GCSE Maths – Geometry and Measures

Circle Theorems (Higher only)

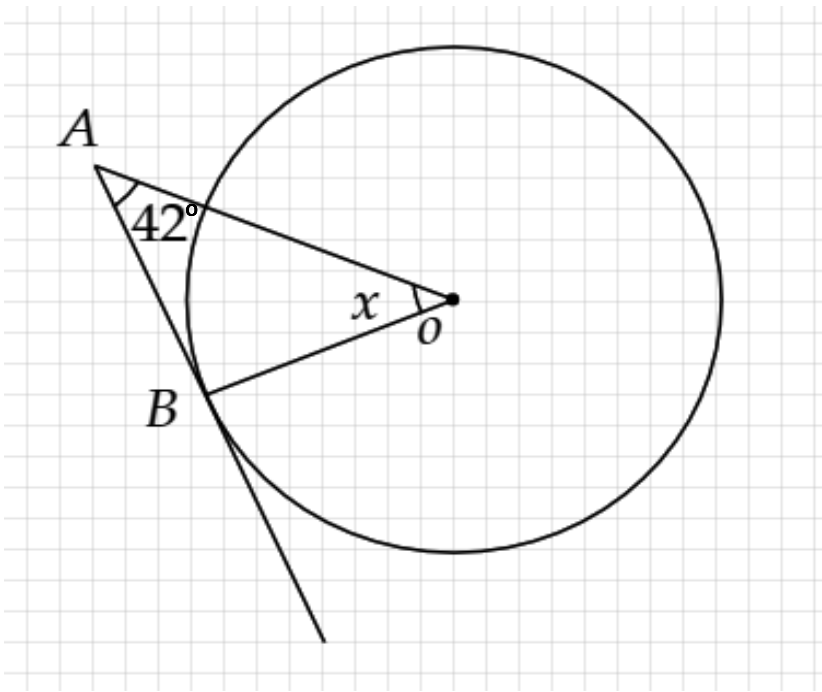
Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work with different types of circle theorem questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Worked Example

In the following diagram, find the value of angle x.
Diagram is not drawn to scale.



Step 1: Identify which circle theorem can be used in the given question.

We can use the theorem which says the radius meets the tangent a $90°$ *angle:*

$$OBA = 90°$$

Step 2: Work out the angle being asked for in the question.

Since all the angles in a triangle add up to $180°$*, we have:*

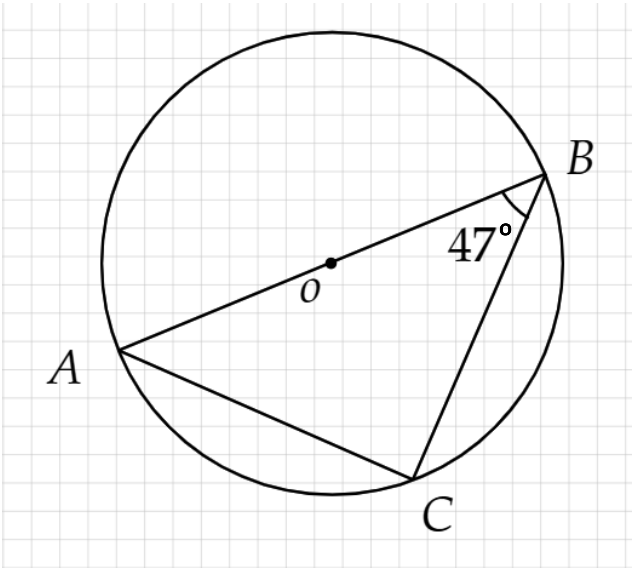
$$42° + 90° + x = 180°$$

$$x = 180° - 90° - 42°$$

$$x = 48°$$

Guided Example

In the following diagram, work out angle BAC.
The diagram is not drawn to scale.



Step 1: Identify which circle theorem can be used in the given question.

Angle in semicircle is 90°

Step 2: Work out the angle being asked for in the question.

$\angle ACB = 90°$

$90 + 47 + \angle BAC = 180$

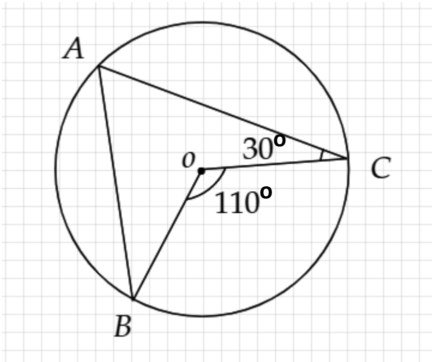
-90

-47

$\angle BAC = 43°$

Worked Example

Find the value of angle ABO. The diagram is not drawn to scale.



Step 1: Identify which circle theorem can be used in the given question.

Using the theorem that the angle at the centre is twice the angle at the circumference:

$$Angle\ BOC = 2 \times Angle\ BAC$$

$$110 = 2 \times Angle\ BAC$$

$$Angle\ BAC = 55$$

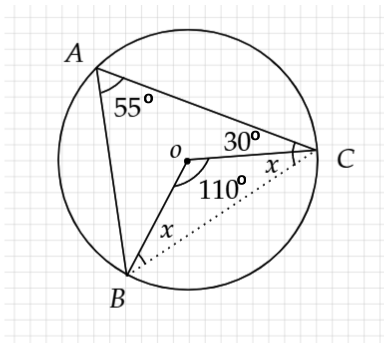
Step 2: Using the diagram, work out all the details not directly stated in the question.

$OB = OC$ because they are radii of the circle.

Therefore, triangle BOC is isosceles.

Property of isosceles triangle: base angles are equal. We will label these angles x.

In triangle BOC, since all the angles in a triangle add up to $180°$, we have:



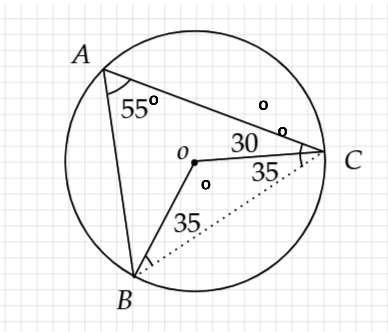
$$110° + 2x = 180°$$

$$2x = 180° - 110° = 70°$$

$$x = \frac{70°}{2} = 35°$$

Step 3: Work out the angle being asked for in the question.

Now looking at triangle ABC:
All angles add to $180°$ in a triangle, so:



$55° + (30° + 35°) + (Angle\ ABO + 35°) = 180°$

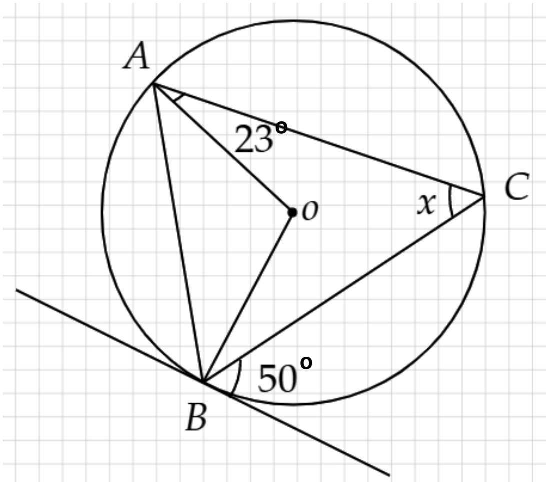
$155° + Angle\ ABO = 180°$

$Angle\ ABO = 180° - 155° = 25°$

Hence, $Angle\ ABO = 25°$.

Guided Example

In the following diagram, work out angle x.
The diagram is not drawn to scale.



Step 1: Identify which circle theorem can be used in the given question.

- Angle at the centre is twice the angle at the circumference
- Alternate segment Theorem

Step 2: Using the diagram, work out all the details not directly stated in the question.

$\angle BAC = 50°$ Alternate Segment

$\angle BAO = 50 - 23 = 27°$

$\angle BAO = \angle ABO$ Isosceles Triangles (same radius)

$\angle AOB = 180 - 27 - 27$
$= 126°$

Step 3: Work out the angle being asked for in the question.

$\angle x = 126 \div 2$ (Angle is twice at origin than circumference).

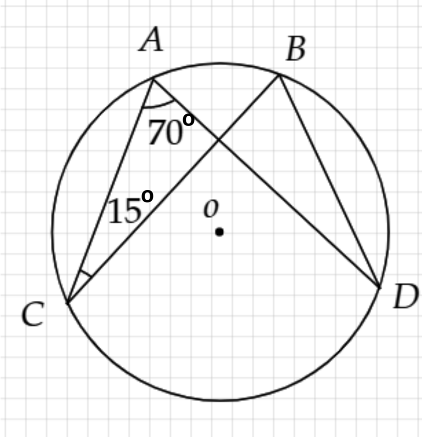
$x = 63°$

Now it’s your turn!

If you get stuck, look back at the worked and guided examples.

1. Use circle theorems to solve the following questions. Diagrams are not drawn to scale.

   a) In the following diagram, find the value of angle CBD and angle BDA. Give reasons for your answers.

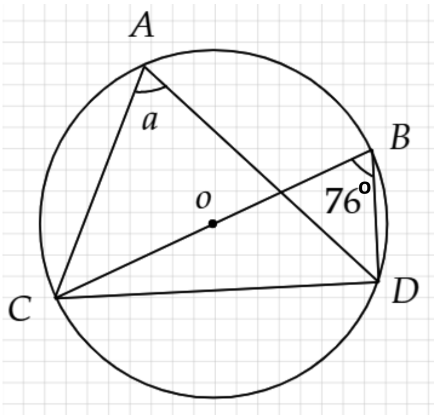


Angles in the same segment are equal:

$CBD = 70°$

$BDA = 15°$

   b) In the following diagram, work out angle BDC and the value of angle a. Give reasons for your answers.



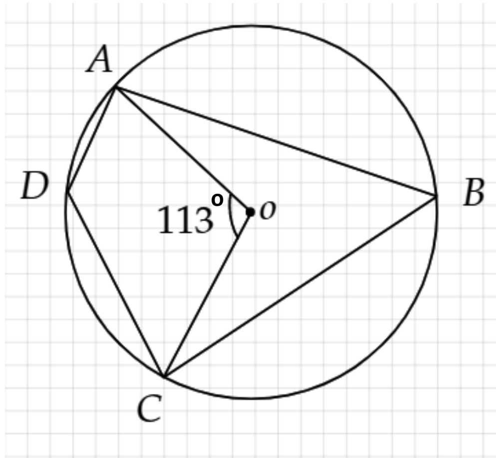
Angles in the same segment are equal:

$\angle a = 76°$

Angle in semicircle is 90°.

$\angle BDC = 90°$

c) In the following diagram, find the value of angle ADC.



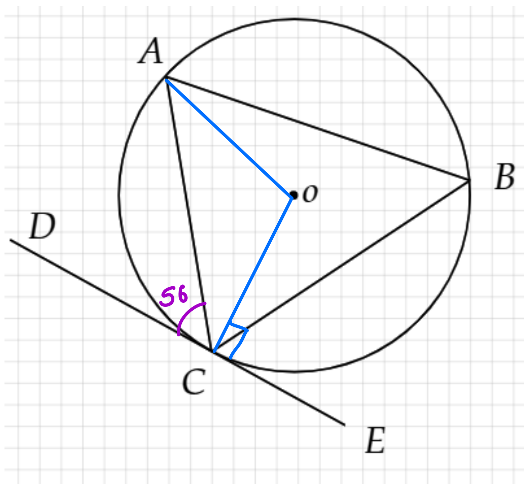
• Angle at the centre is twice the angle at the circumference

$\angle ABC = 113 \div 2$
$= 56.5°$

Opposite angles in cyclic quad add to 180°

$\angle ADC = 180 - 56.5$
$= 123.5°$

d) Given that angle $ACD = 56°$ and DE is a tangent to the circle. Is it possible to calculate angle BAC?



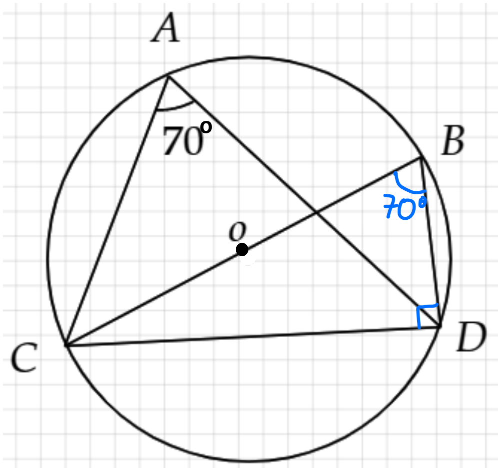
Alternate segment Theorem
$\angle ABC = 56°$

Tangents meet radius at 90°

$\angle OCA = 90 - 56$
$= 34°$

No, it is not possible to calculate $\angle BAC$

e) In the following diagram, find angle BCD.



Angles in the same segment are equal:

$\angle CBD = 70°$

Angle in semicircle is $90°$.

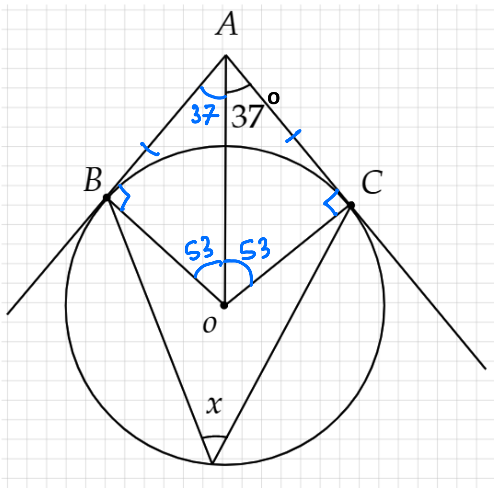
$\angle BDC = 90°$

$90 + 70 + \angle BCD = 180$

$160 + \angle BCD = 180$ $\quad -160$

$\angle BCD = 20°$

f) In the following diagram, AB and AC are tangents to the circle. Find the value of angle x. By RHS, $\triangle AOB$ and $\triangle AOC$ are similar



Tangents meet radius at $90°$

For both $\triangle AOB$ and $\triangle AOC$:

$180 - 90 - 37 = 53°$

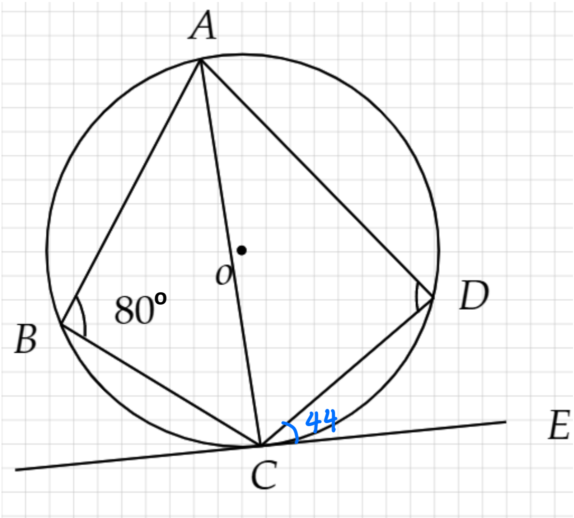
$\angle BOC = 53 \times 2$

Angle at the centre is twice the angle at the circumference

$\angle x = \frac{53 \times \cancel{2}}{\cancel{2}}$

$= 53°$

g) If angle $DCE = 44°$, work out the value of angle ACD.



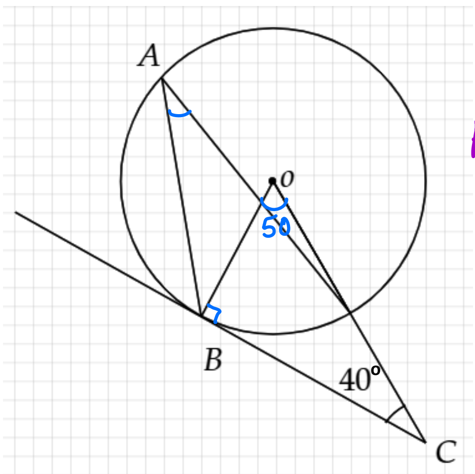
Alternate segment Theorem

$\angle ABC = \angle ACE$

$\angle ACD = 80 - 44$

$= 36°$

h) In the following diagram, BC is a tangent to the circle. Find the angle at A.



Tangent meet radius at 90°.

$\angle BOC = 180 - 90 - 40 = 50°$

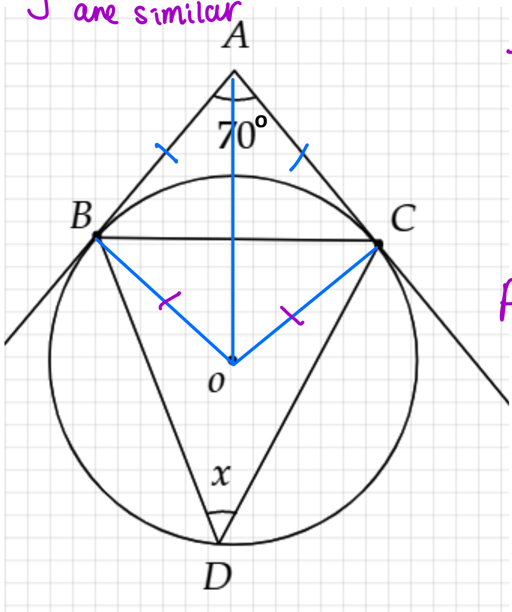
Angle at the centre is twice the angle at the circumference

$\angle A = 50 \div 2$

$= 25°$

i) In the following diagram, AB and AC are tangents to the circle. Find the value of angle x. (similar to f).

By RHS, $\triangle AOB$ and $\triangle AOC$ are similar



AO bisects $\angle BAC$ so $\angle BAO$ and $\angle CAO = 35°$

Tangents meet radius at 90°

For both $\triangle AOB$ and $\triangle AOC$:

$180 - 90 - 35 = 55°$

$\angle BOC = 55 \times 2$

Angle at the centre is twice the angle at the circumference

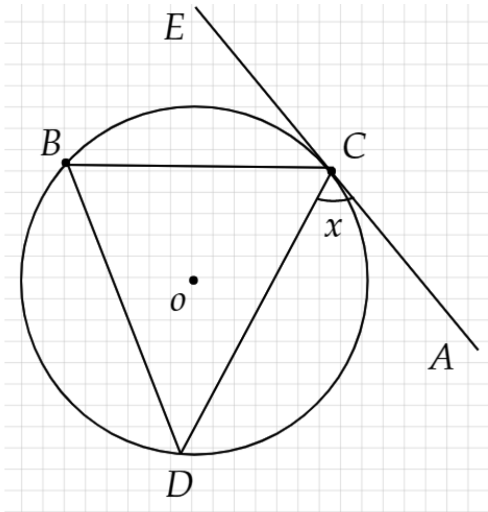
$\angle x = \frac{55 \times \cancel{2}}{\cancel{2}}$

$= 55°$

Section B - Proof Questions

Worked Example

In the following diagram, AC is a tangent to the circle and CD is a bisector of angle BCA. Prove that $BD = CD$.



Step 1: Using information provided in the question, find new details not directly stated in the question.

As length CD is a bisector of angle BCA:

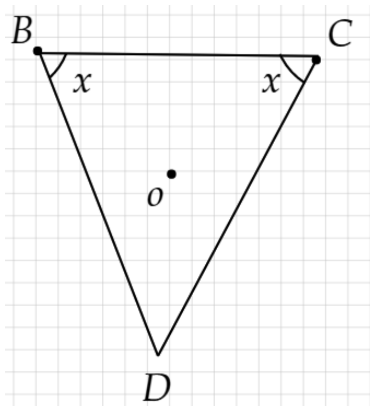
$$Angle\ ACD = Angle\ DCB = x$$

Step 2: Identify the circle theorem that fits the given scenario.

By the alternate segment theorem,

$$Angle\ DBC = x$$

Therefore, the following is the triangle DBC.



Step 3: Using the information found, prove the required statement.

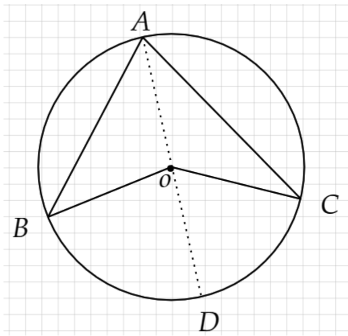
Triangle DBC is an isosceles triangle because the base angles are equal.

Therefore, $BD = CD$ which are the opposite sides.

Hence, we have proved the required statement that $BD = CD$.

Worked Example

Using the following diagram, prove that angle BOC is two times angle BAC.

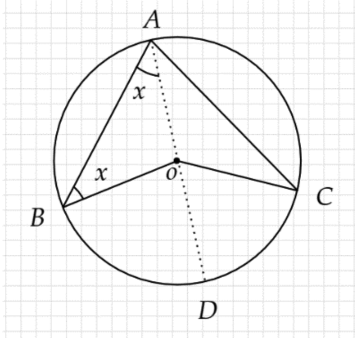


Step 1: Using information provided in the question, find new details not directly stated in the question.

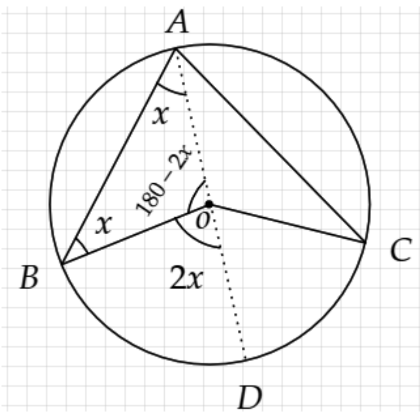
Since OA, OB and OC are all radii, we have:

$$OA = OB = O$$

This means triangle AOB and triangle AOC are isosceles.
Property of isosceles triangles: base angles are equal.

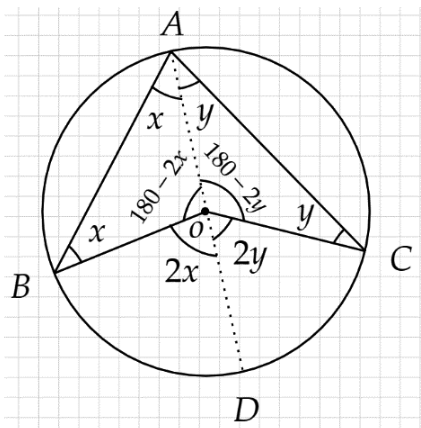


Step 2: With the information found, create a new labelled diagram.



All angles in a triangle and a straight line add to $180°$.

Similarly, in triangle AOC:

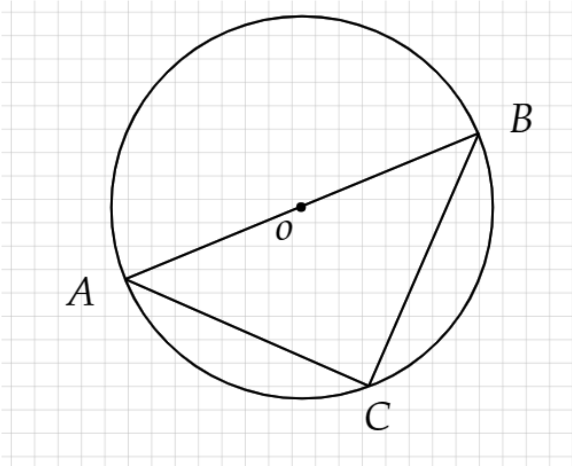


Step 3: Using the information found, prove the required statement.

$Angle\ BOC = 2x + 2y = 2(x + y)$
$Angle\ BAC = x + y$
Hence, we have proved $Angle\ BOC = 2 \times Angle\ BAC$.

Guided Example

Using the following diagram, prove that angle $BCA = 90°$.



Step 1: Using information provided in the question, find new details not directly stated in the question.

AOB is a straight line.

Therefore $\angle AOB = 180°$

Step 2: Find the circle theorem that fits the scenario.

Angle at the centre is twice the angle at the circumference

Step 3: Using the information found, prove the required statement.

$180 \div 2 = 90°$

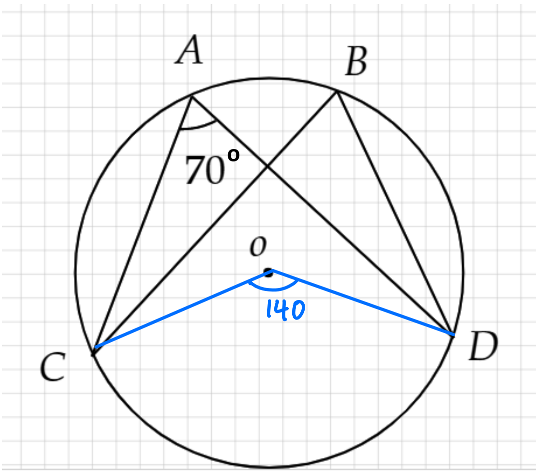
Therefore, $\angle BCA = 90°$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

2. The following are proof questions. Write down reasons for each step. The diagrams are not drawn to scale.

a) Prove that angle $CBD = 70°$.



By joining OC and OD we can use the Angle at the centre is twice the angle at the circumference theorem.

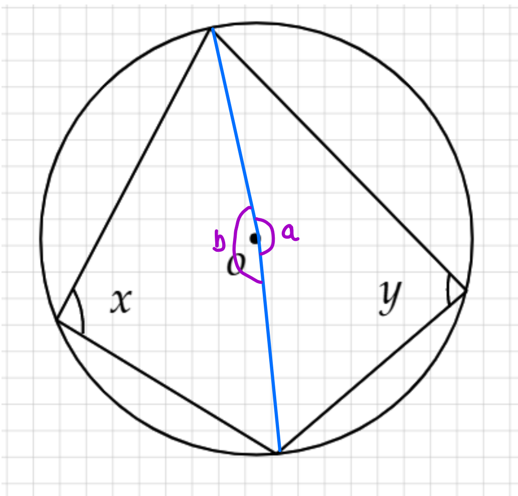
$\angle COD = 70 \times 2 = 140°$ (from theorem)

Also,

$\angle CBD = 140 \div 2 = 70°$ (from same theorem)

$\therefore \angle CBD = 70°$

b) Prove that $x + y = 180°$.



① Split the cyclic quadrilateral.

② Use the theorem:
Angle at the centre is twice the angle at the circumference

From this theorm:

$\frac{a}{2} = x$ and $\frac{b}{2} = y$

③ Angles around a point add to 360

$a + b = 360°$

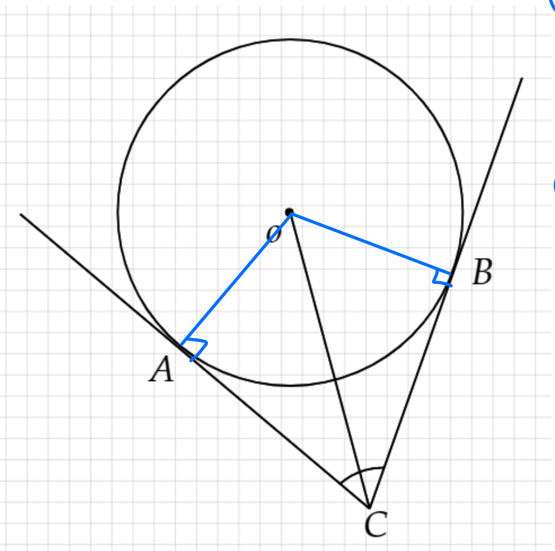
④ Divide equation by 2

$\frac{a}{2} + \frac{b}{2} = 180$

⑤ Substitute in ②

$\therefore x + y = 180$

c) If AB and BC are tangents to the circle, prove that angle ACO is equal to angle BCO.

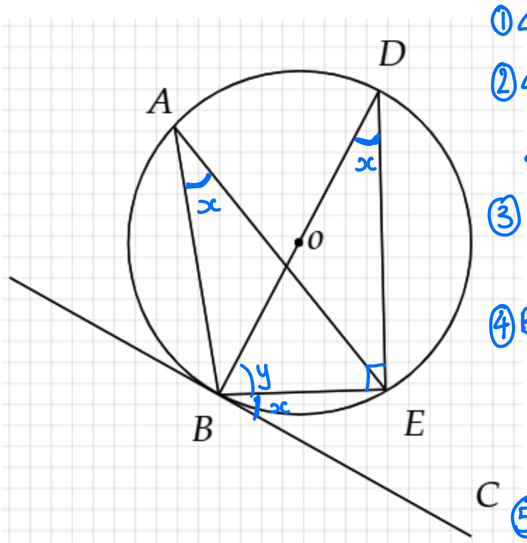


① Use theorem
Tangent meet radius at 90°
Therefore, $\angle CAO$ and $\angle CBO = 90°$

② $\triangle OAC$ and $\triangle OBC$ are similar by RHS
R: right angled
H: share hypotenuse OC
S: $OA = OB$ (both radii)

③ As the triangles are similar,
$\angle ACO = \angle BCO$.

d) Using the following diagram, prove that $angle\ EBC = angle\ BAE$.



① $\angle OBC = 90°$ (Tangent meet radii at 90°)
② $\angle BED = 90°$ (Angle in semicircle = 90°)
$\angle EBC = x$ and $\angle EBD = y$
③ By ①, $x + y = 90°$
$x = 90 - y$
④ By ②, $\angle EDB + y + 90 = 180$
$\angle EDB = 90 - y$
By ④, $\angle EDB = x$
⑤ Angle is same segment are equal
$\angle EDB = \angle BAE = x$

⑥ $\therefore \angle EBC = \angle BAE$ (both equal x)
$\angle EBC = \angle EDB = \angle BAE$.