GCSE Maths – Geometry and Measures

Circle Theorems (Higher only)

Worksheet

NOTES

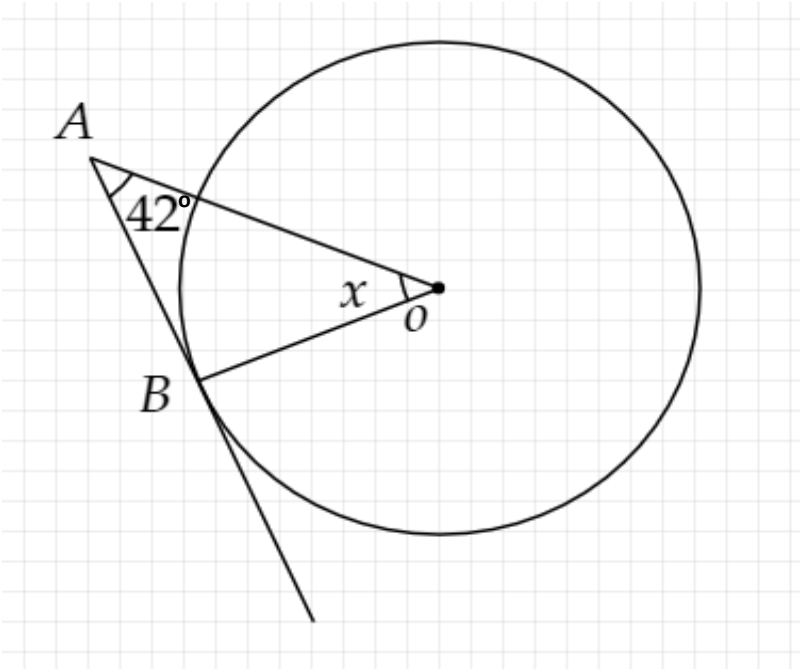
SOLUTIONS

This worksheet will show you how to work with different types of circle theorem questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A - Using Circle Theorems

Worked Example

In the following diagram, find the value of angle x.
Diagram is not drawn to scale.



Step 1: Identify which circle theorem can be used in the given question.

We can use the theorem which says the radius meets the tangent a $90°$ *angle:*

$$OBA = 90°$$

Step 2: Work out the angle being asked for in the question.

Since all the angles in a triangle add up to $180°$*, we have:*

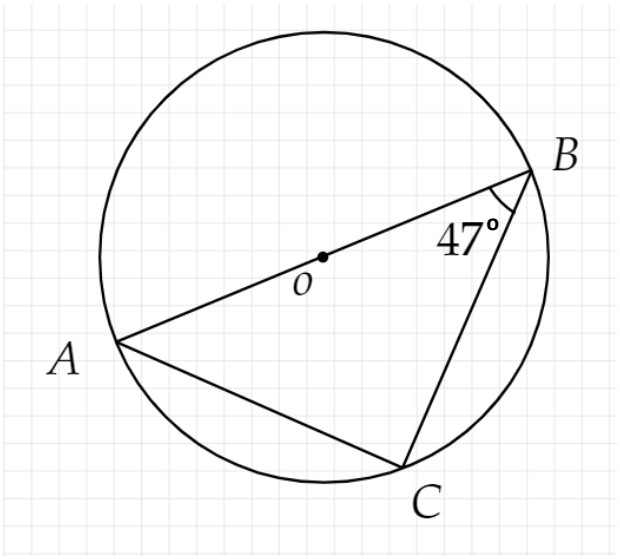
$$42° + 90° + x = 180°$$

$$x = 180° - 90° - 42°$$

$$x = 48°$$

Guided Example

In the following diagram, work out angle BAC.
The diagram is not drawn to scale.

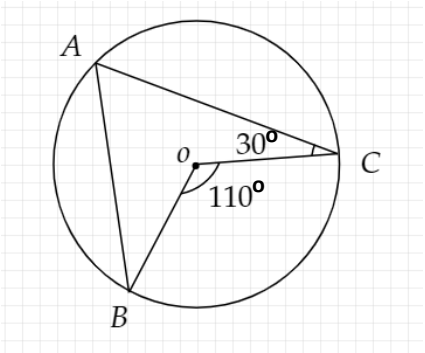


Step 1: Identify which circle theorem can be used in the given question.

Step 2: Work out the angle being asked for in the question.

Worked Example

Find the value of angle ABO. The diagram is not drawn to scale.



Step 1: Identify which circle theorem can be used in the given question.

Using the theorem that the angle at the centre is twice the angle at the circumference:

$$Angle\ BOC = 2 \times Angle\ BAC$$

$$110 = 2 \times Angle\ BAC$$

$$Angle\ BAC = 55$$

Step 2: Using the diagram, work out all the details not directly stated in the question.

$OB = OC$ *because they are radii of the circle.*

Therefore, triangle BOC *is isosceles.*

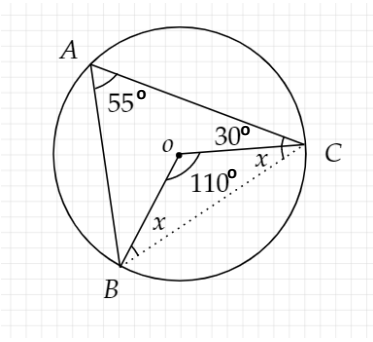
Property of isosceles triangle: base angles are equal. We will label these angles x.

In triangle BOC, *since all the angles in a triangle add up to* $180°$, *we have:*

$$110° + 2x = 180°$$

$$2x = 180° - 110° = 70°$$

$$x = \frac{70°}{2} = 35°$$



Step 3: Work out the angle being asked for in the question.

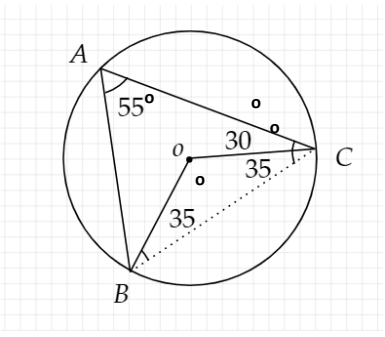
Now looking at triangle ABC:
All angles add to $180°$ *in a triangle, so:*

$$55° + (30° + 35°) + (Angle\ ABO + 35°) = 180°$$

$$155° + Angle\ ABO = 180°$$

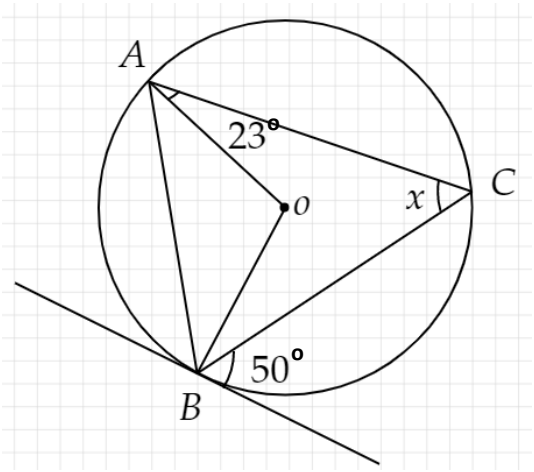
$$Angle\ ABO = 180° - 155° = 25°$$

Hence, $Angle\ ABO = 25°$.



Guided Example

In the following diagram, work out angle x.
The diagram is not drawn to scale.



Step 1: Identify which circle theorem can be used in the given question.

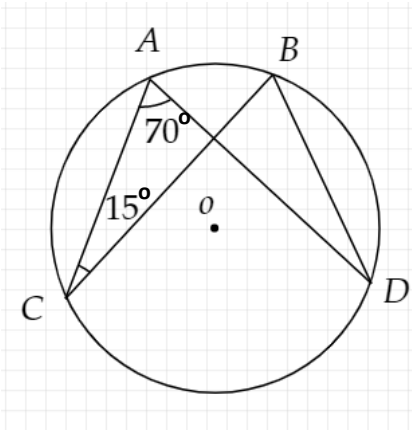
Step 2: Using the diagram, work out all the details not directly stated in the question.

Step 3: Work out the angle being asked for in the question.

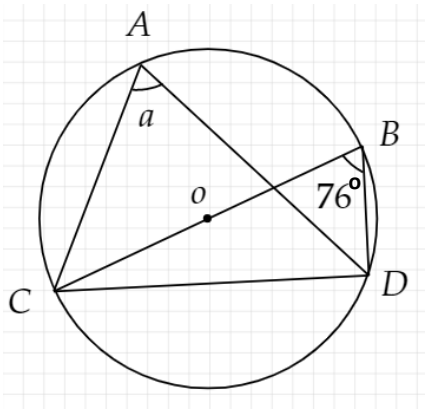
Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Use circle theorems to solve the following questions. Diagrams are not drawn to scale.

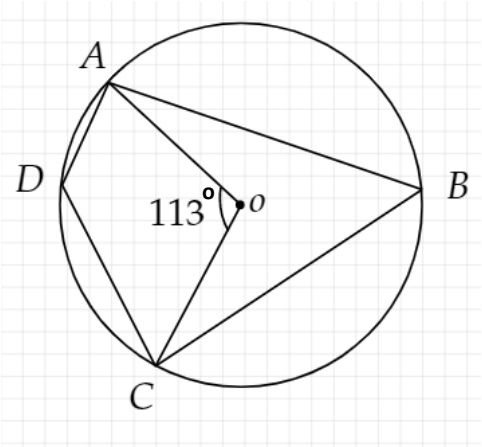
   a) In the following diagram, find the value of angle CBD and angle BDA. Give reasons for your answers.



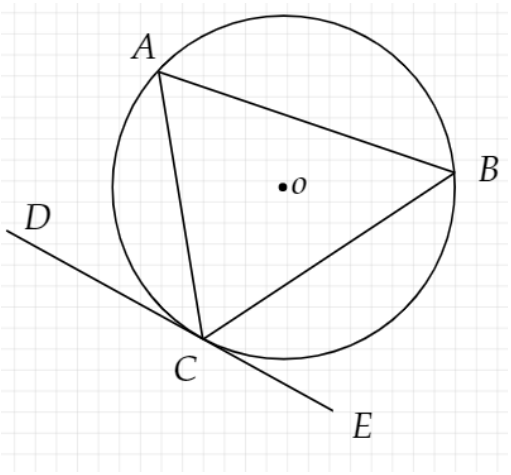
   b) In the following diagram, work out angle BDC and the value of angle a. Give reasons for your answers.



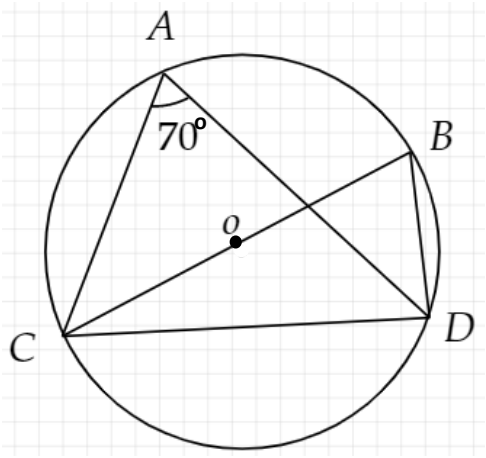
c) In the following diagram, find the value of angle ADC.



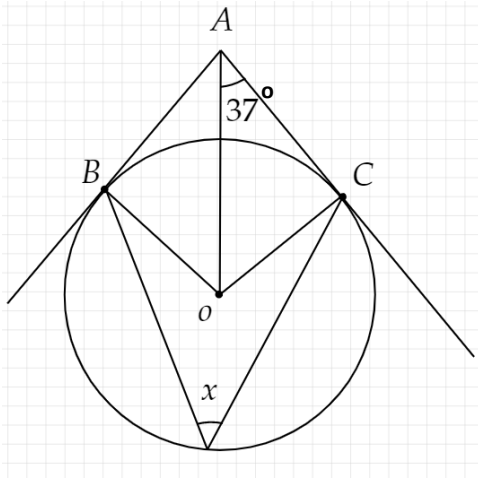
d) Given that angle $ACD = 56°$ and DE is a tangent to the circle. Is it possible to calculate angle BAC?



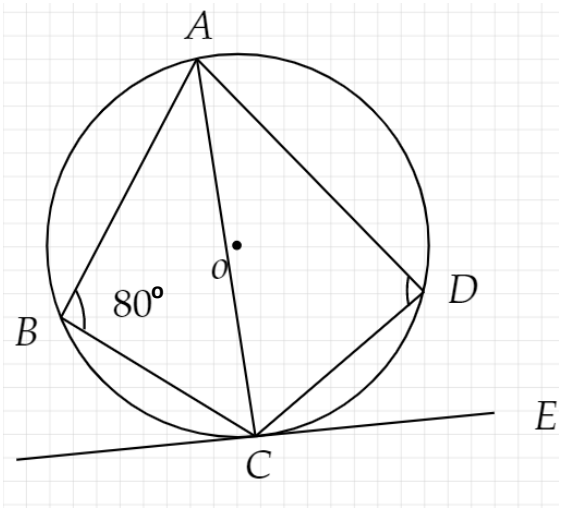
e) In the following diagram, find angle BCD.



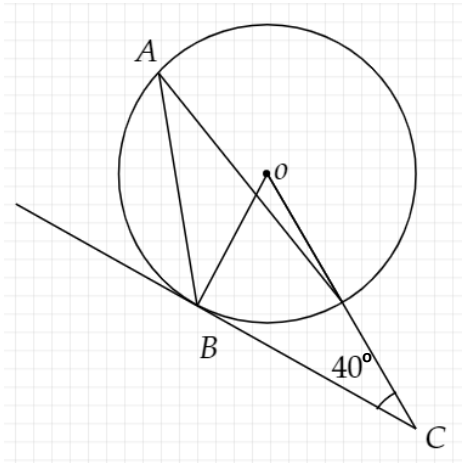
f) In the following diagram, AB and AC are tangents to the circle. Find the value of angle x.



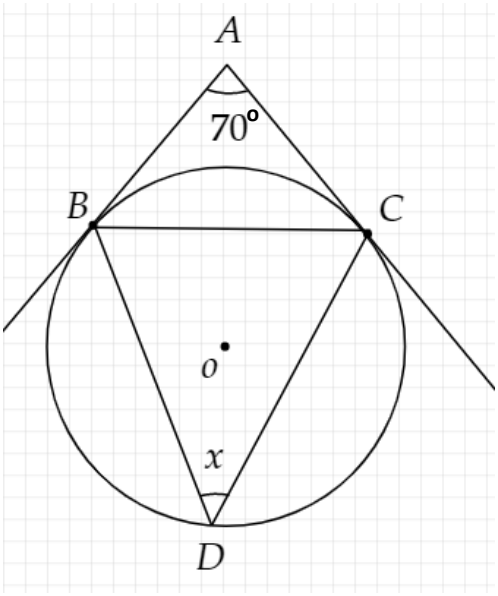
g) If angle $DCE = 44°$, work out the value of angle ACD.



h) In the following diagram, BC is a tangent to the circle. Find the angle at *A*.



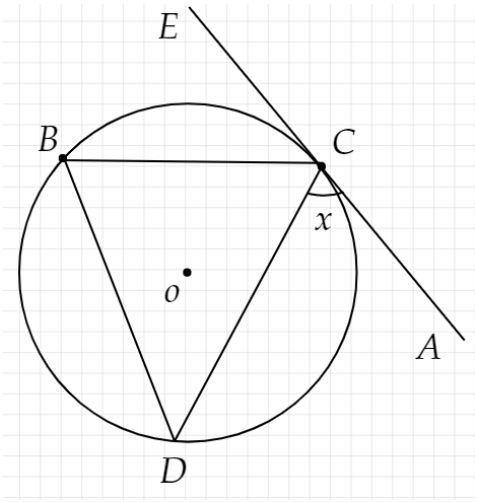
i) In the following diagram, AB and AC are tangents to the circle. Find the value of angle x.



Section B - Proof Questions

Worked Example

In the following diagram, AC is a tangent to the circle and CD is a bisector of angle BCA. Prove that $BD = CD$.



Step 1: Using information provided in the question, find new details not directly stated in the question.

As length CD is a bisector of angle BCA:

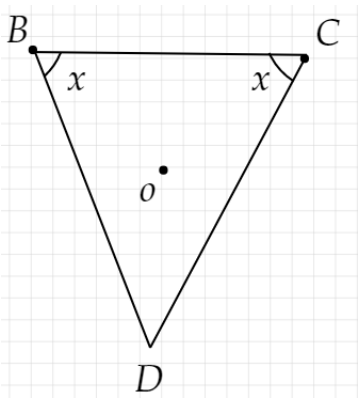
$$Angle\ ACD = Angle\ DCB = x$$

Step 2: Identify the circle theorem that fits the given scenario.

By the alternate segment theorem,

$$Angle\ DBC = x$$

Therefore, the following is the triangle DBC.



Step 3: Using the information found, prove the required statement.

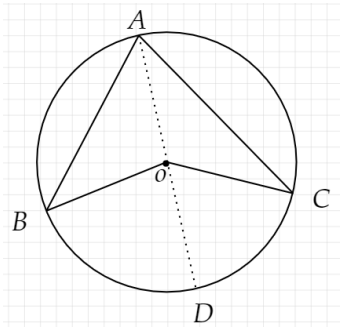
Triangle DBC is an isosceles triangle because the base angles are equal.

Therefore, $BD = CD$ which are the opposite sides.

Hence, we have proved the required statement that $BD = CD$.

Worked Example

Using the following diagram, prove that angle BOC is two times angle BAC.

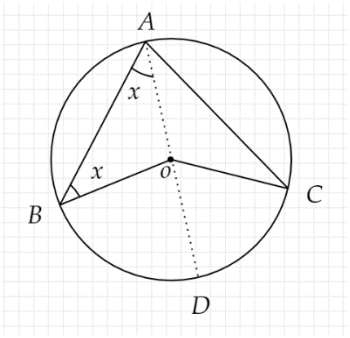


Step 1: Using information provided in the question, find new details not directly stated in the question.

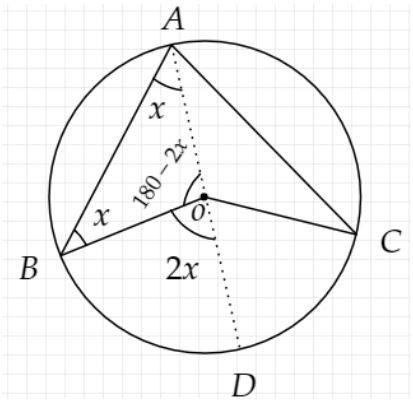
Since OA, OB and OC are all radii, we have:

$$OA = OB = O$$

This means triangle AOB and triangle AOC are isosceles.
Property of isosceles triangles: base angles are equal.

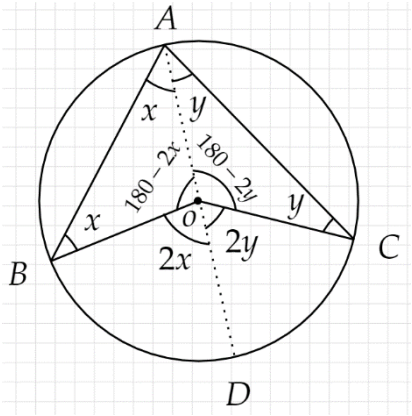


Step 2: With the information found, create a new labelled diagram.



All angles in a triangle and a straight line add to 180°.

Similarly, in triangle AOC:

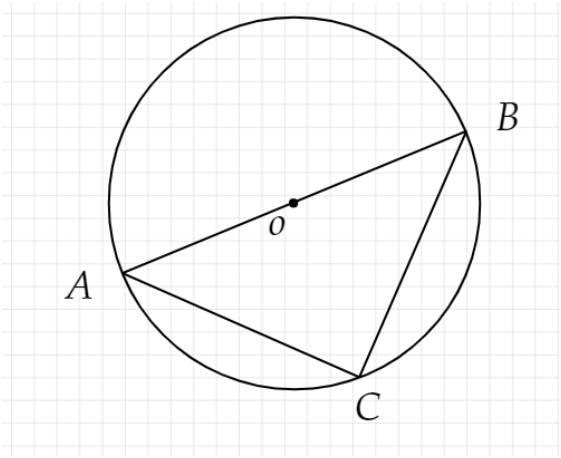


Step 3: Using the information found, prove the required statement.

$Angle\ BOC = 2x + 2y = 2(x + y)$
$Angle\ BAC = x + y$
Hence, we have proved $Angle\ BOC = 2 \times Angle\ BAC$.

Guided Example

Using the following diagram, prove that angle $BCA = 90°$.



Step 1: Using information provided in the question, find new details not directly stated in the question.

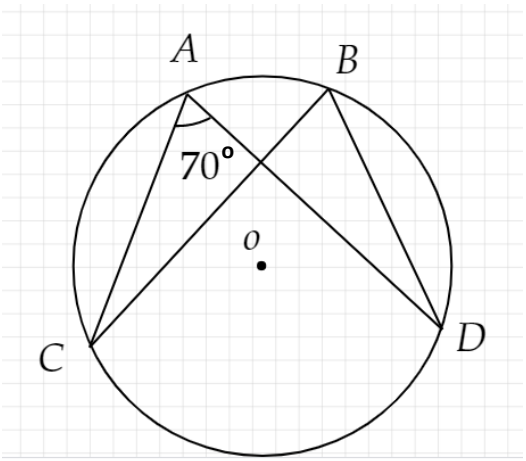
Step 2: Find the circle theorem that fits the scenario.

Step 3: Using the information found, prove the required statement.

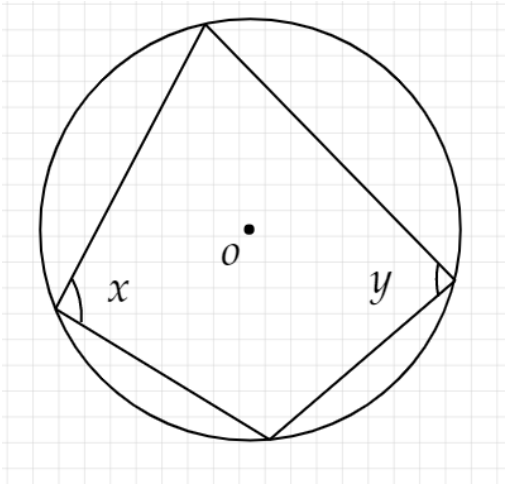
Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. The following are proof questions. Write down reasons for each step. The diagrams are not drawn to scale.

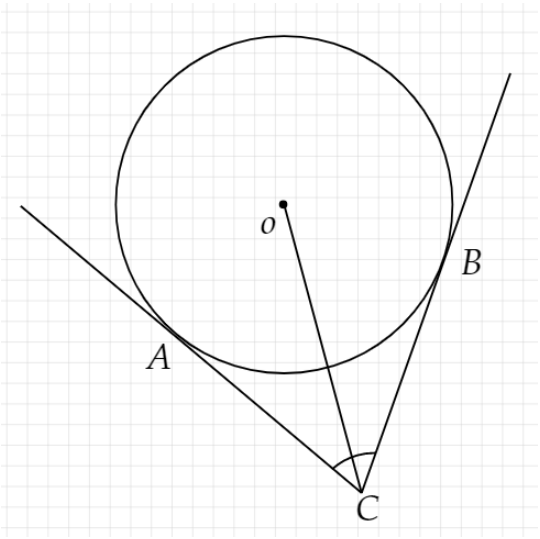
a) Prove that angle $CBD = 70°$.



b) Prove that $x + y = 180°$.



c) If AB and BC are tangents to the circle, prove that angle ACO is equal to angle BCO.



d) Using the following diagram, prove that $angle\ EBC = angle\ BAE$.