GCSE Maths – Ratio, Proportion and Rates of Change

Ratio and Similar Shapes

Worksheet

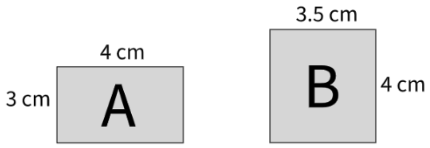
WORKED SOLUTIONS

This worksheet will show you how to work out different types of ratio and similar shapes questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Find the ratio of the area of A to the area of B



Step 1: Find the area of each rectangle.

The formula for the area of a rectangle is $base \times height$.

$$\text{Area of shape A: } 4 \times 3 = 12\ cm^2$$
$$\text{Area of shape B: } 3.5 \times 4 = 14\ cm^2$$

Step 2: Note these areas as a ratio in the form asked for in the question.

$$Area\ of\ Shape\ A : Area\ of\ Shape\ B$$
$$12 : 14$$

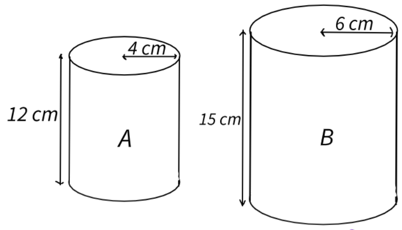
Step 3: Simplify the ratio.

Divide both sides by the same number until the ratio is in its simplest form.

$$12 : 14$$
$$6 : 7$$

Guided Example

Find the ratio of the volume of A to the volume of B



Step 1: Find the volume of cylinder A and B.

volume of cylinder = $\pi r^2 h$

volume of cylinder A = $\pi \times 4^2 \times 12 = 192\pi$

volume of cylinder B = $\pi \times 6^2 \times 12 = 432\pi$

leave π in its original form since it'll make the calculation easier

Step 2: Form these volumes into a ratio of A:B

volume cylinder A : volume cylinder B

$192\pi : 432\pi$

Step 3: Simplify the ratio by eliminating π from either side, and then dividing both sides by the same number.

$$192\pi : 432\pi$$

$\div 192\pi$ $\div 192\pi$

$$4 \times 1 : \frac{9}{4} \times 4$$

$$\boxed{4 : 9}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. The base of cone P has a radius of 4 cm and is 12 cm high. Cone Q is 10 cm high and has a base with radius 5 cm. Work out the ratio of the volume of cone P to cone Q.

Volume of cone : $\frac{1}{3} \times \pi r^2 h$

volume of cone P : $\frac{1}{\cancel{3}_1} \times \pi \times (4)^2 \times \cancel{12}^4$

: $16\pi \times 4$

: 64π

volume of cone Q : $\frac{1}{3} \times \pi \times (5)^2 \times 10$

: $\frac{250\pi}{3}$

volume of cone P : volume of cone Q

$64\pi : \frac{250}{3}\pi$ ($\div \pi$)

$64 : \frac{250}{3}$ ($\times 3$)

$192 : 250$ ($\div 2$)

$96 : 125$

Volume ratio of P to Q : **96 : 125**

2. The ratio of the size of Anna's garden to Brandon's garden is 11:7. If Anna's garden is 169.4 m², what is the area of Brandon's garden?

Ratio : Anna's garden : Brandon's garden
11 : 7

Area : 169.4 : x

$11 : 7$
$169.4 : x$ can also be written as → $\frac{11}{169.4} = \frac{7}{x}$ (cross multiply) $= \quad 11x = 169.4\,(7)$

$11x = 1185.8$

$x = 107.8$

The area of Brandon's garden is **107.8 m²**.

3. The surface area of the moon is 14.6 square miles.

The surface area of Earth is 196.9 square miles.

Write the ratio of the surface area of the moon to the surface area of the Earth in the form 1:n , where n is an integer rounded to the nearest whole number.

Ratio of surface area : moon : Earth

$14.6 : 196.9$ ($\div 14.6$)

$1 : 13.49$

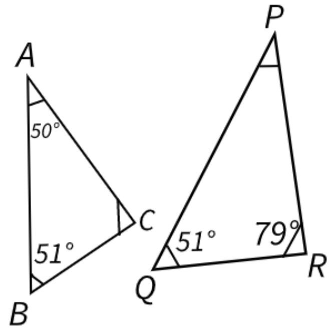
$1 : 13$ → round to the nearest whole number

The ratio of surface area of the moon to the Earth is **1 : 13**

Section B

Worked Example

Are triangle ABC and triangle PQR similar? Explain your answer.



Step 1: Calculate the missing angles of the triangles, to deduce as much information about the triangles as possible.

$$\angle ACB = 180 - 50 - 51 = 79°$$

$$\angle QPR = 180 - 51 - 79 = 50°$$

Step 2: Look at the information we have about each triangle and draw similarities between them.

$$\angle ABC = 51° = \angle PQR$$

$$\angle ACB = 79° = \angle QRP$$

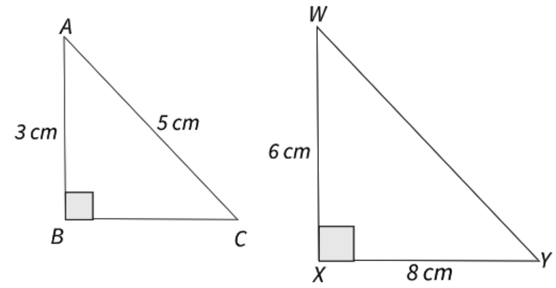
$$\angle BAC = 50° = \angle QPR$$

Step 3: Decide which condition you can prove with the sides/angles you have.

*You know that triangle ABC and PQR have three pairs of equal angles. So, the triangles satisfy the condition **AAA,** and can be said to be similar.*

Guided Example

Are triangles ABC and WXY similar? Explain your answer.



Step 1: Calculate the missing sides of the triangles in order to explore whether we can prove any of the conditions for similarity.

Use Pythagoras theorem : $c = \sqrt{a^2+b^2}$

Triangle ABC : $5 = \sqrt{(3)^2 + (BC)^2}$

$5^2 = 9 + (BC)^2$

-9: $16 = (BC)^2$

square root: $4 = BC$

Triangle WXY : $WY = \sqrt{(6)^2+(8)^2}$

$= \sqrt{36+64}$

$= \sqrt{100}$

$= 10$

Step 2: Make links between the triangles, by looking for scale factors between pairs of sides or identifying pairs of angles of the same size.

Ratio (ABC) : AB : BC : AC

3 : 4 : 5

Ratio (WXY) : WX : XY : WY

6 : 8 : 10

$\times 2$

The sides of triangle WXY are twice the length of the sides of triangle ABC

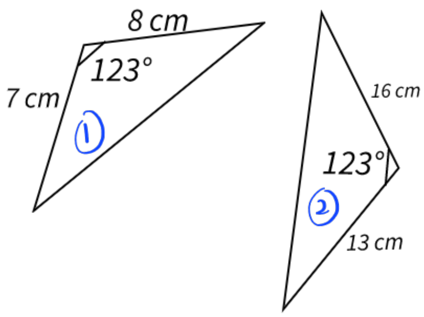
Step x: With the information of both triangles, decide whether the conditions for similarity between triangles (AAA, SAS, SSS) have been met.

The triangle ABC and WXY have 3 pairs of sides with their lengths of the same ratio. Hence, they satisfy the condition SSS, and can be said to be similar.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

4. Are the following pairs of triangles similar? Explain your answers

a)



Since the angle is similar, now we compare both sides nearest to the angle to see if they have the same ratio.

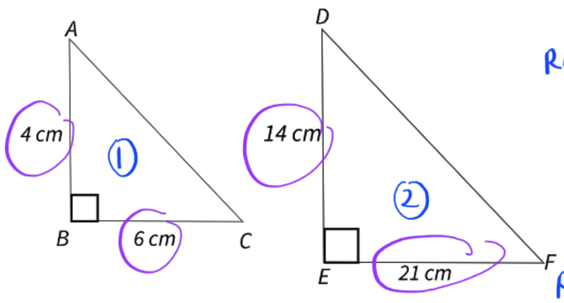
Ratio for triangle 1 : 7 : 8

Ratio for triangle 2 : 13 : 16

Since one angle and 2 sides are given, we can use condition SAS to determine if the triangles are similar.

The triangles do not have the same ratio. Hence, they are not similar.

b)



Ratio for triangle 1 : 4 : 6
2 : 3 ÷ 2

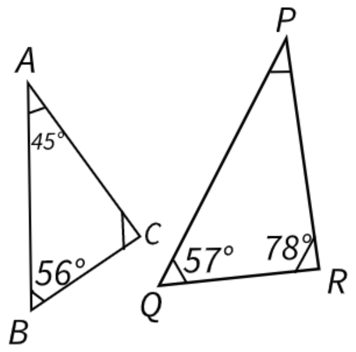
simplify the ratio

Ratio for triangle 2 : 14 : 21
÷ 7 2 : 3 ÷ 7

Compare the 2 sides nearest to the angle to work out their ratio based on condition SAS.

The triangles satisfy the condition SAS. Hence, they are similar.

c)



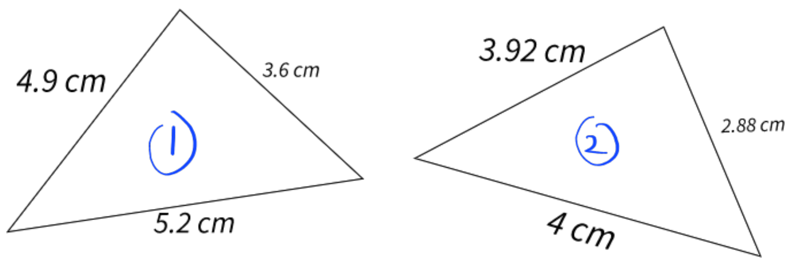
All three pairs of angles in both triangle is not the same.
The triangles do not meet the condition AAA, hence they are not similar.

Calculate the missing angles:

$\angle ACB$: $180° - 56° - 45°$
$= 79°$

$\angle QPR = 180° - 78° - 57°$
$= 45°$

d)



Use condition SSS.

Ratio for triangle 1 : $4.9 : 5.2 : 3.6$

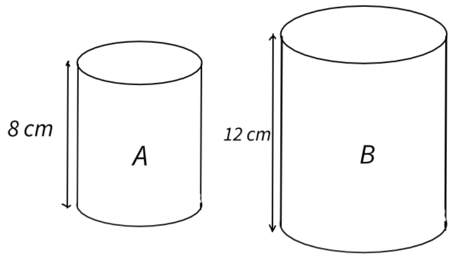
Ratio for triangle 2 : $3.92 : 4 : 2.88$

Both triangles do not have sides of the same ratio.
Condition SSS is not met, hence, the triangles are not similar.

Section C

Worked Example

Find the linear scale factor between cylinders A and B.



Step 1: Find the scale factor by dividing the length of the similar sides in cylinder A and B.

$$Linear\ Scale\ Factor = \frac{Height\ of\ Cylinder\ B}{Height\ of\ Cylinder\ A} = \frac{12}{8} = 1.5$$

This means the length and circumference measurements of cylinder B are 1.5 *times greater than the corresponding measurements in cylinder A.*

Guided Example

Rectangle S has a height of 3 cm. Rectangle T has a height of 15 cm. Find the area scale factor between rectangle S and T.

Step 1: Find the linear scale factor by dividing the length of the similar sides in rectangle A and B.

$$\text{Linear scale factor } (k) = \frac{\text{height of rectangle T}}{\text{height of rectangle S}} = \frac{15}{3} = 5$$

$$k = 5$$

Step 2: To find the area scale factor (k^2), square the linear scale factor you have found.

$$\text{Area scale factor} = k^2$$
$$= (5)^2 = 25$$

The area scale factor between rectangle S and T is 25.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. A standard teacup holds 150 ml of liquid. An enlarged teacup holds 216 ml of liquid. What is the volume scale factor to go from the standard teacup to the enlarged version?

volume standard teacup holds : 150 ml

volume enlarged teacup holds : 216 ml

given value is already in volume

volume scale factor (k^3) : $\frac{\text{enlarged teacup}}{\text{standard teacup}} = \frac{216}{150} = \frac{36}{25} = 1.44$

volume scale factor from standard teacup to enlarged teacup is 1.44.

6. A hexagon has an area of 22.1 cm^2. If it is enlarged to an area of 47.736 cm^2, by what factor has the area, and lengths, increased?

Hexagon area : 22.1

enlarged area : 47.736

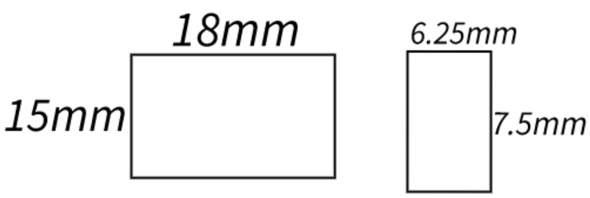
area scale factor (k^2) = $\frac{\text{enlarged area}}{\text{hexagon area}} = \frac{47.736}{22.1} = 2.16$

square root of area scale factor

linear scale factor (k) = $\sqrt{2.16} = 1.47$

The area increased by 2.16 while the lengths are increased by 1.47

7. These two rectangles are similar. What is the length scale factor?



length scale factor : $\frac{18}{7.5} = 2.4$

compare the same sides (in this case, we compare the longer lengths)

Alternatively, you can compare the shorter sides :

$\frac{15}{6.25} = 2.4$

Section D

Worked Example

A company is modelling a prototype of its newest candle. The model is 7 cm high and weighs 50 g. If the actual prototype will be 21 cm high, how much can they expect the prototype to weigh?

Step 1: Calculate the linear scale factor (k) between the model and the prototype.

$$\frac{Prototype\ Height}{Model\ Height} = k$$

$$21 \div 7 = 3$$

The height of the prototype is three times the height of the model.

Step 2: Cube the linear scale factor to find the volume scale factor (k^3).

$$k^3 = 3^3 = 27$$

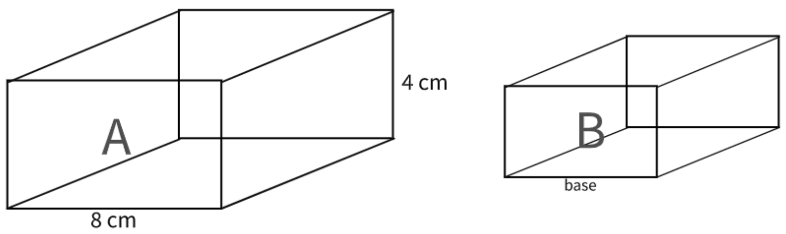
Step 3: Multiply the mass of the model by the volume scale factor, to find the volume of the prototype.

$$50g \times k^3 = 50g \times 27 = 1350g$$

The actual prototype weighs 1.35kg.

Guided Example

Tank A has a surface area of 280 cm². If tank B has a surface area of 70 cm², what is the length of the base of tank B.



Step 1: Divide the surface area of tank A by that of tank B, to find the area scale factor.

$$\text{Area scale factor } (k^2) = \frac{\text{Area of tank A}}{\text{Area of tank B}} = \frac{280}{70} = 4$$

Step 2: Find the length scale factor by square root the area scale factor.

$$\text{length scale factor} = \sqrt{\text{area scale factor}}$$
$$= \sqrt{4}$$
$$= 2$$

Step 3: Divide the base of tank A by the length scale factor, to find the base of tank B.

$$\text{base of tank B} = \frac{\text{base of tank A}}{\text{length scale factor}} = \frac{8}{2} = 4$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

8. A carpenter makes a chest for his daughter, and a miniature version for his daughter's doll house. The version for the doll house takes 50 cm² of wallpaper for decoration, whilst the real-life version takes 450 cm². If the chest for the doll house is 1.2 cm high, how tall is the chest for his daughter?

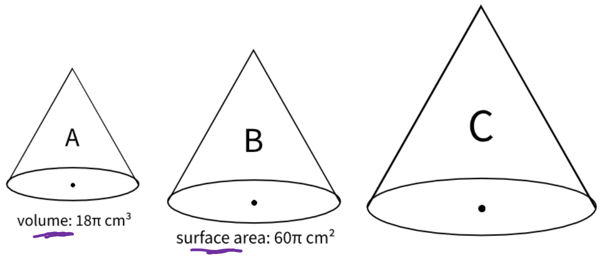
Area scale factor (k^2) = $\dfrac{\text{surface area of chest}}{\text{surface area of doll house}} = \dfrac{450}{50} = 9$

linear scale factor (k) = $\sqrt{9} = 3$

Height of the real-life version chest = $1.2 \times 3 = 3.6$ cm

9. Cone A has a height of 6 cm, cone B has a height of 9 cm and cone C is 15 cm high.

Using the information, what is the volume of cone B and the surface area of cone C?



Compare cone A with B:

linear scale factor (k) : $\dfrac{\text{height of cone B}}{\text{height of cone A}}$

: $\dfrac{9}{6} = \dfrac{3}{2} = 1.5$

volume scale factor (k^3) = $(1.5)^3 = 3.375$

volume of cone B = $18\pi \times 3.375 = 60.75\pi$ cm³

Compare cone B with C:

linear scale factor (k) : $\dfrac{\text{height of cone C}}{\text{height of cone B}}$

: $\dfrac{15}{9} = \dfrac{5}{3}$

area scale factor : $\left(\dfrac{5}{3}\right)^2 = \dfrac{25}{9}$

Surface area of cone C : $60\pi \times \dfrac{25}{9} = \dfrac{500\pi}{3} = 166.67\pi$ cm²

10. Mike is trying to measure a tree. When he is 50 m from the tree and holds his middle finger to it, his finger completely covers the tree. His finger is 7 cm long.

He moves backwards, and now his index finger, which is 6 cm long, completely covers the tree.

How far did he move backwards?

linear scale factor = $\dfrac{7}{6} = 1.167$

distance from the tree : $50 \times 1.167 = 58.33$

difference in the distance : 58.3 m $- 50$ m $= 8.33$ m

Mike move backwards by 8.33 m.