GCSE Maths - Geometry and Measures

Trigonometric Ratios and Exact Trig Values

Worksheet

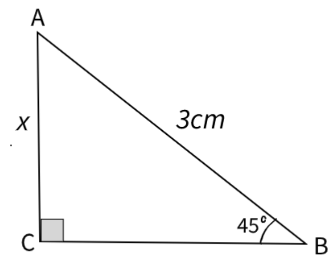
WORKED SOLUTIONS

This worksheet will show you how to work out different types of trigonometry-based questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

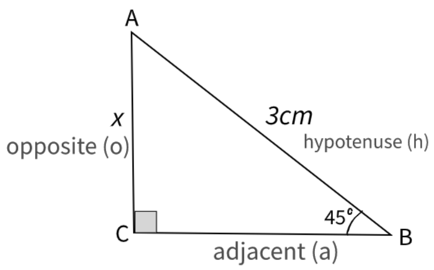
Worked Example

Work out the length of side x in the right-angled triangle ABC, to 3 significant figures.



Step 1: Label the sides of the triangle according to the known angle.

We label the sides according to the $45°$ *angle, not the* $90°$ *angle.*



Step 2: Decide which trigonometric ratio to use depending on which sides we know.

We know the length of the hypotenuse and we are looking for the opposite side. So, we use the sine ratio:

$$\sin\theta = \frac{opposite}{hypotenuse}$$

Step 3: Substitute the known values into the correct trigonometric ratio, and solve the equation.

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\sin 45 = \frac{x}{3}$$

$$x = 3 \times \sin 45$$

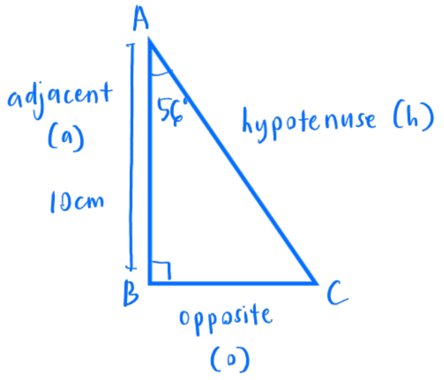
$$x = 2.12 \text{ cm}$$

The length of side x is **2.12 cm.**

Guided Example

Triangle ABC is a right-angled triangle.
Side AB is 10 cm, angle ABC is 90° and angle BAC is 56°.
Find the length of side BC to 3 significant figures.

Step 1: Use the information to construct triangle ABC and label the sides according to the angles we know.



Step 2: Decide which trigonometric ratio to use depending on the side we know and the side we are trying to find. Use the phrase SOH CAH TOA to help choose the right one.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

since we were given adjacent and we need to find the opposite length, TOA is the best option to use.

Step 3: Substitute the side and angle we know into the ratio and solve the equation. Round the final answer to 3 significant figures.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

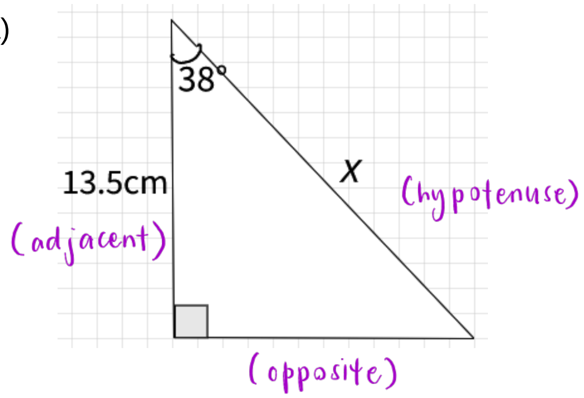
$$\tan 56^\circ = \frac{x}{10}$$

$$\begin{aligned} x &= 10 \times \tan 56^\circ \\ &= 10 \times 1.4825\ldots \\ &= 14.825\ldots \\ &\approx 14.8 \text{ cm} \quad (3 \text{ s.f.}) \end{aligned}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Find side x:

a)



$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\cos 38° = \dfrac{13.5}{x}$

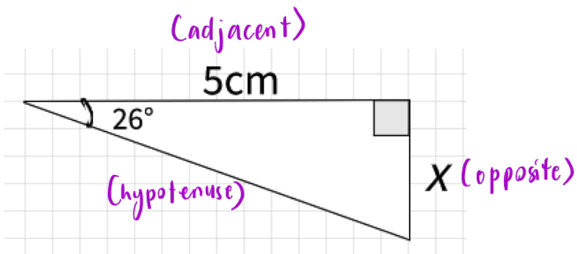
$x = \dfrac{13.5}{\cos 38°}$

$= \dfrac{13.5}{0.788...}$

$= 17.13...$

$= 17.1\text{ cm}$ (3 s.f.)

b)



$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

$\tan 26° = \dfrac{x}{5}$

$x = 5 \times \tan 26°$

$= 5 \times 0.4877...$

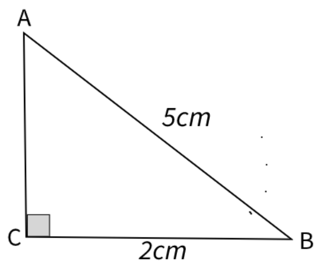
$= 2.4386...$

$= 2.44\text{ cm}$ (3 s.f.)

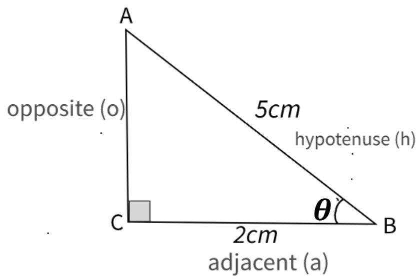
Section B

Worked Example

Find the size of angle ABC in the triangle below, to the nearest degree.



Step 1: Label the sides of the triangle according to the unknown angle.



Step 2: Decide which trigonometric ratio to use depending on which sides we know.

We know the lengths of the adjacent side and hypotenuse, so we know to use the cosine ratio:

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

Step 3: Substitute our known values into the cosine ratio.

$$cos\ \theta\ =\ \frac{adjacent}{hypotenuse}$$

$$cos\ \theta\ =\ \frac{2}{5}$$

Step 4: In this example, we are finding an unknown angle, so we are required to use the inverse function, for which there is a button on the calculator.

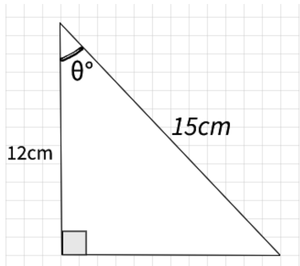
$$cos\ \theta\ =\ \frac{2}{5}$$

$$\theta\ = cos^{-1}\left(\frac{2}{5}\right) = 66.4218\ldots°$$

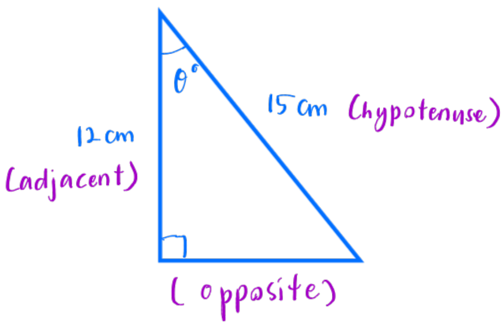
$\theta\ =\ \mathbf{66°}$ (to the nearest degree)

Guided Example

Find the size of angle θ in the triangle to the right, to 1 decimal place.



Step 1: Label the sides of the triangle according to the unknown angle.



Step 2: Based on the sides we know, choose the correct trigonometric ratio, and substitute the known values in to form an equation.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{12}{15}$$

Step 3: Solve the equation using the inverse trigonometric function on the calculator. Round the final answer to 1 decimal place.

$$\cos\theta = \frac{12}{15}$$

$$\theta = \cos^{-1}\left(\frac{12}{15}\right)$$

$$= 36.869\ldots$$

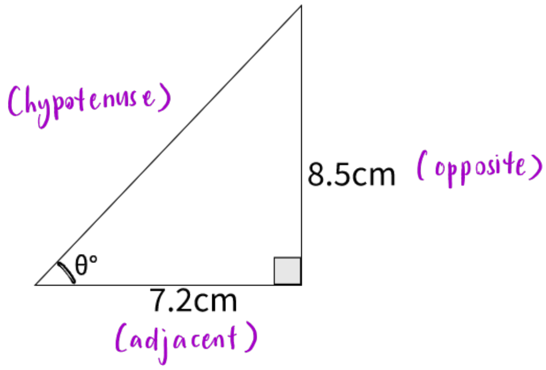
$$= 36.9^\circ \quad (1\ \text{d.p})$$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

2. Find the size of angle θ to 1 decimal place.

a)



$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

$\tan\theta = \dfrac{8.5}{7.2}$

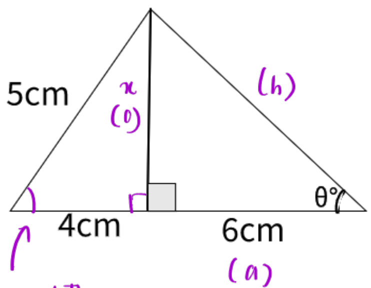
$\theta = \tan^{-1}\left(\dfrac{8.5}{7.2}\right)$

$= \tan^{-1}(1.1805)$

$= 49.733\ldots$

$= 49.7^\circ$ (1 d.p)

b)



use Pythagoras' Theorem in this triangle to find the opposite length, x

① Pythagoras' Theorem:

$5^2 = (4)^2 + x^2$

$25 = 16 + x^2$

$25 - 16 = x^2$

$9 = x^2$

$\sqrt{9} = x$

$x = 3$

② $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

$\tan\theta = \dfrac{3}{6}$ the value of x found from previous step

$\theta = \tan^{-1}\left(\dfrac{3}{6}\right)$

$= \tan^{-1}(0.5)$

$= 26.565\ldots$

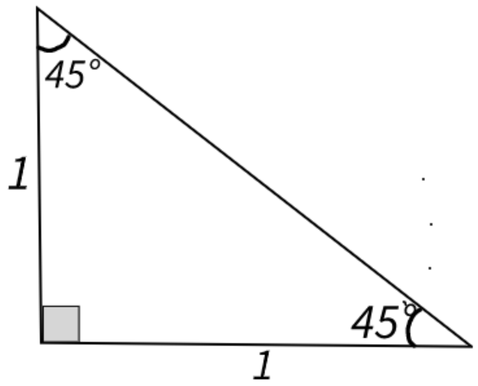
$= 26.6^\circ$ (1 d.p)

Section C

Worked Example

Find the exact value of $\cos 45$.

Step 1: Draw the right-angled isosceles triangle with two sides of length one unit.



Step 2: Use Pythagoras' Theorem to calculate the length of the hypotenuse.

$$a^2 + b^2 = c^2 \text{ (Pythagoras' Theorem)}$$

Here, $a = 1$, $b = 1$ and c is the hypotenuse so substituting into Pythagoras' theorem gives:

$$c^2 = 1^2 + 1^2$$

$$c^2 = 2$$

$$Hypotenuse = c = \sqrt{\mathbf{2}}$$

Step 3: Substitute our known values into the cosine ratio to calculate the exact value of $\cos 45$.

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

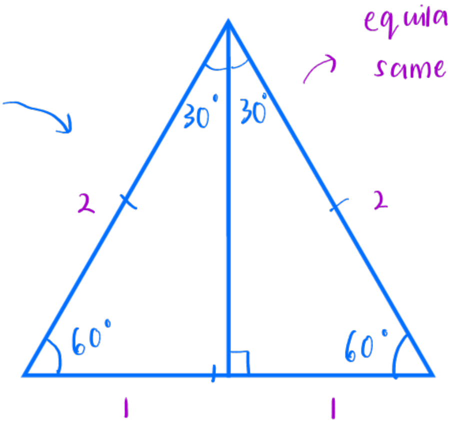
$$\cos 45 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Guided Example

Calculate the exact value of $\tan 30$.

Step 1: Decide which triangle to construct: the right-angled isosceles or the equilateral triangle with a perpendicular bisector.

equilateral triangle with a perpendicular bisector (the side lengths are 2 units)



equilateral triangle has the same angles which are 60°

Step 2: Use Pythagoras' Theorem to calculate the missing lengths.

a = bisector

$$a^2 + b^2 = c^2$$
$$a^2 + (1)^2 = (2)^2$$
$$a^2 + 1 = 4$$
$$a^2 = 4 - 1$$
$$= 3$$
$$a = \sqrt{3}$$

bisector length = $\sqrt{3}$

Step 3: Substitute the sides adjacent and opposite to the 30° angle into the tangent ratio to calculate the exact value of $\tan 30$.

$$\tan 30° = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 30° = \frac{1}{\sqrt{3}}$$

value found in previous step

$$= \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{3}$$

(simplify the surd form)

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

3. Find the exact value of:

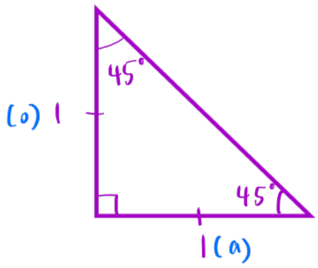
a) sin 90

This value must be memorised as it cannot be derived from the triangles.

$\sin 90° = 1$

b) tan 45

draw isosceles right-angled triangle with equal sides of 1 unit



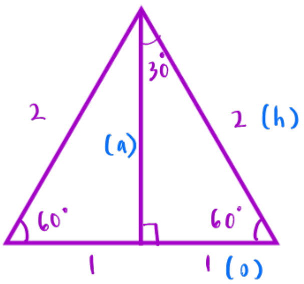
$\tan 45° = \dfrac{\text{opposite}}{\text{adjacent}}$

$= \dfrac{1}{1}$

$\tan 45° = 1$

c) sin 30

draw an equilateral triangle with side lengths 2 units.

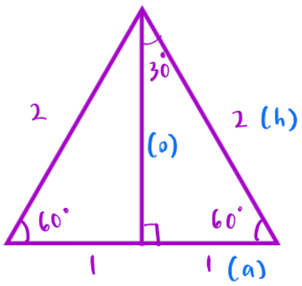


$\sin 30° = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\sin 30° = \dfrac{1}{2}$

d) cos 60

draw an equilateral triangle with side lengths 2 units.

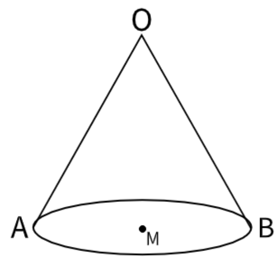


$\cos 60° = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

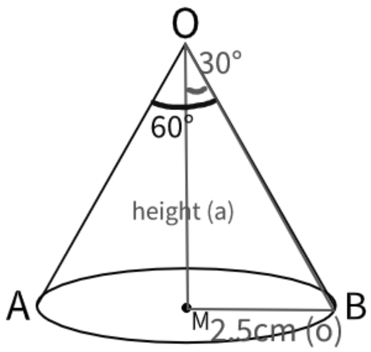
$\cos 60° = \dfrac{1}{2}$

Worked Example

The circular base of the cone has a radius of 2.5 cm. Angle AOB is 60°. Find the height of the cone.



Step 1: Label the cone with the information given and find a right-angled triangle in the 3D shape which you can label with the hypotenuse, opposite and adjacent sides.



Step 2: Working with the right-angled triangle, decide which trigonometric ratio to use and substitute in the values we know.

We know the length of the opposite side and we are trying to find the adjacent side. So, we use the tangent ratio:

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan 30 = \frac{2.5}{height}$$

Step 3: Solve the equation to find the height of the cone.

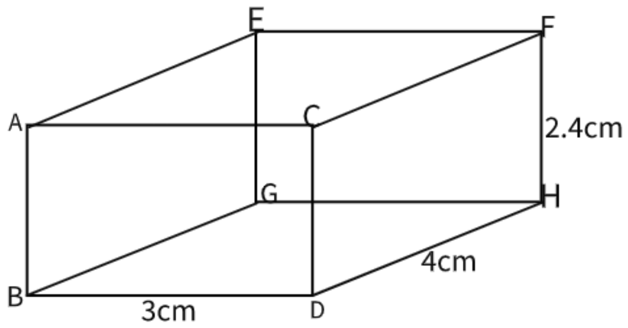
$$\tan 30 = \frac{2.5}{height}$$

$$\tan 30 \times height \;=\; 2.5$$

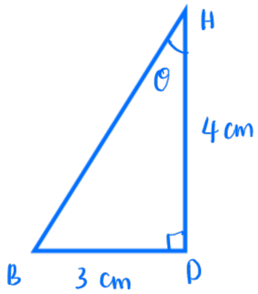
$$height \;=\; \frac{2.5}{\tan 30} = \mathbf{4.3\ cm}\ (to\ 1dp)$$

Guided Example

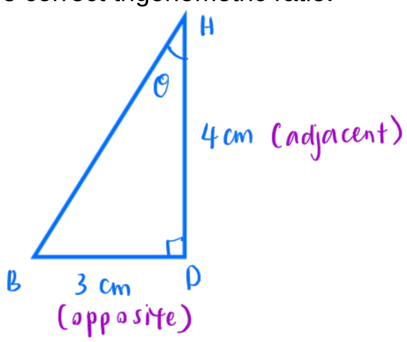
Find the size of angle BHD in the cuboid.



Step 1: Find a right-angled triangle out of the 3D shape which includes the angle BHD.



Step 2: Label the sides according to the angle we are trying to find and use the sides we know to choose the correct trigonometric ratio.



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Step 3: Substitute the known values into the ratio to form an equation. Solve the equation using the inverse trigonometric function.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{3}{4}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\theta = \tan^{-1}(0.75)$$

$$= 36.869\ldots$$

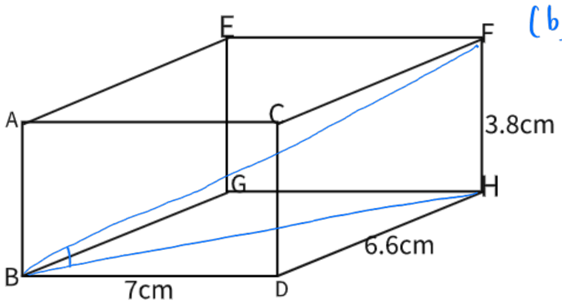
$$= 36.9^\circ \quad (1 \text{ d.p})$$

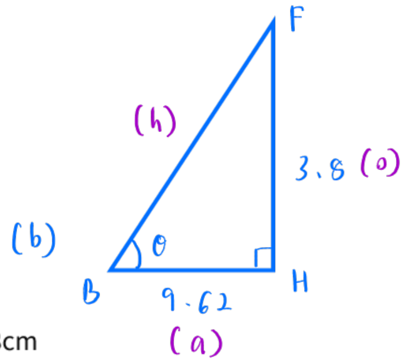
Now it's your turn!

If you get stuck, look back at the worked and guided examples.

4. In the following cuboid find:

   a) the length of side BH (to 3 significant figures)
   b) the value of angle HBF (to 3 significant figures)





(a) $a^2 + b^2 = c^2$ (Pythagoras' Theorem)

$(6.6)^2 + (7)^2 = (BH)^2$

$43.56 + 49 = (BH)^2$

$92.56 = (BH)^2$

$BH = \sqrt{92.56} = 9.62\ cm$ (3 sf)

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

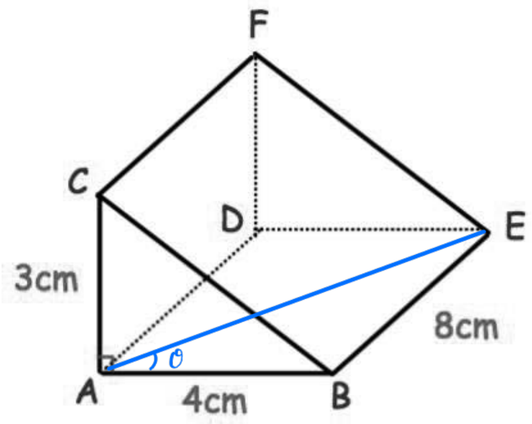
$\tan(\angle HBF) = \dfrac{3.8}{9.62}$

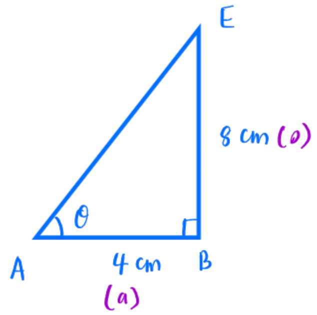
$\angle HBF = \tan^{-1}\left(\dfrac{3.8}{9.62}\right)$

$\angle HBF = 21.554\ldots$

$\angle HBF = 21.6^\circ$ (3 sf)

5. Find the size of angle EAB (to the nearest degree)





$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

$\tan\theta = \dfrac{8}{4}$

$\tan\theta = 2$

$\theta = \tan^{-1}(2)$

$= 63.43\ldots$

$= 63^\circ$ (nearest degree)