GCSE Maths – Probability

Table of Outcomes and Frequency Trees

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions relating to tables of outcomes and frequency trees. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Two fair 6-sided dice are rolled at the same time. Construct a table of outcomes for their product.

Step 1: Identify the two independent events and their individual outcomes.

Two dice are rolled – these are the independent events.
Each die roll can have an outcome of 1-6.

Step 2: Identify the operation.

"…for their product" – so the operation is multiplication of their values.

Step 3: Draw the table. This should include headings for the rows and columns and should contain the individual outcomes of each event as sub-headers.

		Die 1					
		1	2	3	4	5	6
D i e	1						
	2						
	3						
	4						
	5						
2	6						

Step 4: Fill in the table by calculating each outcome.
This is done by multiplying the number in the header of the row and column.

		Die 1					
		1	2	3	4	5	6
D i e	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
2	6	6	12	16	24	30	36

Guided Example

The two fair spinners shown are spun. Their outcomes are added together to produce a result. Show all the possible results in a table of outcomes.
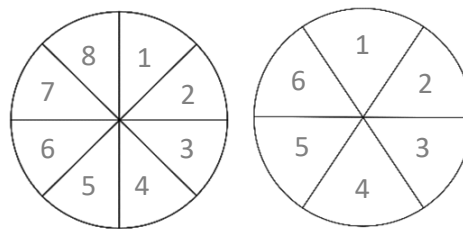
Step 1: Identify the two independent events and their individual outcomes.

Independent events → two spinners spun.

Outcomes → Spinner 1 - 1-8
Spinner 2 - 1-6

Step 2: Identify the operation.

'added together' +

Step 3: Draw the table. This should include headings for the rows and columns and should contain the individual outcomes of each event as headers.

Spinner 1

Spinner 2 \ +	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14

Step 4: Fill in the table by calculating each outcome.

1. A fair coin is flipped, and a 10-sided die is rolled. Record the possible outcome combinations in a table.

Coin → H or T

10 sided die → 1-10

		Die									
		1	2	3	4	5	6	7	8	9	10
Coin	Heads	1H	2H	3H	4H	5H	6H	7H	8H	9H	10H
	Tails	1T	2T	3T	4T	5T	6T	7T	8T	9T	10T

2. Two 6-sided dice are rolled, and their sum calculated. Record the possible outcome combinations in a table.

Two 6 sided dice → 1-6 outcome.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Dice 1 (column header), Dice 2 (row header)

3. A 12-way spinner and a 4-way spinner are spun, and their sum calculated. If their sum is odd, it is recorded as O. If their sum is even, it is recorded as E. Record the possible outcomes in a table.

1 *2*

O + O = even
E + E = even
O + E = odd

Spinner 1

+	1	2	3	4	5	6	7	8	9	10	11	12
1	E	O	E	O	E	O	E	O	E	O	E	O
2	O	E	O	E	O	E	O	E	O	E	O	E
3	E	O	E	O	E	O	E	O	E	O	E	O
4	O	E	O	E	O	E	O	E	O	E	O	E

Spinner 2 (row labels)

4. A restaurant serves four main meals and four desserts, the prices shown in the tables below. Latisha chooses a main meal and a dessert at random. Record in a table of outcomes the possible prices of her meal.

add prices

Main	£
Stir Fry	9.50
Beef Soup	8.75
Roast Lamb	14.00
Carbonara	9.20

Dessert	£
Brownie	4.50
Apple Pie	4.20
Tiramisu	3.25
Trifle	3.90

Main

+	Stir Fry £9.50	Beef Soup £8.75	Roast Lamb £14	Carbonara £9.20
Brownie £4.50	9.50 + 4.50 = £14	8.75 + 4.50 = £13.25	14 + 4.50 = £18.50	9.20 + 4.50 = £13.70
Apple Pie £4.20	9.50 + 4.20 = £13.70	8.75 + 4.20 = £12.95	14 + 4.20 = £18.20	9.20 + 4.20 = £13.40
Tiramisu £3.25	9.50 + 3.25 = £12.75	8.75 + 3.25 = £12	14 + 3.25 = £17.25	9.20 + 3.25 = £12.45
Trifle £3.90	9.50 + 3.90 = £13.40	8.75 + 3.90 = £12.65	14 + 3.90 = £17.90	9.20 + 3.90 = £13.10

Dessert (row labels)

Section B

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Worked Example

Two fair, 6-sided dice are rolled at the same time. Find the probability that their product is an even number.

Step 1: Construct a table of outcomes for the question.

		Die 1					
		1	2	3	4	5	6
D	1	1	2	3	4	5	6
i	2	2	4	6	8	10	12
e	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
2	5	5	10	15	20	25	30
	6	6	12	16	24	30	36

Step 2: Identify the target outcome.

'Find the probability that their product is an <u>even number</u>' so even numbers are the target outcome.

		Die 1					
		1	2	3	4	5	6
D	1	1	2	3	4	5	6
i	2	2	4	6	8	10	12
e	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
2	5	5	10	15	20	25	30
	6	6	12	16	24	30	36

Step 3: Count the appearances of the target outcome.

There are 27 even numbers in the table. Circle or highlight them to help you count without missing any outcomes.

Step 4: Calculate the probability of getting the target outcome.

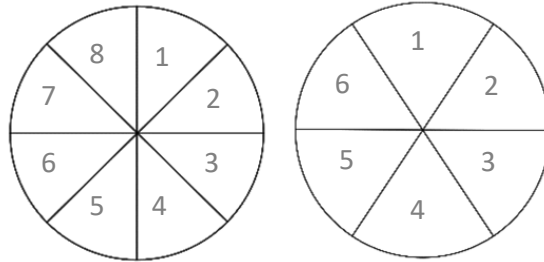*There are 27 appearances of the target outcome, and 36 possible outcomes.*

$$Probability\ of\ target\ outcome\ = \frac{Appearances\ of\ target\ outcome}{Total\ number\ of\ possible\ outcomes}$$

$$Probability\ of\ even\ product = \frac{27}{36}$$

Guided Example

The two fair spinners shown are spun. Find the probability that their sum is more than 10.

Spinner 1: 8, 1, 7, 2, 6, 3, 5, 4

Spinner 2: 1, 6, 2, 5, 3, 4

Step 1: Construct a table of outcomes for the question.

Spinner 1

+	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14

Spinner 2

Step 2: Identify the target outcome.

> 10

Step 3: Count the appearances of the target outcome.

10

Step 4: Calculate the probability of getting the required outcome.

$8 \times 6 = 48$ outcomes.

$> 10 = 10$ outcomes.

$P = \dfrac{10}{48} \begin{smallmatrix} \div 2 \\ = \\ \div 2 \end{smallmatrix} \dfrac{5}{24}$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. A fair coin is flipped, and a 10-sided die is rolled. Find the probability of getting a head and an even number. You can use your answers from Section A to complete this question.

Die

Coin		1	2	3	4	5	6	7	8	9	10
	Heads	1H	2H	3H	4H	5H	6H	7H	8H	9H	10H
	Tails	1T	2T	3T	4T	5T	6T	7T	8T	9T	10T

$2 \times 10 = 20$ outcomes

even + head = 5 outcomes

$$P = \frac{5}{20} \begin{array}{c} \div 5 \\ \div 5 \end{array} = \frac{1}{4}$$

6. Two 6-sided dice are rolled. Find the probability that their sum is more than 10. You can use your answers from Section A to complete this question.

Dice 1

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Dice 2

$6 \times 6 = 36$ outcomes
$>10 = 3$ outcomes

$$P = \frac{3}{36} \begin{array}{c} \div 3 \\ \div 3 \end{array} = \frac{1}{12}$$

7. A 12-way spinner and a 4-way spinner are spun, and their sum calculated. If their sum is odd, it is recorded as O. If their sum is even, it is recorded as E. Calculate the probability of getting E when the 12-sided spinner gives 1. You can use your answers from Section A to complete this question.

only look at this column ↓   Spinner 1

+	1	2	3	4	5	6	7	8	9	10	11	12
1	E	O	E	O	E	O	E	O	E	O	E	O
2	O	E	O	E	O	E	O	E	O	E	O	E
3	E	O	E	O	E	O	E	O	E	O	E	O
4	O	E	O	E	O	E	O	E	O	E	O	E

Spinner 2

The 12 sided spinner gives 1. So there are only 4×1 = 4 outcomes
Even = 2 outcomes.

$P = \dfrac{2 \div 2}{4 \div 2} = \dfrac{1}{2}$

8. Latisha goes into a restaurant that serves four main meals and four desserts. She has a voucher for £12.50. ← Total price is less than/or £12.50

Latisha chooses a main meal and a dessert at random. Find the probability that the voucher covers the cost of her meal. You can use your answers from Section A to complete this question.

Main	£
Stir Fry	9.50
Beef Soup	8.75
Roast Lamb	14.00
Carbonara	9.20

Dessert	£
Brownie	4.50
Apple Pie	4.20
Tiramisu	3.25
Trifle	3.90

Main

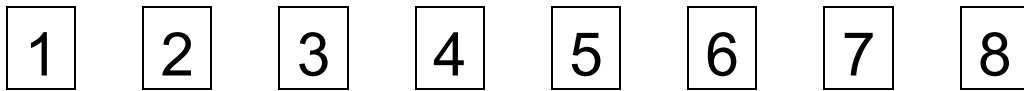| +	Stir Fry £9.50	Beef Soup £8.75	Roast Lamb £14	Carbonara £9.20
Brownie £4.50	9.50+4.50 £14	8.75+4.50 £13.25	14+4.50 £18.50	9.20+4.50 £13.70
Apple Pie £4.20	9.50+4.20 £13.70	8.75+4.20 £12.95	14+4.20 £18.20	9.20+4.20 £13.40
Tiramisu £3.25	9.50+3.25 £12.75	8.75+3.25 £12	14+3.25 £17.25	9.20+3.25 £12.45
Trifle £3.90	9.50+3.90 £13.40	8.75+3.90 £12.65	14+3.90 £17.90	9.20+3.90 £13.10

Dessert

4 × 4 = 16 outcomes.

≤ 12.50 = 2 outcomes.

$P = \dfrac{2 \div 2}{16 \div 2} = \dfrac{1}{8}$

9. Alex has 8 cards:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

She takes one card followed by another to make a two-digit number.

E.g.

$$\boxed{1} + \boxed{2} = 12$$

Find the probability that:

a) The number is more than 50.
b) The number is even.
c) The number is a multiple of 7.

First Digit

	1	2	3	4	5	6	7	8
1	/	21	31	41	51	61	71	81
2	12	/	32	42	52	62	72	82
3	13	23	/	43	53	63	73	83
4	14	24	34	/	54	64	74	84
5	15	25	35	45	/	65	75	85
6	16	26	36	46	56	/	76	86
7	17	27	37	47	57	67	/	87
8	18	28	38	48	58	68	78	/

Second digit

$(8 \times 8) - 8 = 64 - 8 = 56$ outcomes

The 1st + 2nd digit can't be the same as there is only one card

a) Starts with 5,6,7,8
$> 50 = 4 \times 7 = 28$
7 numbers in a row
$P = \frac{28}{56} = \frac{1}{2}$

b) ends in 2,4,6,8
even $= 4 \times 7 = 28$ outcomes
7 numbers in a row
$P = \frac{28}{56} = \frac{1}{2}$

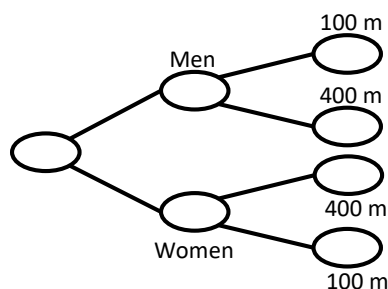c) Multiple of 7 = 8 outcomes
$P = \frac{8}{56} = \frac{1}{7}$

Section C

Worked Example

An athletics club has 100 members. The members must choose which race to do in the upcoming competition. In the club:

- **There are 38 women.**
- **26 women choose the 100m sprint.**
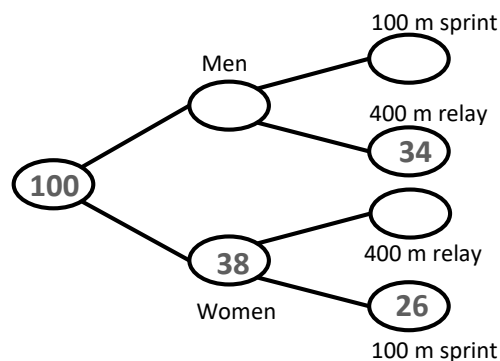- **34 men choose the 400m relay.**

Complete the frequency tree.

Step 1: Fill in any known values that are given.

We know the total value – there are 100 members in the club. This goes in the first circle.

There are 38 women, and that 26 of them do 100 m sprint.

There are 34 men who do 400 m relay.

Step 2: Work out any unknown values by taking the totals and known values and calculating what is left.

There are 100 people in the running club and 38 of them are women, so the rest are men.
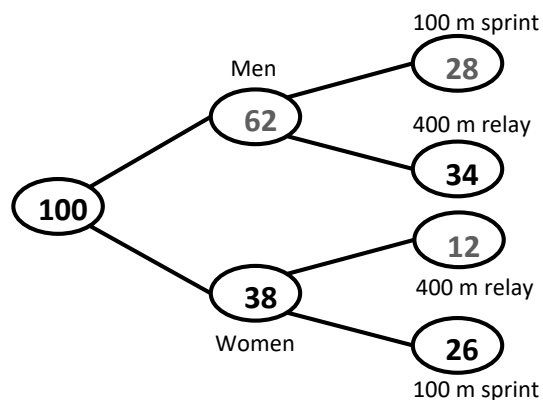
$$Men\ in\ the\ running\ club = 100 - 38 = 62$$

There are 62 men and 34 of them do the 400 m relay, so the rest of them do the 100 m sprint.

$$Men\ who\ do\ 100\ m\ sprint = 62 - 34 = 28$$

There are 38 women and 26 of them do 100 m sprint, so the rest of them do 400 m relay.
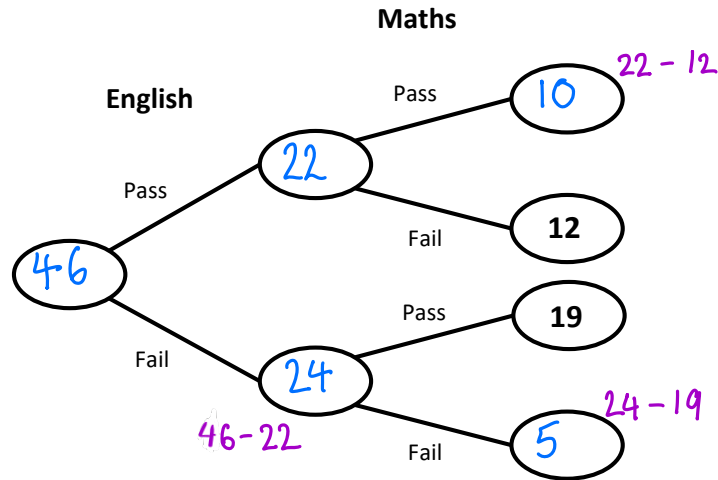
$$Women\ who\ do\ 400\ m\ relay = 38 - 26 = 12$$

Step 3: Complete the tree with the values you have calculated.

Guided Example

46 people took a Maths and an English exam. 22 people passed the English Exam. Complete the frequency tree.

Maths

English

46

Pass

22

Pass

10   22 - 12

Fail

12

Fail

24

46-22

Pass

19

Fail

5   24 - 19

Step 1: Fill in any known values that are given in the question.

Step 2: Work out any unknown values by taking the totals and known values and calculating what is left.

Step 3: Complete the tree with the values you have calculated. Check when you are done that the totals are correct, and all the values are in the correct section.

$10 + 12 + 19 + 5 = 46$ ✓

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

10. 104 students took their Spanish and French speaking exams. Write down the number of students who:
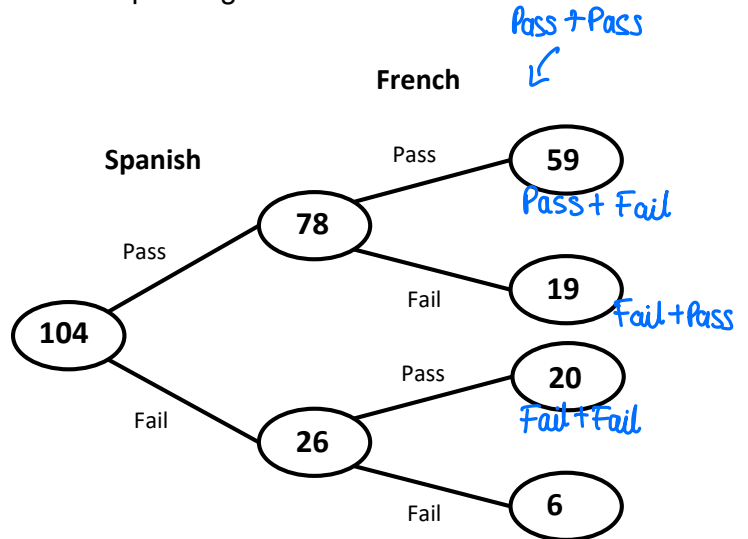
Pass + Pass
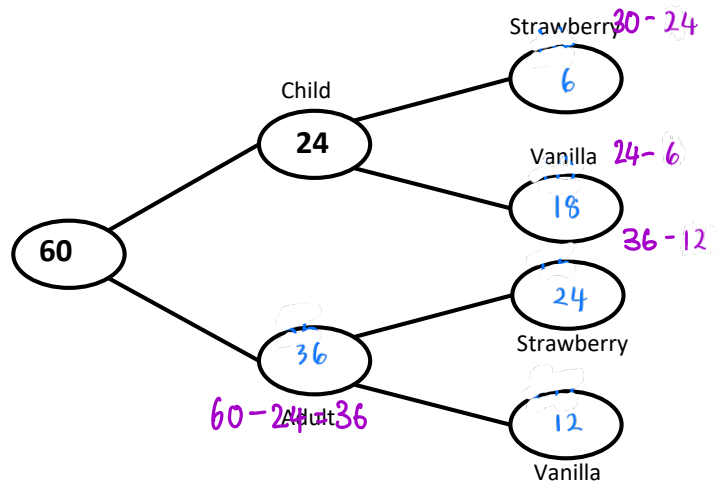
a) Passed Spanish = 78

b) Passed both = 59

c) Passed neither = 6

French

Spanish

Pass

Pass **59** Pass + Fail

78

Fail **19** Fail + Pass

104

Pass **20** Fail + Fail

Fail **26**

Fail **6**

11. 60 people are asked if they prefer strawberry or vanilla ice cream.

- 24 of them are children $\frac{1}{3} \times 36 = 12$
- One third of adults prefer vanilla
- 30 people in total prefer strawberry

a) Complete the frequency tree below.

Strawberry 30 - 24

Child **6**

24 Vanilla 24 - 6

18

60 36 - 12

24

36 Strawberry

60 - 24 Adult 36 **12**

Vanilla

b) Write down the number of adults who prefer strawberry.

24

c) Write down the fraction of children, out of the total population, who prefer vanilla.

18 children like vanilla $\frac{18}{60} = \frac{3}{10}$

60 total