

# GCSE Maths – Number

## Powers, Roots and Fractional Indices

### Worksheet

NOTES



SOLUTIONS



This worksheet will show you how to work out different types of questions on powers, roots and fractional indices. Each section contains a **worked example**, a **question with hints** and then **questions for you to work through on your own**.

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## Section A

### Worked Example 1

Find  $13^3$

**Step 1:** Identify the power and use the number to indicate how many multiples there are.

$$13^3 = 13 \times 13 \times 13$$

**Step 2:** Calculate the product.

$$13^3 = 13 \times 13 \times 13 = 2197$$

### Worked Example 2

Find  $\left(\frac{5}{4}\right)^{-4}$

**Step 1:** Due to the negative sign, flip the base.

$$\left(\frac{5}{4}\right)^{-4} = \left(\frac{4}{5}\right)^4$$

**Step 2:** Apply the remaining power to the numerator and denominator.

$$\left(\frac{5}{4}\right)^{-4} = \left(\frac{4}{5}\right)^4 = \frac{4^4}{5^4} = \frac{4 \times 4 \times 4 \times 4}{5 \times 5 \times 5 \times 5} = \frac{256}{625}$$

### Guided Example 1

Find  $21^2$

**Step 1:** Identify the power and use the number to indicate how many multiples there are.

**Step 2:** Calculate the product.

### Guided Example 2

Find  $\left(\frac{13}{3}\right)^{-2}$

**Step 1:** Due to the negative sign, flip the base.

**Step 2:** Apply the remaining power to the numerator and denominator.



**Now it's your turn!**

If you get stuck, look back at the worked and guided examples.

1. Find  $7^3$

2. Find  $4^4$

3. Find  $5^6$

4. Find  $\left(\frac{3}{5}\right)^2$

5. Find  $\left(\frac{-9}{4}\right)^3$

6. Find  $2^{-3}$

7. Find  $0.5^5$

8. Find  $\left(\frac{7}{11}\right)^{-4}$

9. Find  $\left(\frac{2}{3}\right)^7$

10. Find  $(-11)^{-4}$



## Section B

### Worked Example 1

**Simplify**  $g^5 \times g^3$

**Step 1:** As we are multiplying, we must add the two powers together.

$$g^5 \times g^3 = g^{5+3}$$

**Step 2:** Simplify the addition of the two powers.

$$g^{5+3} = g^8$$

### Worked Example 2

**Simplify**  $(q^6)^{11}$

**Step 1:** As we are raising a power to another power, we must multiply the two powers together.

$$(q^6)^{11} = q^{6 \times 11}$$

**Step 2:** Simplify the multiplication of the two powers.

$$q^{6 \times 11} = q^{66}$$

### Guided Example

**Simplify**  $y^9 \div y^{\frac{1}{2}}$

**Step 1:** As we are dividing, we must subtract the second powers from the first power.

**Step 2:** Simplify the subtraction of the two powers.



**Now it's your turn!**

If you get stuck, look back at the worked and guided examples.

11. Simplify  $x \times x \times x$

12. Simplify  $a^3 \times a^4$

13. Simplify  $r^{40} \div r^{21}$

14. Simplify  $e^{\frac{3}{4}} \times e^{\frac{1}{2}}$

15. Simplify  $t^{\frac{7}{3}} \div t^2$

16. Simplify  $(a^2)^3$

17. Simplify  $(9b^4)^7$

18. Simplify  $(3f^5)^{\frac{9}{10}}$

19. Simplify  $(p^{-q})^{-r}$

20. Simplify  $\left(\frac{x^{2y}}{xy}\right)^3$



## Section C – Higher Only

### Worked Example 1

Find and simplify  $\sqrt{68}$

**Step 1:** Identify if the number in the root has any square number factors.

$$68 = 4 \times 17 \text{ so } 4 \text{ is a square number factor.}$$

**Step 2:** Simplify the square root using rules of surds.

$$\sqrt{68} = \sqrt{4 \times 17} = \sqrt{4} \times \sqrt{17} = 2 \times \sqrt{17} = 2\sqrt{17}$$

### Worked Example 2

Find and simplify  $\sqrt[4]{625}$

**Step 1:** Without using a calculator, find an integer which factors into 625 exactly 4 times (the same number of times as the root).

$$625 = 5 \times 5 \times 5 \times 5 = 5^4$$

**Step 2:** Deduce the solution to the root expression.

$$\sqrt[4]{625} = \sqrt[4]{5^4} = 5$$

### Guided Example 1

Find and simplify  $\sqrt{126}$

**Step 1:** Identify if the number in the root has any square number factors.

**Step 2:** Simplify the square root using rules of surds.

### Guided Example 2

Find and simplify  $\sqrt[5]{32}$

**Step 1:** Without using a calculator, find an integer which factors into 32 exactly 5 times (the same number of times as the root).

**Step 2:** Deduce the solution to the root expression.



### Now it's your turn!

If you get stuck, look back at the worked and guided examples.

21. Find  $\sqrt{81}$

22. Find  $\sqrt{24}$

23. Find  $\sqrt{900}$

24. Find  $\sqrt{612}$

25. Find  $2\sqrt{128}$

26. Find  $13\sqrt{338}$

27. Find  $\sqrt[3]{64}$

28. Find  $\sqrt[4]{16}$

29. Find  $\sqrt[3]{125}$

30. Find  $\sqrt[5]{243}$



## Section D – Higher Only

### Worked Example

Find and simplify  $2^{-\frac{3}{2}}$

**Step 1:** Due to the negative sign, flip the base.

$$2^{-\frac{3}{2}} = \left(\frac{1}{2}\right)^{\frac{3}{2}}$$

**Step 2:** Apply the remaining index to the numerator and denominator.

$$\left(\frac{1}{2}\right)^{\frac{3}{2}} = \frac{1^{\frac{3}{2}}}{2^{\frac{3}{2}}}$$

**Step 3:** Simplify the remaining powers and roots. Rationalise the denominator if necessary.

$$\frac{1^{\frac{3}{2}}}{2^{\frac{3}{2}}} = \frac{1}{\sqrt{2^3}} = \frac{1}{\sqrt{8}} = \frac{1}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{\sqrt{8}}{8} = \frac{\sqrt{4 \times 2}}{8} = \frac{\sqrt{4} \times \sqrt{2}}{8} = \frac{2\sqrt{2}}{8} = \frac{\sqrt{2}}{4}$$

### Guided Example

Find and simplify  $\left(\frac{4}{5}\right)^{\frac{1}{2}}$

**Step 1:** Apply the power to the numerator and denominator. As it is a fraction, we will get a root.

**Step 2:** Simplify the remaining powers and roots. Rationalise the denominator if necessary.





**Now it's your turn!**

If you get stuck, look back at the worked and guided examples.

31. Find and simplify where possible  $9^{\frac{1}{2}}$

32. Find and simplify where possible  $12^{\frac{3}{2}}$

33. Find and simplify where possible  $\left(\frac{4}{3}\right)^{\frac{5}{2}}$

34. Find and simplify where possible  $\left(\frac{27}{64}\right)^{-\frac{1}{3}}$

35. Find and simplify where possible  $9^{-\frac{1}{2}}$



36. Find and simplify where possible  $\left(\frac{7}{8}\right)^{-\frac{2}{3}}$

37. Find and simplify where possible  $\left(\frac{16}{81}\right)^{-\frac{5}{4}}$

38. Find and simplify where possible  $\left(\frac{8}{27}\right)^{-\frac{4}{3}}$

39. Find and simplify where possible  $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

40. Find and simplify where possible  $(32)^{-\frac{2}{5}}$



## Section E – Higher Only

### Worked Example

**Estimate**  $6.5^2$

**Step 1:** Recognise that 6.5 is between two integers, 6 and 7.

$$6 < 6.5 < 7$$

**Step 2:** Due to this,  $6.5^2$  is between  $6^2$  and  $7^2$ .

$$6^2 < 6.5^2 < 7^2$$

**Step 3:** Simplify this inequality.

$$36 < 6.5^2 < 49$$

**Step 3:** Using this we can estimate  $6.5^2$ .

$$6.5^2 \approx 40$$

### Guided Example

**Estimate**  $\sqrt{14}$

**Step 1:** Recognise that 14 is between two square numbers, 9 and 16.

**Step 2:** Due to this the square root of 14 lies between the square root of 9 and the square root of 16.

**Step 3:** As 14 is closer to 16 than to 9, square root of 14 is close to the square root of 16.



**Now it's your turn!**

If you get stuck, look back at the worked and guided examples.

41. Estimate  $4.3^2$

42. Estimate  $1.4^3$

43. Estimate  $2.1^5$

44. Estimate  $0.823^2$

45. Estimate  $\sqrt{39}$

46. Estimate  $\sqrt{35}$

47. Estimate  $\sqrt{140}$

48. Estimate  $\sqrt{18.2}$

49. Estimate  $\sqrt[3]{61}$

