GCSE Maths - Geometry and Measures

Properties of Angles
Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of nth term questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.
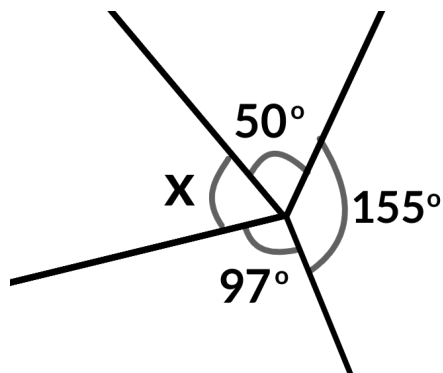
Section A

Worked Example

Find the value of angle x in the diagram below:

Step 1: Sum the angles around the point and equate the sum to 360°.

$$x + 50° + 155° + 97° = 360°$$

Step 2: Solve the equation to find x.

$$x = 360° - 50° - 155° - 97°$$

$$x = 58°$$

Guided Example

Find the value of angle x in the diagram below:
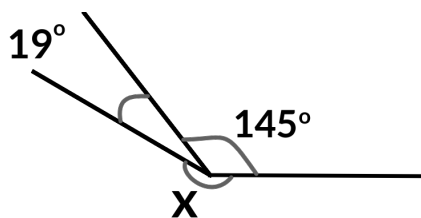
Step 1: Sum the angles around the point and equate the sum to 360°.

$$x + 19 + 145 = 360$$

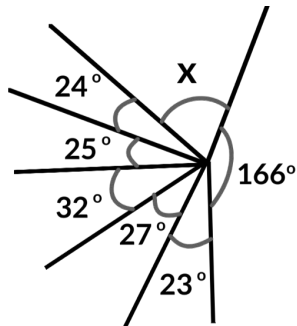**Step 2:** Solve the equation to find x.

$$x = 196°$$

Now it's your turn!

If you get stuck, look back at the worked and guided examples for this section.

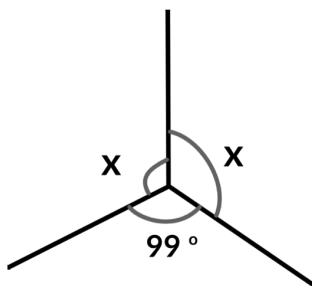1. Find angle x in each of the following diagrams:

a)

24° x
25°
32°   166°
27°
23°

Sum the angles and equate to 360°

$x + 166 + 23 + 27 + 32 + 25 + 24 = 360$

$x + 297 = 360$ $\Big) -297$

$x = 63°$

b)

x x
99°

Sum the angles and equate to 360°

$x + x + 99 = 360$

$2x + 99 = 360$ $\Big) -99$

$2x = 261$ $\Big) \div 2$

$x = 130.5°$

c)

x
51°
x 172°
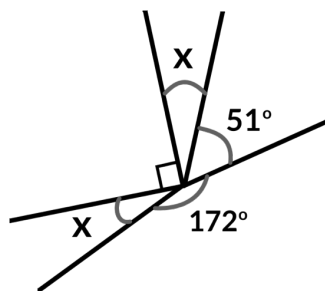
Sum the angles and equate to 360°

$x + 51 + 172 + x + 90 = 360$

$2x + 313 = 360$ $\Big) -313$

$2x = 47$ $\Big) \div 2$

$x = 23.5°$

d)

$90° - x$

x x
187°
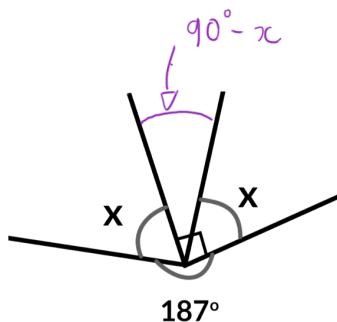
Sum the angles and equate to 360°

$x + 187 + x + 90 - x = 360$ $\Big)$ Simplify

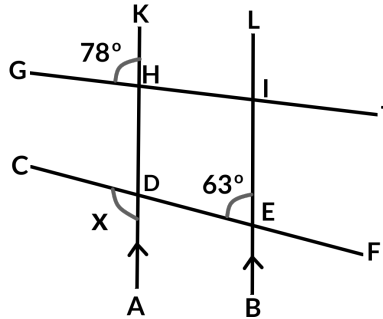$x + 187 + 90 = 360$

$x + 277 = 360$ $\Big) -277$

$x = 83°$

Section B

Worked Example

Find the value of angle x in the diagram below:

Step 1: Find relevant angles using alternating and corresponding angles.

Angle CDH = Angle DEI since they are corresponding angles.
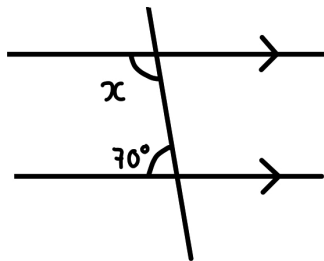So,
$$Angle\ CDH = 63°.$$

Step 2: Find angle x.

Here, find x by using the idea that angles on a straight line add up to 180°*:*

$$x + 63^o = 180^o$$
$$x = 180^o - 63^o = 117^o$$

Guided Example

Find the value of angle x in the diagram below:

Step 1: Find relevant angles using alternating and corresponding angles.

$x + 70 = 180$ As interior angles add up to 180°
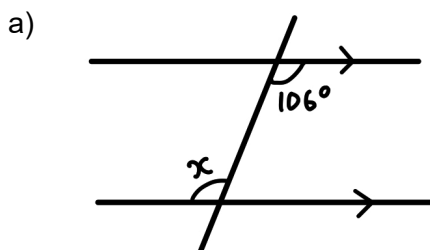
Step 2: Find angle x.
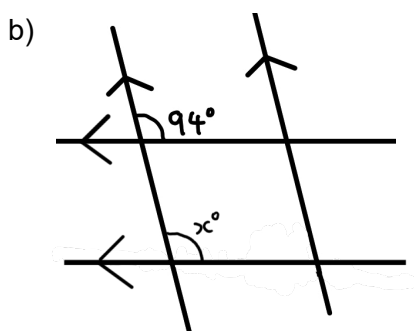
$x = 180 - 70$

$x = 110°$

Now it's your turn!
If you get stuck, look back at the worked and guided examples for this section.

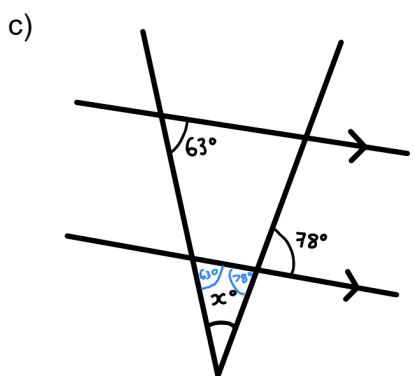2. Find angle x in each of the following diagrams:

a)

$x = $ **106°**

As alternate angles are the same
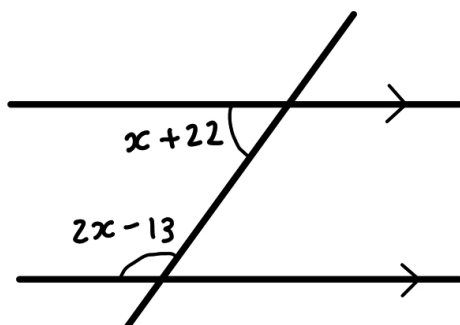
b)

$x = $ **94°**

As corresponding angles are the same

c)

Opposite angles are equal $\rightarrow 78°$

Corresponding angles are equal $\rightarrow 63°$

$x + 63 + 78 = 180 \rightarrow$ Angles in a triangle add up to $180°$.

$x = $ **39°**

d)   Interior angles add up to $180°$

$x + 22 + 2x - 13 = 180$

$3x + 9 = 180$ ⎫ -9

$3x = 171$ ⎭

$x = $ **57°** ⎫ $\div 3$

$x + 22$

$2x - 13$

Section C

Worked Example

Calculate the sum of interior angles in a decagon.

Step 1: Count out how many triangles the polygon can be split into.

A decagon is a 10-sided polygon.

As can be seen in the diagram, the decagon can be split into 8 triangles.

Step 2: Multiply the number of triangles by 180° to obtain the sum of the interior angles.

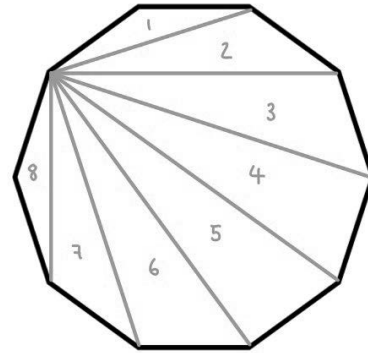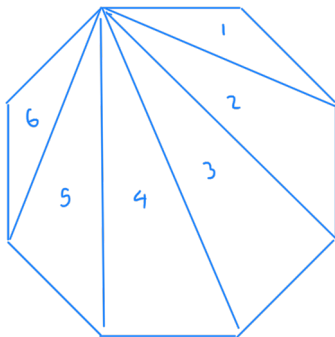$$8 \times 180° = 1440°$$

Guided Example

Calculate the sum of interior angles in an octagon.

Step 1: Count out how many triangles the polygon can be split into.

An octagon has 8 sides.
The octagon can be split into 6 triangles.

Step 2: Multiply the number of triangles by 180° to obtain the sum of the interior angles.

$$6 \times 180° = 1080°$$

(Formula: $180(n-2) = $ Sum of int. angles, $n = $ no. of sides)

3. Calculate the sum of the interior angles in a polygon which has 23 sides.

Use Formula: $180(n-2)$

$= 180(23-2)$

$= 180(21) = 3780°$

4. Calculate the sum of the interior angles in a polygon which has 14 sides.

Use Formula: $180(n-2)$

$= 180(14-2)$

$= 180(12) = 2160°$

5. The sum of the interior angles of a polygon is 12240°. How many sides does the polygon have?

Use Formula in reverse: $180(n-2) = 12240$

$180n - 360 = 12240$

$180n = 12600$

$n = 70$ sides

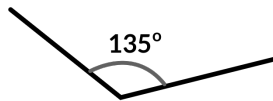6. The sum of the interior angles of a polygon is 89640°. How many sides does the polygon have?

Use Formula in reverse: $180(n-2) = 89640$

$180n - 360 = 89640$

$180n = 9000$

$n = 500$ sides

Section D

Worked Example

The diagram shows part of a regular polygon. Calculate how many sides the polygon has.

135°

Step 1: Work out the value of the exterior angle by extending the side of the polygon and using the fact that angles on a line add up to 180°.

135°

45°

$$180^o - 135^o = 45^o$$

Step 2: Use the fact that the exterior angles add up to 360° to find the number of sides, n:

$$45^o \times n = 360^o$$
$$n = \frac{360°}{45°}$$
$$n = 8$$

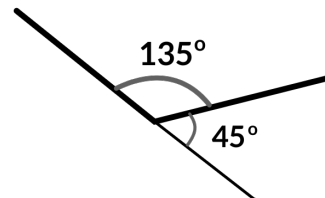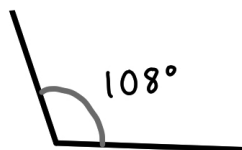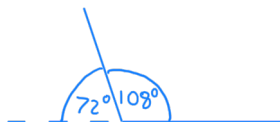*So, the polygon has 8 sides and is therefore an octagon.*

Guided Example

The diagram shows part of a regular polygon. Calculate how many sides the polygon has.

108°

Step 1: Work out the value of the exterior angle by extending the side of the polygon and using the fact that angles on a line add up to 180°.

$$180 - 108 = 72°$$

72° 108°

Step 2: Use the fact that the exterior angles add up to 360° to find the number of sides, n:

$$72° \times n = 360°$$
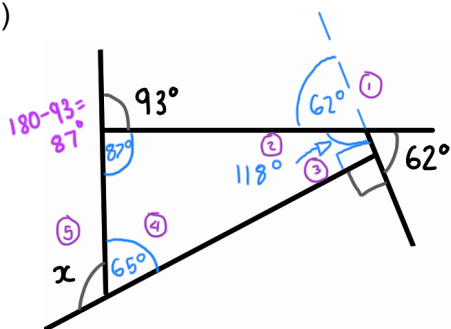$$n = \frac{360°}{72°}$$
$$n = 5 \text{ sides}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples for this section.

7. In the following diagrams, find angle x:

① Extending one of the lines, and opposite angles are equal → 62°

a)

180-93= 87°

93°

87°

118°

⑤

x

65°

62° ①

② ③

62°

② Angles on a straight line add up to 180° → $u + 62 = 180$ so $u = 118°$

③ Angles on a straight line add up to 180° → $y + 90 = 180$ so $y = 90°$

④ Angles in a quadrilateral add up to 360° → $z + 118 + 90 + 87 = 360°$ so $z = 65°$

⑤ Angles on a straight line add up to 180° → $x + 65 = 180$ so $x = \boxed{115°}$

b)

① 111°

69°

2x

② z= 133-x

$x + 47$
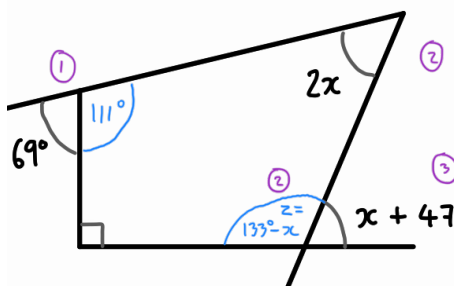
① Angles on a straight line add up to 180° → $y + 69 = 180$ so $y = 111$

② Angles on a straight line add up to 180° → $z + x + 47 = 180$ so $z = 133° - x$

③ Angles in a quadrilateral add up to 360° → $2x + 133 - x + 90 + 111 = 360$

so $x + 334 = 360$

$x = \boxed{26°}$

c)

① 32°

50° ②

98°

98° ③

⑤

x

148°

④ 140°

40°
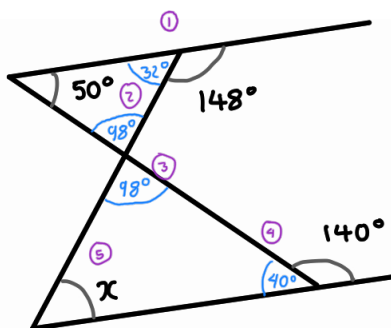
① Angles on a straight line add up to 180° → $y + 148 = 180$ so $y = 32°$

② Angles in a triangle add up to 180° → $z + 50 + 32 = 180$ so $z = 98°$

③ Opposite angles are equal so $w = 98°$

④ Angles on a straight line add up to 180° → $u + 140 = 180$ so $u = 40°$

⑤ Angles in a triangle add up to 180° → $x + 98 + 40 = 180$ so $x = \boxed{42°}$

8. The size of each interior angle of a regular polygon is 156°. Work out the number of sides of the polygon.

$180 - 156 = 24°$

$24° \times n = 360$

$n = \dfrac{360°}{24°} = \boxed{15 \text{ sides}}$

156°