GCSE Maths – Geometry and Measures

Vector Operations

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of vector operation questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Find $a + b$ when $a = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$, $b = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$.

Step 1: Write $a + b$ as one column vector by adding each of the rows.

$$a + b = \begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 + (-3) \\ -2 + 0 \end{pmatrix}$$

Step 2: Sum each row and calculate the total.

$$5 + -3 = 2$$

$$-2 + 0 = -2$$

$$a + b = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

Guided Example

Let $a = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $b = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$. Write $2a + b$ as a column vector.

Step 1: Multiply both components of a by 2.

$$2 \times \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

Step 2: Add the components of $2a$ to the components of b.

$$\underset{2a}{\begin{pmatrix} 6 \\ 8 \end{pmatrix}} + \underset{b}{\begin{pmatrix} 7 \\ 1 \end{pmatrix}} = \begin{pmatrix} 13 \\ 9 \end{pmatrix}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Three vectors are listed below with some missing values

$$a = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad b = \begin{pmatrix} d \\ e \end{pmatrix} \quad c = \begin{pmatrix} 1 \\ f \end{pmatrix}$$

Use the following calculations to find the value of d, e and f:

$\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} d \\ e \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ $\quad a + b = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$3 + d = 3$ $\quad 2 + e = 0$

$d = 0$ $\quad e = -2$

$2c + b = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$2 \times \begin{pmatrix} 1 \\ f \end{pmatrix} + \begin{pmatrix} 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$2 + 0 = 2$ ✓ (checking)

$2f - 2 = 2$

$2f = 4$

$f = 2$

$d = 0$
$e = -2$
$f = 2$

2. Let $a = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

Write the following as column vectors:

a) **2a** – 3**b**

$2 \times \begin{pmatrix} 1 \\ 6 \end{pmatrix} - 3 \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 12 \end{pmatrix} - \begin{pmatrix} 9 \\ -6 \end{pmatrix} = \begin{pmatrix} -7 \\ 18 \end{pmatrix}$

b) **a** – **b**

$\begin{pmatrix} 1 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$

c) 2**b** – 3**a**

$2 \begin{pmatrix} 3 \\ -2 \end{pmatrix} - 3 \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -4 \end{pmatrix} - \begin{pmatrix} 3 \\ 18 \end{pmatrix} = \begin{pmatrix} 3 \\ -22 \end{pmatrix}$

d) 4**a** + 2**b**

$4 \begin{pmatrix} 1 \\ 6 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 24 \end{pmatrix} + \begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \end{pmatrix}$

Section B

Worked Example

What is vector b if $b = 3a$ and $a = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$?

Step 1: Multiply each row of vector a by the scalar 3.

$$3a = 3\begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3(-2) \\ 3(5) \end{pmatrix}$$

Step 2: Calculate each new value and write the new vector b.

$$3 \times -2 = -6$$
$$3 \times 5 = 15$$

$$b = \begin{pmatrix} -6 \\ 15 \end{pmatrix}$$

Guided Example

Let $a = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $b = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$ and $c = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Write $a - 2b + c$ as a column vector.

Step 1: Calculate $2b$ by multiplying each component of b by 2.

$$2 \times \begin{pmatrix} -2 \\ 7 \end{pmatrix} = \begin{pmatrix} -4 \\ 14 \end{pmatrix}$$

Step 2: Calculate $a - 2b$.

$$\begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -4 \\ 14 \end{pmatrix} = \begin{pmatrix} 7 \\ -13 \end{pmatrix}$$

Step 3: Add the vector **c** to $a - 2b$ to find $a - 2b + c$.

$$\begin{pmatrix} 7 \\ -13 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ -10 \end{pmatrix}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

3. Given the vectors $a = \begin{pmatrix}3\\2\end{pmatrix}$ $b = \begin{pmatrix}1\\5\end{pmatrix}$, draw and label the following vectors from the origin on the axes below:

$\begin{pmatrix}x\\y\end{pmatrix}$ means x to the right, y up
if negative, x left
y down

a) **a**

$\begin{pmatrix}3\\2\end{pmatrix}$ → 3 right 2 up

b) **a** + **b**

$\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}1\\5\end{pmatrix} = \begin{pmatrix}4\\7\end{pmatrix}$ → 4 right 7 up

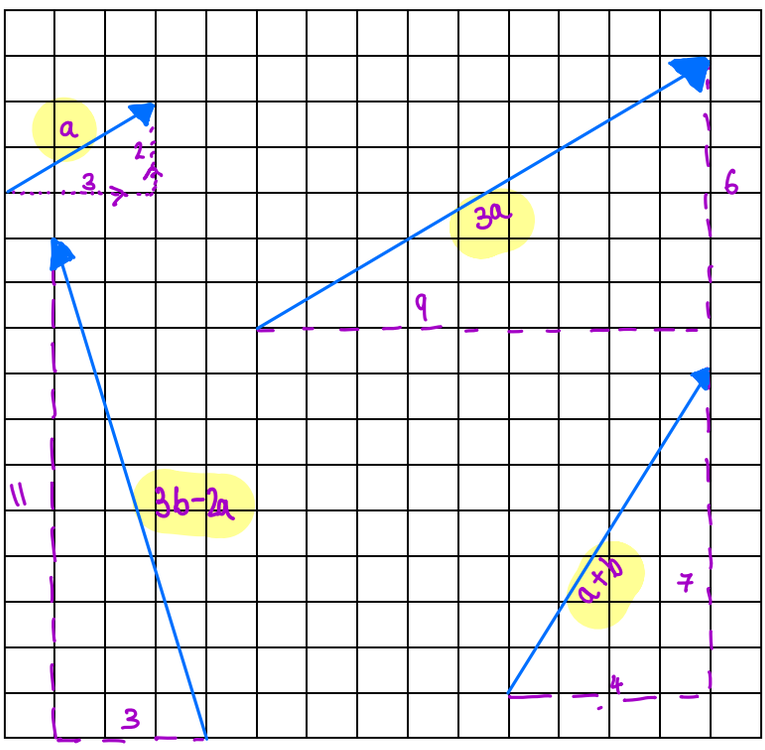
c) 3**a**

$3\begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}9\\6\end{pmatrix}$ → 9 right 6 up

d) 3**b** – 2**a**

$3\begin{pmatrix}1\\5\end{pmatrix} - 2\begin{pmatrix}3\\2\end{pmatrix}$

$= \begin{pmatrix}3\\15\end{pmatrix} - \begin{pmatrix}6\\4\end{pmatrix} = \begin{pmatrix}-3\\11\end{pmatrix}$ → 3 left 11 up



4. Given the vectors $a = \begin{pmatrix}1\\4\end{pmatrix}$ $b = \begin{pmatrix}-3\\6\end{pmatrix}$ $c = \begin{pmatrix}-1\\-2\end{pmatrix}$, write the following expressions as a single column vector:

a) **a** + **b**

$\begin{pmatrix}1\\4\end{pmatrix} + \begin{pmatrix}-3\\6\end{pmatrix} = \begin{pmatrix}-2\\10\end{pmatrix}$

b) 2**c** + **b**

$2\begin{pmatrix}-1\\-2\end{pmatrix} + \begin{pmatrix}-3\\6\end{pmatrix}$

$= \begin{pmatrix}-2\\-4\end{pmatrix} + \begin{pmatrix}-3\\6\end{pmatrix} = \begin{pmatrix}-5\\2\end{pmatrix}$

c) 3**a** – 2**c**

$3\begin{pmatrix}1\\4\end{pmatrix} - 2\begin{pmatrix}-1\\-2\end{pmatrix}$

$= \begin{pmatrix}3\\12\end{pmatrix} - \begin{pmatrix}-2\\-4\end{pmatrix} = \begin{pmatrix}5\\16\end{pmatrix}$

d) **a** + **b** – 1.4**c**

from a

$\begin{pmatrix}-2\\10\end{pmatrix} - 1.4\begin{pmatrix}-1\\-2\end{pmatrix}$

$= \begin{pmatrix}-2\\10\end{pmatrix} - \begin{pmatrix}-1.4\\-2.8\end{pmatrix} = \begin{pmatrix}-0.6\\12.8\end{pmatrix}$

e) 4**a** – **b** + 3**c**

$4\begin{pmatrix}1\\4\end{pmatrix} - \begin{pmatrix}-3\\6\end{pmatrix} + 3\begin{pmatrix}-1\\-2\end{pmatrix}$

$= \begin{pmatrix}4\\16\end{pmatrix} - \begin{pmatrix}-3\\6\end{pmatrix} + \begin{pmatrix}-3\\-6\end{pmatrix} = \begin{pmatrix}4+3-3\\16-6-6\end{pmatrix}$

$= \begin{pmatrix}4\\4\end{pmatrix}$