GCSE Maths – Probability

Independent and Dependent Events

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of independent and dependent events questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Suppose events A and B are mutually exclusive.
Given $P(A) = 0.2$ and $P(B) = 0.75$, find $P(A \cup B)$.

Step 1: Choose the correct formula.

Here, the $\cup$ means union (the OR rule). This means we are looking at the probability of A OR B occurring. Mutually exclusive events cannot happen at the same time. This means $P(A \cap B) = 0$ so the formula you should use is when events are mutually exclusive is:

$$P(A \cup B) = P(Event\ A\ or\ event\ B) = P(A) + P(B)$$

Step 2: Substitute the values given in the question into the formula.

You have already been given the values of $P(A)$ and $P(B)$. Substitute them into the formula to find $P(A \cup B)$.
$$P(A \cup B) = 0.2 + 0.75$$

Step 3: Calculate the probability.

$$P(A \cup B) = 0.2 + 0.75 = 0.95$$

Guided Example

Suppose events A and B are independent. Given $P(A) = 0.7$ and $P(B) = 0.5$, calculate $P(A \cap B)$.

Step 1: Choose the correct formula.

$P(A \cap B)$ is an AND rule. Hence, both of the probabilities need to be multiplied together: $P(A) \times P(B)$

Step 2: Substitute the values given in the question into the formula.

$$P(A \cap B) = P(A) \times P(B)$$
$$= 0.7 \times 0.5$$

Step 3: Calculate the probability.

$$P(A \cap B) = 0.7 \times 0.5 = 0.35$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Calculate the following given $P(A) = 0.35$ and $P(B) = 0.65$:

   a) $P(A \cap B)$ when A and B are independent.

   $$P(A \cap B) = P(A) \times P(B)$$
   AND rule
   $$= 0.35 \times 0.65$$
   $$= \boxed{0.2275}$$

   b) $P(A \cup B)$ when A and B are mutually exclusive.

   $$P(A \cup B) = P(A) + P(B)$$
   OR rule
   $$= 0.35 + 0.65$$
   $$= \boxed{1}$$

   c) $P(A \cup B)$ when A and B are not mutually exclusive.

   For non-mutually exclusive event: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

   $$P(A \cap B) = 0.35 \times 0.65 = 0.2275$$

   $$P(A \cup B) = 0.35 + 0.65 - 0.2275 = \boxed{0.7725}$$

2. Find $P(A \cup B)$ when $P(A) = 0.2$ and $P(B) = 0.4$, given A and B are independent events.

   $$P(A \cup B) = P(A) + P(B)$$
   OR rule
   $$= 0.2 + 0.4$$
   $$= \boxed{0.6}$$

3. Suppose $P(A \cup B) = 0.85$. Given $P(A) = 0.7$ and $P(B) = 0.05$, determine if events A and B are independent.

   $$P(A \cup B) = P(A) + P(B)$$
   $$= 0.7 + 0.05$$
   $$= 0.75$$

   If the events are independent, $P(A \cup B)$ should be 0.75. However, the question states that $P(A \cup B) = 0.85$. Hence, events A and B are dependent.

Worked Example

Amelie flips a coin and rolls a 6-sided die. Find the probability of her obtaining a head on the coin and a 2 on the die.

Step 1: Work out which probability formula to use.

Let A be the event that a head is obtained on the coin.
Let B be the event that Amelia gets a 2 on the die.
The events are independent because what is obtained on the dice has no effect on what is obtained on the coin, and vice versa.

We are interested in the probability of getting a head AND getting a 2, so we use the formula for $P(A \cap B)$ for independent events: $P(A \cap B) = P(A) \times P(B)$.

Step 2: Find the probability of each individual event.

$$P(A) = P(Getting\ a\ head) = \frac{1}{2}$$

$$P(B) = P(Getting\ a\ 2) = \frac{1}{6}$$

Step 3: Put the relevant probabilities into the formula and calculate the probability.

$$P(Getting\ a\ head\ AND\ Getting\ a\ 2) = P(A \cap B) = P(A) \times P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

Guided Example

Jason rolls two 6-sided dice. Find the probability that he gets a 6 on both dice.

Step 1: Work out which probability formula to use.

Let A be the event that Jason gets a 6 on die 1.
Let B be the event that Jason gets a 6 on die 2.
The probability of getting 6 on both dice : $P(A \cap B) = P(A) \times P(B)$

Step 2: Find the probability of each individual event.

$P(A) = $ Getting a 6 on die 1 $= \frac{1}{6}$

$P(B) = $ Getting a 6 on die 2 $= \frac{1}{6}$

Step 3: Put the relevant probabilities into the formula and calculate the probability.

$P(A \cap B) = P(A) \times P(B)$
$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

4. Marley rolls two 6-sided dice. Find the probability that he:

a) Rolls a 2 on both dice.

AND rule

$P(A) = $ rolls a 2 on die 1 $= 1/6$
$P(B) = $ rolls a 2 on die 2 $= 1/6$
$P(A \cap B) = P(A) \times P(B) = 1/6 \times 1/6 = \boxed{1/36}$

b) Rolls a 1 on only one of the dice.

OR rule

$P(A) = $ rolls a 1 on die 1 $= 1/6$
$P(B) = $ rolls a 1 on die 2 $= 1/6$
$P(A \cap B) = $ getting 1 on both dice $= 1/6 \times 1/6 = 1/36$
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/6 + 1/6 - 1/36 = \boxed{11/36}$

c) Rolls a 1 or a 2 on either die.

$P(A) = $ rolls 1 or 2 on die 1 $= 2/6 = 1/3$
$P(B) = $ rolls 1 or 2 on die 2 $= 2/6 = 1/3$
$P(A \cup B) = P(A) + P(B) = 1/3 + 1/3 = \boxed{2/3}$
OR rule

5. Lucy rolls three 6-sided dice and flips a fair coin. Find the probability that she:

a) Rolls a 6 on all three dice and obtains a head on the coin.

$P(A) = $ rolls a 6 on die 1 $= 1/6$
$P(B) = $ rolls a 6 on die 2 $= 1/6$
$P(C) = $ rolls a 6 on die 3 $= 1/6$
$P(D) = $ obtaining a head $= 1/2$
$P(A \cap B \cap C \cap D) = P(A) \times P(B) \times P(C) \times P(D)$
$= 1/6 \times 1/6 \times 1/6 \times 1/2$
$= \boxed{1/432}$

b) Rolls exactly one 3 across the three dice rolls and obtains a tail on the coin.

$P(A) = $ rolls a 3 on die 1 $= 1/6$
$P(B) = $ do not roll a 3 on die 2 $= 5/6$
$P(C) = $ do not roll a 3 on die 3 $= 5/6$
$P(D) = $ obtaining a tail $= 1/2$
$P(A \cap B \cap C \cap D) = P(A) \times P(B) \times P(C) \times P(D)$
$= 1/6 \times 5/6 \times 5/6 \times 1/2$
$= \boxed{\dfrac{25}{432}}$

c) Rolls no 4s on the dice and obtains a head on the coin.

$P(A) = $ do not roll a 4 on die 1 $= 5/6$
$P(B) = $ do not roll a 4 on die 2 $= 5/6$
$P(C) = $ do not roll a 4 on die 3 $= 5/6$
$P(D) = $ obtaining a head $= 1/2$
$P(A \cap B \cap C \cap D) = P(A) \times P(B) \times P(C) \times P(D)$
$= 5/6 \times 5/6 \times 5/6 \times 1/2$
$= \boxed{\dfrac{125}{432}}$

6. Emma has a bag containing twenty coloured balls: 3 of the balls are red, 5 of the balls are blue and 12 of the balls are yellow.

independent event

a) She pulls a ball from the bag and then replaces it. What is the probability that the ball was red or blue?

OR rule

$P(A) = $ getting a red ball $= 3/20$

$P(B) = $ getting a blue ball $= 5/20$

$P(A \cup B) = P(A) + P(B) = 3/20 + 5/20 = 8/20 = 2/5$

b) She pulls a yellow ball from the bag and doesn't replace it. What is the probability that the next ball will also be yellow? (dependent event)

Remaining yellow balls: $12 - 1 = 11$

Remaining balls in the bag: $20 - 1 = 19$

Probability of getting the next ball yellow: $\dfrac{11}{19}$

c) She adds another four red balls to the bag. What is the probability that the next four balls she pulls from the bag without replacing them each time will all be red?

Total red balls now: $3 + 4 = 7$ dependent event
Total balls now: $20 + 4 = 24$

Probability of getting next 4 balls red: $\dfrac{7}{24} \times \dfrac{6}{23} \times \dfrac{5}{22} \times \dfrac{4}{21} = \dfrac{5}{1518}$ or 0.0033

the probability changes because this is a dependent event

d) She pulls one ball from the bag and then replaces it. What is the probability that it is not blue?

Probability of getting a non-blue ball $= \dfrac{20-5}{20} = \dfrac{15}{20} = \dfrac{3}{4}$