GCSE Maths – Probability

Theoretical Probability, Frequency and Expected Outcomes

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of probability questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

The probability that a biased coin will land on heads is 0.6. Jessie is going to flip the coin 200 times. Calculate an estimate for the number of times the coin will land on tails. Explain why this value is only an estimate.

Step 1: The question gives you the probability of the coin landing on heads. Use this to work out the probability of the coin landing on tails.

The total probability of all possible events must add up to 1. This means the sum of the probability of getting a head and the probability of getting a tail is 1.

$$P(T) + P(H) = 1$$
$$P(T) + 0.6 = 1$$
$$P(T) = 1 - 0.6$$
$$P(T) = 0.4$$

Step 2: Multiply the probability by the number of trials to find the expected outcomes.

There are 200 trials, as she will flip the coin 200 times.

$$200 \times 0.6 = 120$$

The coin should land on tails 120 times.

Step 3: Explain the reason for this value being an estimate.

120 is an estimate because the probability is theoretical not experimental, and each trial is a new random event.

Guided Example

The probability that a biased die will land on four is 0.2. Elisha is going to roll the die 560 times. Calculate an estimate for the number of times the die will land on four.

When she rolls the die, it lands on four 137 times. Explain why this is different from your value.

Step 1: Multiply the probability by the number of trials to find the expected outcomes.

probability to land on 4 = 0.2

$0.2 \times 560 = 112$

Estimate number of times the die will land on 4 = 112 times

Step 2: Explain why your calculated value is not the same as the experimental value.

112 is just an estimate because the probability is theoretical not experimental. Each trial is a new random event, hence, the different value.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. The probability of a biased coin landing on heads is 0.72. Percy flips the coin 1400 times. Estimate the number of times it lands on tails.

$P(H) = 0.72$
$P(H) + P(T) = 1$
$0.72 + P(T) = 1$
$P(T) = 0.28$

$= P(T) \times$ number of events
$= 0.28 \times 1400$
$= 392$
The number of times it lands on tails : 392 times

2. Jack plants 200 trees. The probability of a tree growing is 0.95. Estimate the number of trees that will not grow.

P(trees growing) $= 0.95$
P(trees not growing) $= 1 - 0.95$
$= 0.05$

$= 0.05 \times 200 = 10$

The estimate number of trees that will not grow is 10 trees

3. Anderson LTD makes 10,000 doors a day. Each door costs £5.10 to make and sells for £13.99.

The probability of a door being faulty is 0.0045.

a) Estimate the number of faulty doors made in one day.

$0.0045 \times 10{,}000 =$ 45 doors

b) Calculate an estimate for the profit that Anderson LTD makes in one working week (5 days) (assuming no faulty doors are made during the week).

Profit : £13.99 − £5.10 = £8.89
= £8.89 × 10,000 × 5 = £444,500

c) On Wednesday, a machine breaks down. Anderson LTD makes 138 faulty doors. Calculate the theoretical probability of a door being faulty on Wednesday.

Thereotical probability $= \frac{138}{10{,}000} = \frac{69}{5000} = 0.0138$

d) Anderson LTD wants to start making 500 kettles a day. Each kettle costs £4.99 to make and sells for £7.10. The probability of a kettle being faulty is in the range $0.25 \leq P(Faulty) \leq 0.3$. The new item will only be approved if it will make the company a daily profit. Should Anderson LTD start selling kettles?

Profit : £7.10 − £4.99 = £2.11
Lowest non-faulty kettle produced = $0.7 \times 500 = 350$
Profit in a day : £2.11 × 350 = £738.5
Yes, they should since an estimate daily profit of £738.5 can be obtained.

Section B

Worked Example

A 6-sided die is rolled 100 times. Complete the table:

Number	1	2	3	4	5	6
Frequency	15		16		19	
Relative frequency		0.2		0.15		

Step 1: Work out the missing relative frequencies using:

$$Relative\ frequency = \frac{Number\ of\ times\ outcome\ happened}{Number\ of\ times\ experiment\ was\ carried\ out}$$

The relative frequency of rolling a 1 = $\frac{15}{100} = 0.15$

The relative frequency of rolling a 3 = $\frac{16}{100} = 0.16$

The relative frequency of rolling a 5 = $\frac{19}{100} = 0.19$

Number	1	2	3	4	5	6
Frequency	15		16		19	
Relative frequency	0.15	0.2	0.16	0.15	0.19	

Step 2: Work out the missing frequencies by multiplying the relative frequency by the total number of trials (100).

Frequency of 2 = $100 \times 0.2 = 20$

Frequency of 4 = $100 \times 0.15 = 15$

Number	1	2	3	4	5	6
Frequency	15	20	16	15	19	
Relative frequency	0.15	0.2	0.16	0.15	0.19	

Step 3: Calculate the rest of the values using the known total number of trials.

Frequency of 6 = $100 - 15 - 20 - 16 - 15 - 19 = 15$

The relative frequency of rolling a 6 = $\frac{15}{100} = 0.15$

Number	1	2	3	4	5	6
Frequency	15	20	16	15	19	15
Relative frequency	0.15	0.2	0.16	0.15	0.19	0.15

Guided Example

A biased coin is flipped 400 times. It lands on heads 350 times. Find the relative frequency of the coin landing on tails.

Step 1: Work out the number of times the coin lands on tails.

Number of times the coin lands on tails : 400 − 350

= 50 times

Step 2: Use the formula to calculate the relative frequency.

relative frequency the coins land on tails : $\frac{50}{400} = \frac{1}{8} = 0.125$

Guided Example 2

A spinner has four sections labelled A, B, C and D. Cara spins the spinner 32 times. The following information gives information about the results of the spins. Complete the table.

	A	B	C	D
Frequency	12	6	8	6
Relative Frequency	0.375	0.2	0.25	0.19

Step 1: Fill in the missing frequency values.

Frequency for B = $0.2 \times 32 = 6.4 \approx 6$ times

Frequency for D = $32 - 6 - 8 - 12 = 6$ times

Step 2: Use the formula to calculate the missing relative frequency values.

Relative frequency for A = $\frac{12}{32} = \frac{3}{8} = 0.375$

Relative frequency for C = $\frac{8}{32} = \frac{1}{4} = 0.25$

Relative frequency for D = $6/32 = 3/16 = 0.1875 \approx 0.19$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

4. Lanais asks some people in her village what their favourite sport is.
   22 people said hockey
   56 people said football
   14 people said rugby

   a) Work out the relative frequency of someone in the town liking hockey.

   Total number of people in the village : $22 + 56 + 14 = 92$

   relative frequency of someone liking hockey $= \frac{22}{92} = \frac{11}{46} = 0.239$

   ≈ 0.24

   b) There are 2000 people living in Lanais' village. Using your answer to part a), estimate the number of people whose favourite sport is hockey.

   relative frequency x number of people

   $0.24 \times 2000 = 480$ people

   There are approximately 480 people in Lanais' village whose favourite sport is hockey.

5. A tin contains some biscuits. The flavours of the biscuits and the amount of each flavour are shown in the table.

Flavour	Chocolate	Lemon	Ginger	Caramel
Frequency	14	16	28	8

   a) How many biscuits are in the tin?

   Total biscuits : $14 + 16 + 28 + 8 = 66$

   b) What is the relative frequency of a chocolate biscuit?

   relative frequency of chocolate biscuit : $\frac{14}{66} = \frac{7}{33} = 0.21$

   (dependent events)

   c) Jordan takes two biscuits from the tin one at a time without replacing them. Find the probability that they are both not lemon biscuits.

   Probability of getting the first non-lemon biscuit : $\frac{(14+28+8)}{66} = \frac{50}{66} = \frac{25}{33}$

   Probability of getting the second non-lemon biscuit : $\frac{49}{65}$

   probability of getting both non-lemon biscuits : $\frac{25}{33} \times \frac{49}{65} = 0.57$

6. A bag contains only red, green and yellow counters. The relative frequency of each counter is shown in the table.

Colour	Red	Green	Yellow
Relative Frequency	0.35	0.6	0.05

a) Find the relative frequency of a yellow counter.

Total relative frequency = 1

relative frequency of yellow counter $= 1 - 0.35 - 0.6 = 0.05$

b) Jolene takes a counter from the bag. What is the probability that it is not red?

Probability of not being red $= 0.6 + 0.05 = 0.65$

c) There are 5 yellow counters in the bag. How many green counters are in the bag?

$0.05 = \frac{5}{\text{total number of counters}}$

total number of counters $= \frac{5}{0.05} = 100$

Number of green counters $= 0.6 \times 100$

$= 60$

7. A spinner is labelled with numbers 1 to 5. Tarig spins the spinner 52 times, and the results are shown in the following table.

	1	2	3	4	5
Frequency	12	6	9	7	18
Relative frequency	0.23	0.12	0.17	0.13	0.35

a) Complete the table

Frequency number 5 $= 52 - 12 - 6 - 9 - 7 = 18$

Relative frequency number 1 : $\frac{12}{52} = \frac{3}{13} = 0.23$

Relative frequency number 2 : $\frac{6}{52} = \frac{3}{26} = 0.12$

Relative frequency number 3 : $9/52 = 0.17$

Relative frequency number 4 : $\frac{7}{52} = 0.13$

Relative frequency number 5 $= \frac{18}{52} = 0.35$

b) Estimate the probability that an odd number is obtained when the spinner is spun.

Probability of getting an odd number : add all relative frequency with odd numbers

$= 0.23 + 0.17 + 0.35$

$= 0.75$

Section C

Worked Example

A bag contains 26 green balls, 14 red balls and 10 blue balls. What is the probability of picking a red or blue ball from the bag?

Step 1: Identify the target groups.

The question asks you to find the probability of choosing a red OR a blue ball. Red and blue balls are the target groups.

Step 2: Find the probability of picking each of the target groups.

There are $14 + 26 + 10 = 50$ *balls in total.*
There are 14 *red balls so the probability of choosing a red ball is* $\frac{14}{50}$.
There are 10 *blue balls so the probability of choosing a blue ball is* $\frac{10}{50}$.

Step 3: Apply the formula for the OR rule.

The OR rule is: $P(A \cup B) = P(A) + P(B)$

Let A *be the event of picking a red ball.*
Let B be the event of picking a blue ball.

$$P(A \cup B) = \frac{14}{50} + \frac{10}{50} = \frac{24}{50}$$

The probability of choosing a red or blue ball is $\frac{24}{50}$.

Guided Example

In Maia's village, there are 200 people. 68 of them play football, and 120 of them play tennis. Maia picks two people at random. What is the probability that neither of them plays football?

Step 1: Find the number of people who do not play football (remember that some people play neither sport).

$200 - 68 = 132$

Step 2: Find the probability of one person not playing football.

$\frac{132}{200} = \frac{33}{50} = 0.66$

Step 3: Apply the AND probability rule.

Probability of choosing 2 people not playing football $= 0.66 \times 0.66 = 0.44$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

8. A fair 6-sided die is rolled.

a) What is the probability of rolling an odd number?

Odd number : $\{1, 3, 5\}$

Probability of getting an odd number $= \frac{3}{6} = \frac{1}{2}$

b) If the die is rolled 100 times, how many times would you expect to get an odd number?

$1/2 = 0.5$

$0.5 \times 100 = 50$

We would expect to get an odd number 50 times.

c) Mariana rolls the die 100 times. She gets an odd number 68 times. Explain why this is not the same as the value that you calculated in part b).

The value calculated is only an estimate since the probability that was used was theoretical and not experimental. Each trial is a new random event, which explains why the value is different than calculated.

9. An unbiased 10-sided die is rolled. Complete the sentences below using the words:

Likely Unlikely Even Chance Certain Impossible

a) It is unlikely that the die will land on 1.

b) It is even chance that the die will land on an even number.

c) It is impossible that the die will land on 11.

d) It is likely that the die will land on a number bigger than 1.

e) It is certain that the die will land on a number between 1 and 10.

10. Harriet flips a drawing pin 100 times. It lands with the point upwards 68 times.

Harriet says:

"Because the pin can only land point up or point down, the probability of it landing point up should be 0.5."

Comment on her statement.

Her statement is correct. The theoretical probability of the pin landing point up should be $\frac{1}{2}$ if the pin is unbiased. This is because the drawing pin only has 2 expected outcome which is point up or point down. However, the experimental value may be different as every trial is a new random event.