GCSE Maths – Probability

Conditional Probability (Higher)

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of conditional probability questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

The table below shows information about students' pets. A boy is chosen at random from the class. Find the probability that this boy also has a dog.

	Cat	Dog	No Pet	Total
Boy	14	3	10	
Girl	11	4	8	
Total	25		18	50

Step 1: Find how many boys are in the class

Sum together the values in the row corresponding to pets of boys.

$$14 + 3 + 10 = 27$$

There are 27 boys in total.

As we know that the chosen student is a boy, the probability is out of the total number of boys. Thus, the probability will be out of 27.

Step 2: Using the table, identify how many boys also have a dog.

Looking at the row corresponding to boys' pets, there are 3 boys who have a dog.

Step 3: Calculate the required probability.

Out of the 27 boys, 3 have a dog.

$$P(Chosen\ student\ has\ a\ dog\ given\ they\ are\ a\ boy) = \frac{3}{27} = \frac{1}{9}$$

Guided Example

The table below shows information about students' pets. A student who owns a cat is chosen at random in the class. Find the probability that this student is also a boy.

	Cat	Dog	No Pet	Total
Boy	14	3	10	
Girl	11	4	8	
Total				50

Step 1: Calculate how many students in the class own a cat.

$$14 + 11 = 25$$

Step 2: Identify how many boys own a cat.

14 boys have a cat

Step 3: Calculate the required probability.

$$P(\text{Boy given they have a cat}) = \frac{14}{25}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. The table below shows information about students' pets. A student who doesn't have a pet is chosen at random. Calculate the probability that they are a boy.

	Cat	Dog	No Pet	Total
Boy	14	3	10	
Girl	11	4	8	
Total				50

Students with No Pet: 10 + 8 = 18
Boys with No Pet: 10

P(Boy given they don't have a pet) = $\frac{10}{18}$ = $\frac{5}{9}$

2. The table below gives information about the results of peoples' driving tests. A boy is chosen at random. Calculate the probability that he passed his test.

	Pass	Fail	Total
Boy	42-3 39	3	72-30 42
Girl	21	9	30
Total	60		72

42 boys.
39 boys who passed.
P(Passed given a boy) = $\frac{39}{42}$ = $\frac{13}{14}$

3. The table below gives information about the sports people play. A girl is chosen at random. Calculate the probability that she plays tennis.

	Football	Tennis	Netball	Total
Boy		18		30
Girl		12		70−30 **40**
Total	15	30	25	15+30+25 **70**

40 girls

12 girls who play tennis

$P(\text{plays tennis given a girl}) = \dfrac{12}{40} = \dfrac{3}{10}$

4. The table below gives information about the languages that students study. A boy is chosen at random. Calculate the probability that he studies French.

	French	German	Spanish	Total
Boy	10 − 7 **3**	2		40 − 22 **18**
Girl	7	5−2 **3**	12	7+3+12 **22**
Total	10	5	25	10+5+25 **40**

18 boys

3 boys who study French
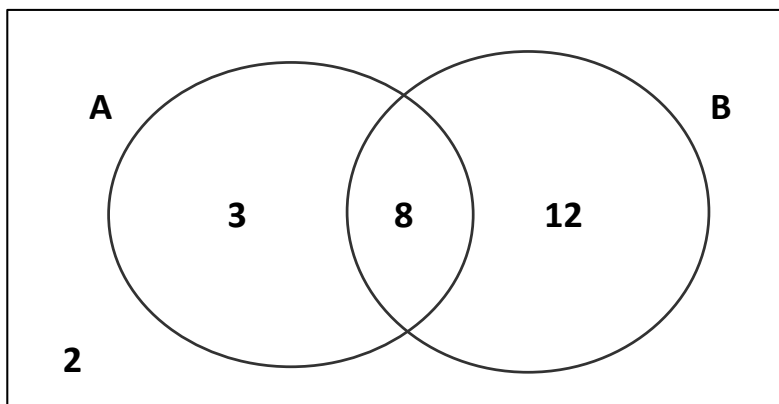
$P(\text{study French given a boy}) = \dfrac{3}{18} = \dfrac{1}{6}$

Worked Example

Below is a Venn diagram showing events A and B. Calculate $P(B|A)$

Step 1: Identify the probability which is being asked for.

The question is asking you to find $P(B|A)$ which is the probability of an outcome being in B given that it is already in A.

Note, there are $2 + 3 + 8 + 12 = 25$ units allocated in the Venn diagram.

Step 2: Calculate $P(A)$.

$$8 + 3 = 11$$

There are 11 units in A so $P(A) = \frac{11}{25}$.

Step 3: Calculate $P(A \cap B)$.

There are 8 in the intersection between A and B, so

$$A \cap B = 8.$$

Hence, $P(A \cap B) = \frac{8}{25}$.

Step 4: Calculate the required probability using the formula for conditional probability.
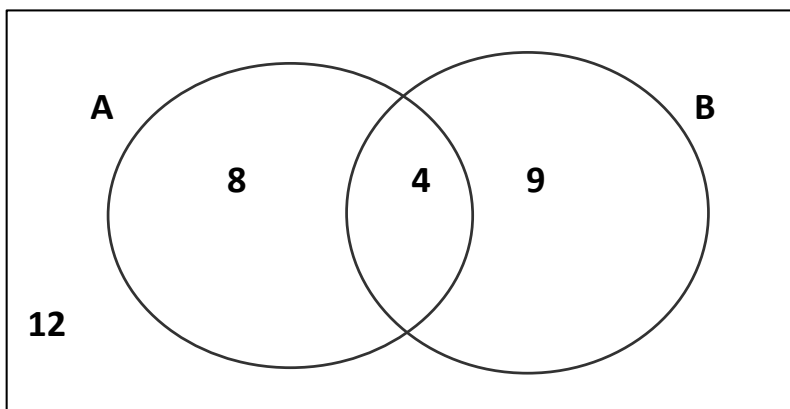
$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B|A) = \frac{\frac{8}{25}}{\frac{11}{25}} = \frac{8}{11}$$

Guided Example

Let A denote the event that someone plays rugby and let B denote the event that someone plays football. Calculate the probability that someone plays rugby given that they play one of the two sports.

Step 1: Identify the probability which is being asked for.

The question is asking for the probability of those who play rugby P(A) given they play a sport P(A∪B)
$$= P(A \mid A \cup B)$$

Step 2: Calculate $P(A)$.

$$P(A) = \frac{8+4}{8+4+9+12} = \frac{12}{33}$$

Step 3: Calculate $P(A \cup B)$.

$$P(A \cup B) = \frac{8+4+9}{33} = \frac{21}{33}$$

Step 4: Calculate the required probability using the formula for conditional probability.
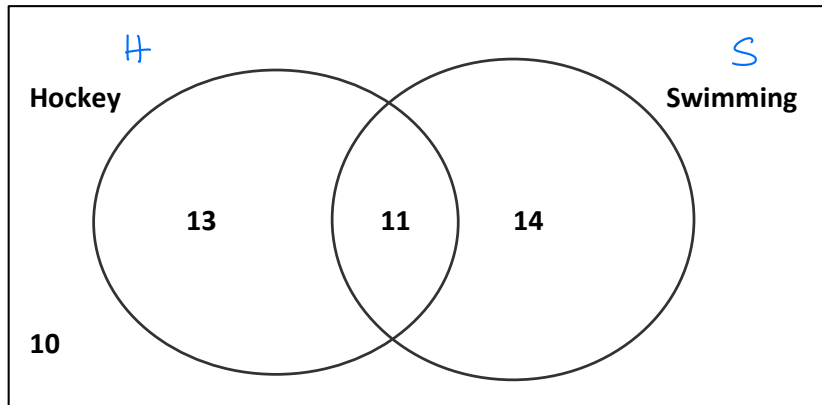
$$\frac{P(A)}{P(A \cup B)} = \frac{\frac{12}{33}}{\frac{21}{33}} = \frac{12}{21} = \frac{4}{7}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Consider the following Venn diagram which shows the number of people partaking in hockey club and swimming club. Karen attends swimming club. What is the probability that she does not attend hockey club?
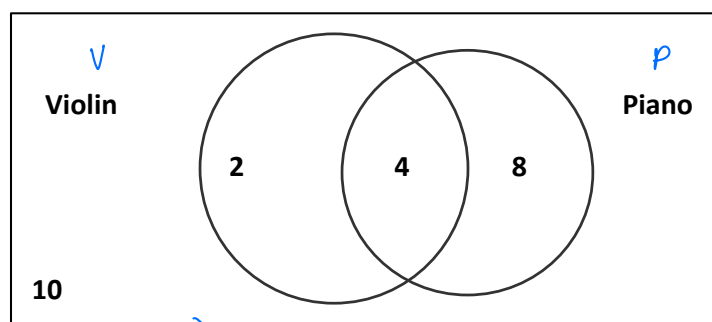
H

Hockey   S   **Swimming**

13   **11**   **14**

10

$P(H' \cap S | S)$

$P(H' \cap S) = \dfrac{14}{14+10+11+13} = \dfrac{14}{48}$

$P(S) = \dfrac{14+11}{48} = \dfrac{25}{48}$

$P(H' \cap S | S) = \dfrac{14/48}{25/48} = \dfrac{14}{25}$

6. The following Venn diagram shows the number of people who play violin and piano. Calculate the probability someone plays neither instrument, given they don't play piano.
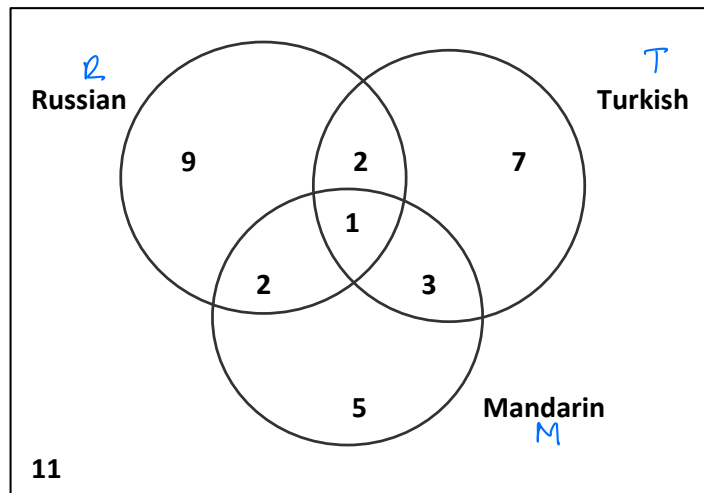
V

Violin   **Piano**   P

2   **4**   **8**

10

$P((V' \cap P') | P')$

$P(V' \cap P') = \dfrac{10}{10+2+4+8} = \dfrac{10}{24}$

$P(P') = \dfrac{2+10}{24} = \dfrac{12}{24}$

$P(V' \cap P' | P') = \dfrac{10/24}{12/24} = \dfrac{10}{12} = \dfrac{5}{6}$

7. The following Venn diagram shows the numbers of people who study languages in a class. A student is selected and found to study a language. Find the probability that they study Russian.
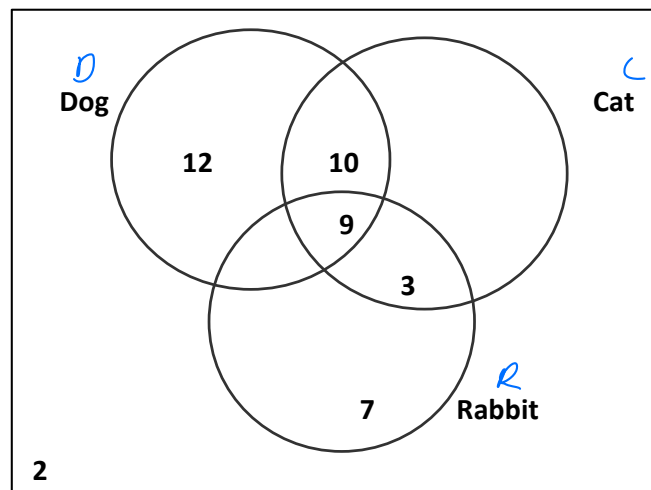
$$= P(R \mid R \cup T \cup M)$$

Total: 40

$$P(R) = \frac{9+2+1+2}{40} = \frac{14}{40}$$

$$P(R \cup T \cup M) = \frac{40-11}{40} = \frac{29}{40}$$

$$P(R \mid R \cup T \cup M) = \frac{14/40}{29/40} = \boxed{\frac{14}{29}}$$

8. The following Venn diagram shows the numbers of people have certain pets. A student is selected and found to own a cat. What is the probability that they also own a dog?

$$P(D \cap C \mid C)$$

Total: 43

$$P(D \cap C) = \frac{10+9}{43} = \frac{19}{43}$$

$$P(C) = \frac{10+9+3}{43} = \frac{22}{43}$$

$$P(D \cap C \mid C) = \frac{19/43}{22/43} = \boxed{\frac{19}{22}}$$

Worked Example

There are 3 blue balls and 6 red balls in a bag. Calculate the probability of picking out a red ball given that a red ball was just taken out.
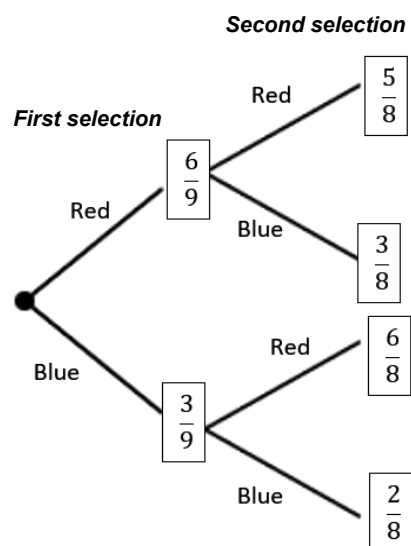
Step 1: Draw the required tree diagram.

Step 2: Identify the branches required for the given question.

The question asks about a red ball being chosen, given a red ball has already been chosen. This means we are interested in the path of 'Red' followed by 'Red'.

Step 3: Find the probability of obtaining a red ball after a red ball is taken out by reading the probability of the required branch.
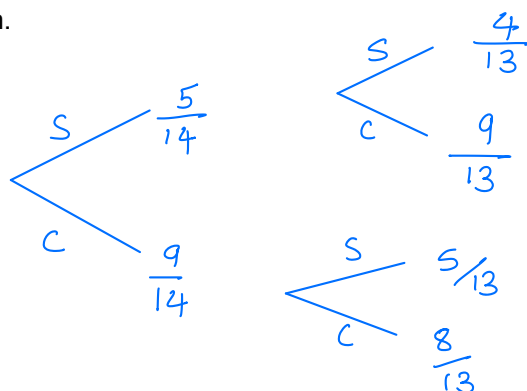
Second selection

First selection

$$P(Red\ ball\ given\ a\ red\ ball\ just\ taken) = \frac{5}{8}$$

Tree diagram:
First selection Red $\frac{6}{9}$ → Second selection Red $\frac{5}{8}$, Blue $\frac{3}{8}$
First selection Blue $\frac{3}{9}$ → Red $\frac{6}{8}$, Blue $\frac{2}{8}$

Guided Example

There are 5 sweets and 9 chocolates in a bag. Calculate the probability of obtaining a sweet given that a chocolate was just taken out.

(annotations: S above "sweets", C above "chocolates")

Step 1: Draw a tree diagram.

Tree diagram:
S $\frac{5}{14}$ → S $\frac{4}{13}$, C $\frac{9}{13}$
C $\frac{9}{14}$ → S $\frac{5}{13}$, C $\frac{8}{13}$

Step 2: Identify the branches required for the given question.

We are interested in the path chocolate then sweet.

Step 3: Find the probability of obtaining a sweet after a chocolate is taken out by reading the probability of the correct path.
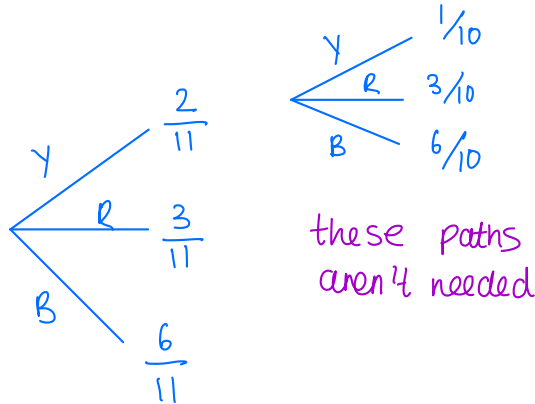
$$\frac{5}{13}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. There are 2 yellow marbles, 3 red marbles and 6 blue marbles in a bag. A marble is selected and found to be yellow. Given that this marble is not returned to the bag, find the probability that a yellow marble is chosen in a second selection.

Follow path Yellow then Yellow

$$P(\text{Yellow} \mid \text{Yellow}) = \frac{1}{10}$$

2. There are 2 yellow marbles, 11 red marbles and 17 blue marbles in a bag. A marble is chosen from the bag. Given that it is not a blue marble, what is the probability that it is a yellow marble?

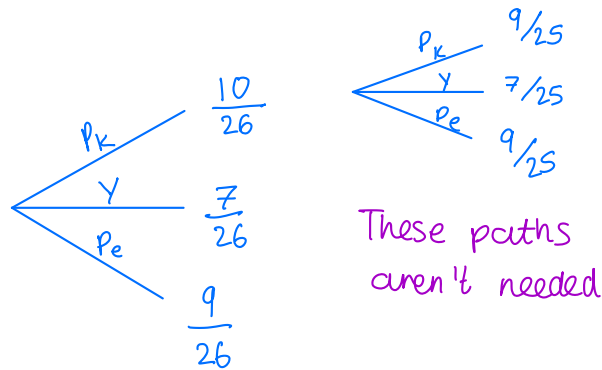$$P(B') = \frac{2+11}{2+11+17} = \frac{13}{30}$$

$$P(Y) = \frac{2}{30}$$

$$P(Y \mid B') = \frac{\frac{2}{30}}{\frac{13}{30}} = \frac{2}{13}$$

3. There are 10 pink, 7 yellow and 9 purple mini eggs in a bag. Calculate the probability that 2 pink mini eggs are chosen in a row. Note, the first mini egg chosen is eaten and so does not get returned to the bag.

P_k Y P_e

P_k — $\dfrac{10}{26}$

P_k — $\dfrac{9}{25}$
Y — $\dfrac{7}{25}$
P_e — $\dfrac{9}{25}$

Y — $\dfrac{7}{26}$
P_e

$\dfrac{9}{26}$

These paths aren't needed

Follow the P_k and P_k path.

$$P(\text{Pink twice}) = P(\text{Pink}_1) \times P(\text{Pink}_2)$$
$$= \frac{10}{26} \times \frac{9}{25} = \frac{9}{65}$$

4. There are 10 pink, 7 yellow, 4 blue and 9 purple mini eggs in a bag. A mini egg is chosen from the bag. Given that it is not blue or pink, calculate the probability that the chosen egg is purple.

P_k Y B P_e

$$P(B' \cap P_k') = \frac{9+7}{10+7+4+9} = \frac{16}{30}$$

$$P(P_e) = \frac{9}{30}$$

$$P(P_e \mid B' \cap P_k') = \frac{9/30}{16/30} = \frac{9}{16}$$