

GCSE Maths – Probability

Sample Spaces

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of sample space questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

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Section A

Worked Example

Two fair 6-sided dice are rolled at the same time. Construct a sample space diagram showing their combined outcomes.

Step 1: Identify the two independent events and their individual sample spaces.

*Two dice are rolled – these are the independent events.
Each die roll has a sample space of {1, 2, 3, 4, 5, 6}.*

Step 2: Draw the table. This should include labels for the rows and columns and contain the sample spaces of each event as headers.

		Die 1					
		1	2	3	4	5	6
Die 2	1						
	2						
	3						
	4						
	5						
	6						

Step 3: Fill in the table by calculating each outcome.

Work out what goes in each box by taking the first number from the row and the second number from the column.

		Die 1					
		1	2	3	4	5	6
Die 2	1	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6



Worked Example 2

Two fair 6-sided dice are rolled at the same time. Find the probability of rolling one even number and one odd number.

Step 1: Identify the two independent events and their individual sample spaces.

*Two dice are rolled – these are the independent events.
Each die roll has a sample space of {1, 2, 3, 4, 5, 6}.*

Step 2: Construct the table of outcomes.

		Die 1					
		1	2	3	4	5	6
Die 2	1	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

Step 3: Identify the target outcome.

The target outcome is a roll with one even and one odd number. There are 18 appearances of this in the table.

		Die 1					
		1	2	3	4	5	6
Die 2	1	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

Step 4: Calculate the required probability.

The diagram shows there are 18 ways rolling one odd number and one even number out of 36 possible outcomes.

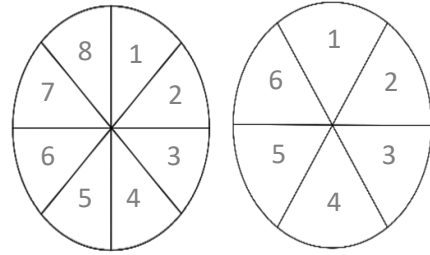
$$18 \text{ target combinations out of } 36 \text{ outcomes} = \frac{18}{36} = \frac{1}{2}$$

The probability of rolling an odd and an even number is $\frac{1}{2}$.



Guided Example

The two fair spinners shown are spun. Their outcomes are added together to produce a result. State the sample space for each spinner and show their combined outcome in a sample space diagram.



Step 1: Identify the two independent events and their individual sample spaces.

Outcome for spinner 1 : $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Outcome for spinner 2 : $\{1, 2, 3, 4, 5, 6\}$

Step 2: Draw the table. This should include labels for the rows and columns and contain the sample spaces of each event as headers.

		Spinner 1							
		1	2	3	4	5	6	7	8
Spinner 2	1	2	3	4	5	6	7	8	9
	2	3	4	5	6	7	8	9	10
	3	4	5	6	7	8	9	10	11
	4	5	6	7	8	9	10	11	12
	5	6	7	8	9	10	11	12	13
	6	7	8	9	10	11	12	13	14

Target outcome : Addition of both outcomes.

Step 3: Fill in the table by calculating each outcome.



Now it's your turn!

If you get stuck, look back at the worked and guided examples.

1. Write down the sample space for each of these events.

a) Flipping a coin.

Sample space for flipping a coin : $\{ \text{Head}, \text{Tail} \}$

b) Rolling an 8-sided die.

Sample space : $\{ 1, 2, 3, 4, 5, 6, 7, 8 \}$

c) Running a random number generator numbered 1-10.

sample space : $\{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \}$

2. Lucy is playing a game. She has a 6-sided die and a coin. First, she rolls the die and records her score. Then she flips the coin.

If the coin lands on heads, her score is doubled.

If the coin lands on tails, her score has 2 points deducted from it.

a) Write down the sample spaces for the coin and the die.

Sample space for coin : $\{ \text{Head}, \text{Tail} \}$

Sample space for die : $\{ 1, 2, 3, 4, 5, 6 \}$

b) Construct a sample space diagram showing the possible scores she could get.

		Die					
		1	2	3	4	5	6
Coin	Head	2	4	6	8	10	12
	Tail	-1	0	1	2	3	4

The score doubles during heads

The score reduces by 2 during tails

c) Work out the probability that she scores higher than 10.

The only score that is higher than 10 is 12.

Total possible outcome : 12

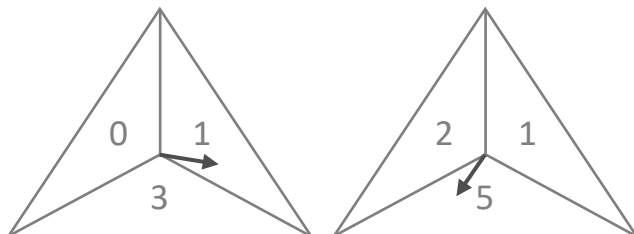
Probability of scoring higher than 10 : $\frac{1}{12}$



3. Jacob has 2 spinners. He spins them both and adds their results together.

Jacob says: "The probability that I score an even number is $\frac{1}{2}$."

Show that he is incorrect.



Sample space for spinner 1 : $\{0, 1, 3\}$

Sample space for spinner 2 : $\{1, 2, 5\}$

There are 5 possible outcomes out of 9 total outcomes to score an even number.

Probability of scoring an even number = $\frac{5}{9}$

Hence, Jacob's statement is incorrect.

add the outcomes together

		Spinner 1		
		0	1	3
Spinner 2				
	1	1	2	4
	2	2	3	5
	5	5	6	8

4. Miles has organised a game to raise money for the local rugby club. Players must roll a 6-sided die and flip a coin.

If the coin lands on heads, the score is doubled.

If the coin lands on tails, the score is halved.

People pay 50p to play.

If they score 10 or more, they win £1.

If they score more than 5 but less than 10, they win 75p.

800 people play his game.

Use probability to estimate how much money he raises.

Sample space for die : $\{1, 2, 3, 4, 5, 6\}$

Sample space for coin : $\{\text{Head}, \text{Tail}\}$

		Die					
		1	2	3	4	5	6
Coin							
	Head	2	4	6	8	10	12
Tail	0.5	1	1.5	2	2.5	3	

Probability of scoring 10 or more : $\frac{2}{12} = \frac{1}{6}$

Probability of scoring >5 but <10 : $\frac{2}{12} = \frac{1}{6}$

Total admission fee :

$$£0.5 \times 800 = £400$$

Money used to pay people

scoring ≥ 10 :

$$\frac{1}{6} \times 800 \times £1 = £133.3$$

Money used to pay people scoring >5 but <10 :

$$\frac{1}{6} \times 800 \times £0.75 = £100$$

Total money Miles made :

$$£400 - £133.3 - £100 = £166.7$$



5. A six-sided die is rolled twice.

a) Construct a sample space diagram to show the possible outcomes.

		Die 1					
		1	2	3	4	5	6
Die 2	1	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

Sample space for 6 sided die :
 $\{1, 2, 3, 4, 5, 6\}$

b) Work out the probability that the second roll is the same as the first roll.

Second roll is the same as first roll : $\{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

Total outcomes : 36 : 6 outcomes

Probability of getting the second roll the same as the first : $\frac{6}{36} = \frac{1}{6}$

c) State the probability that the second roll is NOT the same as the first roll.

Outcomes for the second roll not the same as the first : $36 - 6 = 30$

Probability = $\frac{30}{36} = \frac{5}{6}$

d) Work out the probability that five sequential rolls are all the same.

Probability of getting five sequential rolls the same :

$$\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{7776} = 0.00013$$

6. Enzo flips a fair coin ten times. He gets heads 7 times, and tails 3 times.

Enzo says, "The coin is definitely biased because I did not get the same number of heads and tails."

Is he correct? Explain your answer.

He is incorrect. The $\frac{1}{2}$ probability of getting either a head or tail is only theoretical, not experimental. That theoretical probability is only used as an estimate of an outcome of an event. Every event is considered a new random event hence, it is normal for the outcome of an experiment to be different from the values calculated from the probability. In this case, the coin is not necessarily biased.

