented a significant (and, hopefully, satisfying) mathematical achievement. Although such matters are always best assessed with the benefit of hindsight, our efforts in these areas seem to have proved entirely successful, with the vast majority of candidates concentrating their efforts on four to six questions, as planned. Moreover, marks really did have to be earned: only around 20 candidates managed to gain or exceed a score of 100, and only a third of the entry managed to hit the half-way mark of 60.

As in previous years, the pure maths questions provided the bulk of candidates' work, with relatively few efforts to be found at the applied ones. Questions 1 and 2 were attempted by almost all candidates; 3 and 4 by around three-quarters of them; 6, 7 and 9 by around half; the remaining questions were less popular, and some received almost no "hits". Overall, the highest scoring questions (averaging over half-marks) were 1, 2 and 9, along with 13 (very few attempts, but those who braved it scored very well). This at least is indicative that candidates are being careful in exercising some degree of thought when choosing (at least the first four) 'good' questions for themselves, although finding six successful questions then turned out to be a key discriminating factor of candidates' abilities from the examining team's perspective. Each of questions 4-8, 11 & 12 were rather poorly scored on, with average scores of only 5.5 to 6.6.

Comments on individual questions

Q1 The first question is invariably set with the intention that everyone should be able to attempt it, giving all candidates something to get their teeth into and thereby easing them into the paper with some measure of success. As mentioned above, this was both a very popular question and a high-scoring one. Even so, there were some general weaknesses revealed in the curve-sketching department, as many candidates failed to consider (explicitly or not) things such as the gradient of the curve at its endpoints and, in particular, the shape of the curve at its peak (often more of a vertex than a maximum). It was also strange that surprisingly many who had the correct domain for the curve *and* had decided that the single point of intersection of line and curve was at $x = 1$ still managed to draw the line $y = x + 1$ **not** through the endpoint at $x = 1$. Most other features – domain, symmetry, coordinates of key points, etc. – were well done in (i). Unfortunately, those who simply resort to plotting points are really sending quite the wrong message about their capabilities to the examiners.

Following on in (ii), the majority of candidates employed the expected methods and were also quite happy to plough into the algebra of squaring-up and rearranging; however, there were frequently many (unnecessarily) careless errors involved. The only other very common error was in the sketch of the **half**-parabola $y = 2\sqrt{1-x}$, due to a misunderstanding of the significance of the radix sign.

Q2 Personally, this was my favourite question, even though it was ultimately (marginally) deflected from its original purpose of expressing integers as sums of two rational cubes. Given that the question explicitly involves inequalities (which are, as a rule, never popular) and cubics rather than quadratics, it was slightly surprising to find that it was the most popular question on the paper. However, although the average score on the question was almost exactly 10, these two issues then turned out to be the biggest stumbling-blocks to a completely successful attempt as candidates progressed through the question, both in establishing the given inequalities and then in the use of them. In particular, it was noted that many candidates "proved" the given results by showing that they implied something else that was true, rather than by *deducing* them from something else known to be true; such logical flaws received little credit in terms of marks. The purpose of this preliminary work was to enable the candidates to whittle down the possibilities to a small, finite list and then provide them with some means of testing each possibility's validity. This help was often ignored in favour of starting again. In general, though, part (i) was done reasonably well; as was (ii) by those who used (i)'s methodology as a template.

Only a very few candidates were bold enough to attempt (ii) successfully without any reference to (i)'s methods; indeed, this arithmetic approach was how the question was originally posed (as part (i), of course) before proceeding onto the algebra. Noting that the wording of the question does not demand any particular approach in order to find the required *two* solutions to the equation $x^3 + y^3 = 19z^3$, a reasonably confident arithmetician might easily note that

$$19 \times 2^3 = 152 = 3^3 + 5^3 \quad \text{and} \quad 19 \times 3^3 = 513 = 1^3 + 8^3$$

and it isn't even necessary to look very far for *two* solutions. For 10 marks, this is what our transatlantic cousins would call " a steal".

Q3 Again, despite the obvious presence of inequalities in the question, this was another very popular question, and was generally well-handled very capably in part (i), where the structure of the question provided the necessary support for successful progress to be made here. Part (ii) was less popular and less well-handled, even though the only significant difference between this and (i)(c) was (effectively) that the direction of the inequality was reversed. Although the intervals under consideration were clearly flagged, many candidates omitted to consider that, having shown the function *increasing* on this interval, they still needed to show something simple such as $f(0) = 0$ in order to show that $f(x) \geq 0$ on this interval. A few also thought that $f'(x)$ increasing implied that $f(x)$ was also increasing.

Q4 This question was the first of the really popular ones to attract relatively low scores overall. In the opening part, it had been expected that candidates would employ that most basic of trig. identities, $\sin A = \cos(90^\circ - A)$, in order to find the required values of θ, but the vast majority went straight into double-angles and quadratics in terms of $\sin\theta$ instead, which had been expected to follow the initial work; this meant that many candidates were unable to explain convincingly why the given value of $\sin 18^\circ$ was as claimed.

Despite the relatively straightforward trig. methods that were required in this question, with part (ii) broadly approachable in the same way as the second part of (i), the lack of a clear-minded strategy proved to be a big problem for most attempters, and the connection between parts (ii) and (iii) was seldom spotted – namely, to divide through by 4 and realise that $\sin 5\alpha$ must be $\pm\frac{1}{2}$. Many spotted the solution $\alpha = 6^\circ$, but few got further than this because they were stuck exclusively on $\sin 30^\circ = +\frac{1}{2}$.

Q5 This vectors question was neither popular nor successful overall. For the most part this seemed to be due to the fact that candidates, although they are happy to work with scalar parameters – as involved in the vector equation of a line, for instance – they are far less happy to interpret them geometrically. Many other students clearly dislike non-numerical vector questions. Having said that, attempts generally fell into one of the two extreme camps of 'very good' or 'very poor'. More confident candidates managed the first result and realised that a "similarity" approach killed off the second part also, although efforts to tidy up answers were frequently littered with needless errors that came back to penalise the candidates when they attempted to use them later on. Many candidates noted that D was between A and B, but failed to realise it was actually the midpoint of AB. In the very final part, it was often the case that candidates overlooked the negative sign of $\cos\theta$, even when the remainder of their working was broadly correct.

Q6 This was another very popular question attracting many poor scores. There were several very serious errors on display, including the beliefs that

$$\int \mathrm{f}(x)^n \, \mathrm{d}x = \frac{\mathrm{f}(x)^{n+1}}{(n+1)} \text{ or } \frac{\mathrm{f}(x)^{n+1}}{(n+1)\mathrm{f}'(x)}.$$

The understanding that the original integral needed to be split as $\int \mathrm{f}'(x) \times \left(\mathrm{f}'(x)\mathrm{f}(x)^n\right)\mathrm{d}x$ before attempting to integrate by parts was largely absent, with many substituting immediately for $\mathrm{f}(x)\mathrm{f}'(x)$ in terms of $\mathrm{f}''(x)$, which really wasn't helpful at all. Those who got over this initial hurdle generally coped very favourably with the rest of the question.

In (i), it was quite common for candidates to omit verifying the result for tan x.

Q7 The initial hurdle in this question involved little more than splitting the series into separate sums of powers of λ and μ, leading to easy sums of GPs. Many missed this and spent a lot of wasted time playing around algebraically without getting anywhere useful. In (ii), many candidates applied (i) once, for the inner summation, but then failed to do so again for the second time, and this was rather puzzling. Equally puzzling was the lack of recognition, amongst those who had completed most of the first two parts of the question successfully, that the sum of the odd terms in (iii) was *still* a geometric series. Almost exactly half of all candidates made an attempt at this question, but the average score was only just over 5/20.

Q8 This was the least popular of the pure maths questions, probably with good reason, as it included a lengthy introduction *and* a diagram. In the first part, despite showing candidates that the point where the string leaves the circle is in the second quadrant, the necessary coordinate geometry work provided a considerable challenge. The second part, finding the maximum of x by standard differentiation techniques, proved to be relatively straightforward and a lot of candidates managed to get full marks for this work. The third part presented the core challenge of this question, in the sense that not many candidates seemed to have understood how to set the limits of the parametric integral, and '*benefit of the doubt*' had to be fairly generously applied to those who switched signs when it suited them. The next part of the question involved applying integration by parts in order to evaluate the integrals but surprisingly few candidates managed to do so entirely successfully. Some of the common issues were the signs, that now needed to be fully consistent, and the application of parts twice after using double-angle formulae. The notion of the "total area swept out by the string" was also not so well understood, with only a very few realising that they needed to integrate from $t = 0$ to $t = \frac{1}{2}\pi$ as well. Most remembered to subtract the area of the semi-circle though.

Q9 Almost half of all candidates attempted this question, and scores averaged over 12/20. In the majority of cases, the first two parts of the question proved relatively straightforward conceptually, although there was the usual collection of errors introduced because of a lack of care with signs/directions. It was only the final part of the question that proved to be of any great difficulty: most candidates realised that they had to show that the given expression for *B*'s velocity was always positive, but a lot of their efforts foundered on the lack of appreciation that the term $1-4e^2$ could be positive or negative.

Q10 This was the second most popular of the mechanics questions. The first couple of parts to the question were fairly routine in nature, but then the algebra proved too demanding in many cases, principally when it came to dealing with a quadratic equation in t which had non-numerical coefficients. Candidates also found it a struggle to know when to use g and H instead of u and θ in the working that followed. A good number of candidates understood the nature of the problem as the two particles rose and fell together, although it transpired (unexpectedly) that there was another difficult obstacle to grasp in working with two distances. Even amongst essentially fully correct solutions, very few indeed arrived at the correct final answer for $\tan\theta$.

Q11 This was the least popular of all the questions on the paper, receiving under 40 "hits". The fact that it clearly involved both vectors and 3-dimensions was almost certainly responsible for the reluctance of candidates to give it a go. Those who managed to get past the initial stage of sorting out directions and components usually did very well, but most efforts foundered in the early stages. It was not helpful that some of these efforts confused angles to the vertical with those to the horizontal. Almost no-one verified that the given vector in (i) was indeed a unit vector.

Q12 Around a quarter of all candidates made an attempt at this question, though the average score was very low. Parts (ii) and (iii) were managed quite comfortably, on the whole, but it was (i) that proved to be difficult for most of those who attempted the question. The real difficulty lay in establishing the given result for w, as the event to which it corresponded was defined recurrently. As it happens, most wayward solutions left the straight-and-narrow by misreading the rules of the match to begin with.

Q13 This question was almost as unpopular as question 11, receiving under 70 attempts, very few of which ventured an opening opinion as to what *skewness* might measure. Those who could handle expectations lived up to them and scored well; the rest just found the question a little too overwhelming in its demands.

T F Cross
Principal Examiner

STEP Mathematics III 2011: Report

The percentages attempting larger numbers of questions were higher this year than formerly. More than 90% attempted at least five questions and there were 30% that didn't attempt at least six questions. About 25% made substantive attempts at more than six questions, of which a very small number indeed were high scoring candidates that had perhaps done extra questions (well) for fun, but mostly these were cases of candidates not being able to complete six good solutions.

Section A: Pure Mathematics

1. As might be expected, this was a very popular question, in fact the most popular being attempted by very nearly all the candidates. Fortunately, it was also generally well-attempted, with scores well above those for other questions. Apart from frequent algebraic errors and overlooking terms, especially when using results from a previous part that required adaptation, the main difficulties were in showing that (*) in part (ii) did indeed lead to a first order differential equation in $\frac{dz}{dx}$, and the consequent solution of that equation. Part (iii) was generally well done. At the other end of the scale, some candidates did leave their answers to part (i) in the form $\ln u =$.

2. This was quite a popular question, being attempted by 70% of candidates. Scores were polarized, though overall the mean score was below half marks, much the same as half of the questions on the paper. Most candidates successfully dealt with the stem. Attempts at part (i) were in equal proportions, applying the stem or a variant of the standard proof of the irrationality of the square root of 2, though some of the latter overlooked the fact that it was the *n*th root being discussed. Parts (ii) and (iii) saw three methods employed. One method was to consider the location of the real roots then apply the stem, the second being to re-arrange the expression to equal the integer and consider factors (again applying the stem). In both these cases, failure to consider all cases lost marks, and there were frequent lacks of rigour. However, considering x being odd or even, when used, was particularly slick and successful.

3. The second most popular question, attempted by 80% of the cohort, with a similar level of success to question 2. The significance of the condition $q^2 \neq 4p^3$ was ignored by many candidates, and the fact that it does not apply in the last part of the question was often similarly overlooked. Whilst a and b were generally found correctly, the rest of the first part was often missing. Though there were frequent numerical errors, many candidates correctly found the given solution of the equation, though the other two eluded most, with a common error being to assume that the other two were xw and xw^2.

4. About two thirds of the candidates tried this, with very slightly greater success than questions 2 and 3. They found part (i) tricky, especially understanding the integral of the inverse function. Also, commonly, they thought the condition was that $b = a$. However, part (ii) was done better, most errors being due to taking the inverse incorrectly, and of course, the verification frequently went wrong due to the false condition. Most realised the function to use in part (iii) but there was plenty of inaccuracy in working this part, though the final deduction caused few worries.

5. Less than a third of the candidates attempted this. There were quite a few perfect scores, however the vast majority scored less than a quarter of the marks, which was the

mean mark. The general result at the start of the question was the key to success. Those that stumbled with handling four variables in terms of the fifth one, and the consequent calculus, did not attempt to make further progress into the rest of the question.

6. This was quite popular, with attempts from three quarters of the candidates, and slightly more success than questions like 2 and 3. Needing to prove three equalities, many got close to doing two well and, with the others splitting half and half between getting close to all three or just one. A small number of candidates made several attempts without always having any sense of direction and often proved a particular pair equal both ways round. The other weaknesses were in dealing with the limits when changing variable and evaluating the definite term (which was zero!) when employing integration by parts.

7. The popularity and success rate of this was very similar to question 6. Quite a few failed to realise the importance of ${A_n}^2 = a(a+1){B_n}^2 + 1$ as part of the induction, and even if they did tripped up on that part of the working. Part (ii) generally went well and the result in C_n and D_n was found more easily. Very few had a problem with part (iii) but a small number failed totally to see what it was about.

8. The response rate of this was similar to question 4, but with success rate similar to question 2. Most students did reasonably well getting half to three quarters of the marks by finding u and v and doing part (i), and then getting hold of (ii) and (iii) or not. Part (iv) rightly discriminated the strong candidates from the generality. A few alternative methods were tried but mostly they had their limitations. Details like omitted points from loci and the negative sign that arises when using the cosine double angle formula frequently lost marks.

Section B: Mechanics

9. About a sixth of candidates tried this, and on average with slightly less success than question 2. Of the attempts, about a third were close to completely correct, and nearly all the others were barely doing more than grasping at crumbs, reflecting the fact that candidates either did or did not know what they were doing. There was negligible middle ground.

10. Just under a quarter of candidates offered something on this, with relatively little success and less than 20 candidates earning good marks. As with question 9, it tended to be a case of “all or nothing”. Of the good solutions, half based their working on the motion of and relative to the centre of mass of the system, and the other half on setting up simultaneous differential equations for the displacements of the particles. Of the poor attempts, most usually drew some kind of diagram, but then didn’t use it to identify a sensible coordinate system, or positive direction, and there were common confusions over displacements x and extensions x. Energy approaches usually got nowhere.

11. The least popular question on the paper, attempted by about 4%, but with similar mean score to question 2 (and several others). Mostly, they did pretty well in finding the couple, and using the initial trigonometric relation and its consequences to do so. At that point they tended not to know how to proceed to the last part, though there were some very good and simple solutions from considering energy.

Section C: Probability and Statistics

12. This question ran a close second to number 11 for unpopularity, but reflected the same level of success. Most attempts followed the method of the question, and if they got off on the right foot, often got most of the way through. Some struggled with the algebra for the variance result, and a few tripped up on the standard pgf for the number of tosses to the first head. Strangely, having found the pgf for Y successfully, and used it or the results of the question for expectation and variance, the final probabilities were often wrong, and not merely from overlooking the initial case.

13. This too was fairly unpopular, being attempted by about 10% of the candidates. Of these no more than a dozen got it largely correct, but there was only one totally correct solution as the detail for the non unique case frequently tripped even the better candidates. The mean score was only about a third of the marks available as most candidates got part (i) largely correct, barring some simplifying errors and not obtaining the non unique solution. Fewer candidates had the correct probabilities for part (ii) and so were unable to proceed, though a few were wrong merely by a constant which cancelled to give the correct ratio.

Explanation of Results STEP 2011

All STEP questions are marked out of 20. The mark scheme for each question is designed to reward candidates who make good progress towards a solution. A candidate reaching the correct answer will receive full marks, regardless of the method used to answer the question.

All the questions that are attempted by a student are marked. However, only the 6 best answers are used in the calculation of the final grade for the paper.

There are five grades for STEP Mathematics which are:
S – Outstanding
1 – Very Good
2 – Good
3 – Satisfactory
U – Unclassified

The rest of this document presents, for each paper, the grade boundaries (minimum scores required to achieve each grade), cumulative percentage of candidate achieving each grade, and a graph showing the score distribution (percentage of candidates on each mark).

STEP Mathematics I (9465)

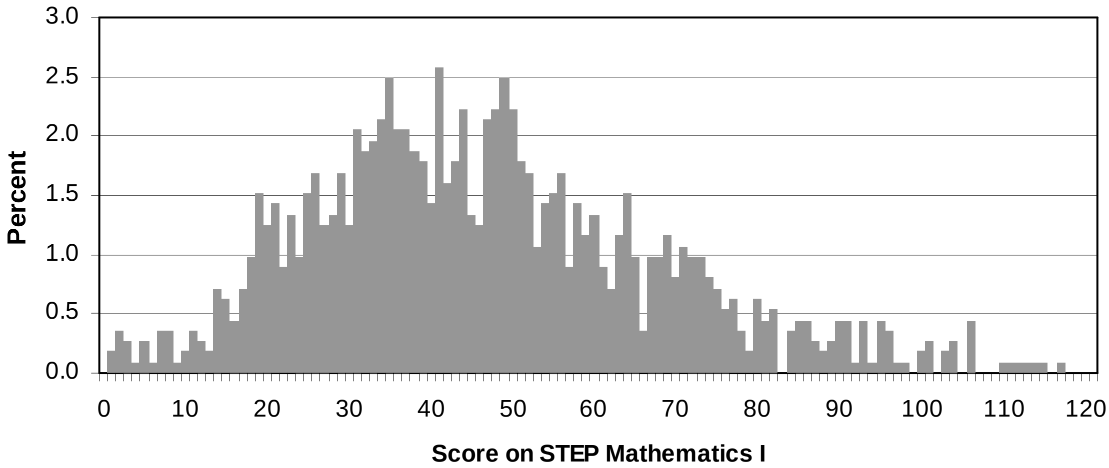
Grade boundaries

Maximum Mark	S	1	2	3	U
120	86	66	47	28	0

Cumulative percentage achieving each grade

Maximum Mark	S	1	2	3	U
120	5.6	18.5	46.9	81.6	100.0

Distribution of scores



STEP Mathematics II (9470)

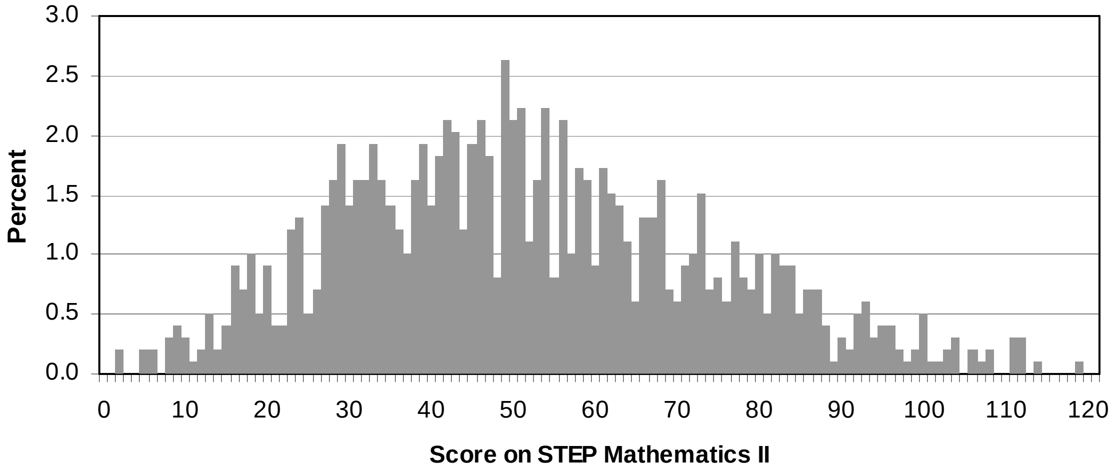
Grade boundaries

Maximum Mark	S	1	2	3	U
120	83	62	49	29	0

Cumulative percentage achieving each grade

Maximum Mark	S	1	2	3	U
120	10.0	30.9	52.8	85.3	100.0

Distribution of scores



STEP Mathematics III (9475)

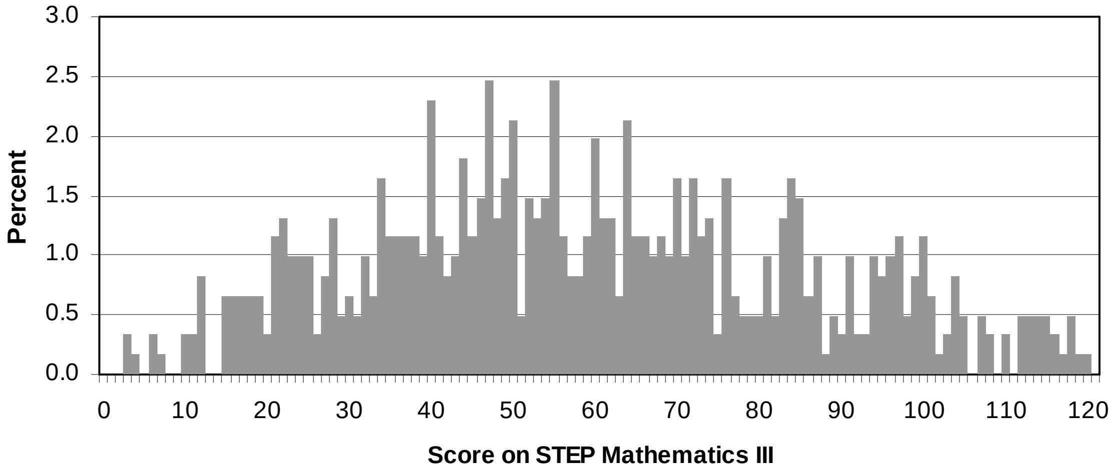
Grade boundaries

Maximum Mark	S	1	2	3	U
120	91	65	52	30	0

Cumulative percentage achieving each grade

Maximum Mark	S	1	2	3	U
120	14.9	39.7	57.8	85.6	100.0

Distribution of scores



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