GCSE Maths – Geometry and Measures

Pythagoras' Theorem

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions related to Pythagoras' theorem. Each section contains a worked example, a question with hints and then questions for you to work through on your own.
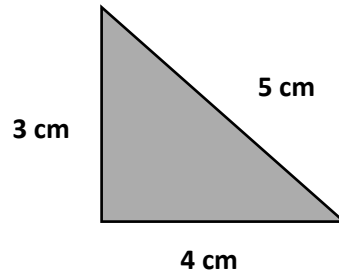
Section A

Worked Example

What is the length of the hypotenuse in this triangle?

3 cm ... 5 cm ... 4 cm

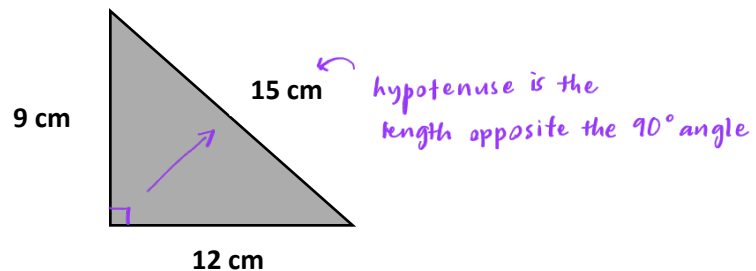**Step 1**: Give the definition of the hypotenuse.

The hypotenuse is located opposite the right angle in the triangle and is the longest side of the triangle.

Step 2: Identify the length of the hypotenuse in the given diagram.

The length opposite the 90° *angle is* 5 cm*.*

Guided Example

What is the length of the hypotenuse in this triangle?

9 cm ... 15 cm ... 12 cm

hypotenuse is the length opposite the 90° angle

Step 1: Give the definition of the hypotenuse.

The hypotenuse is located opposite the right angle, in the triangle and is the longest side of the triangle.

Step 2: Identify the length of the hypotenuse in the given diagram.
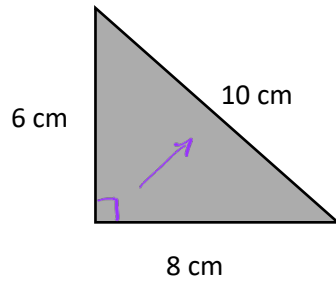
15 cm

Now it's your turn!
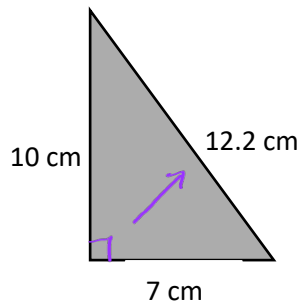If you get stuck, look back at the worked and guided examples.

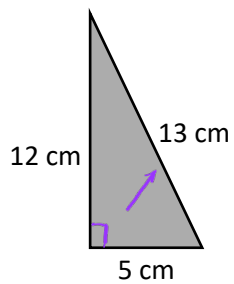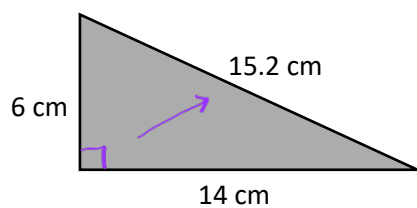1. What is the length of the hypotenuse in these triangles?

a)

6 cm
10 cm
8 cm

10 cm

b)

10 cm
12.2 cm
7 cm

12.2 cm

c)

12 cm
13 cm
5 cm

13 cm
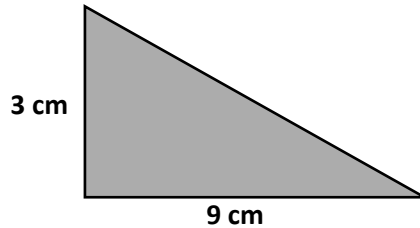
d)

6 cm
15.2 cm
14 cm

15.2 cm

Section B

Worked Example

Calculate the length of the hypotenuse. Give your answer to 3 significant figures.

3 cm

9 cm

Step 1: Use the formula and substitute in the given values of a and b.

$$a^2 + b^2 = c^2$$
$$3^2 + 9^2 = c^2$$

Step 2: Calculate the length of the hypotenuse (the value of c) to 3 significant figures.

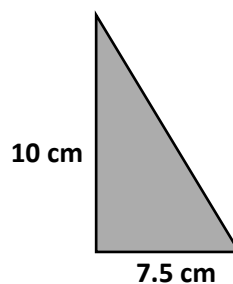$$c^2 = 3^2 + 9^2$$
$$c^2 = 9 + 81$$
$$c^2 = 90$$
$$c = \sqrt{90} = 9.486\ ...$$

The length of the hypotenuse is 9.49 cm.

Guided Example

Calculate the length of the hypotenuse. Give your answer to 3 significant figures.

10 cm

7.5 cm

Step 1: Use the formula and substitute in the given values of a and b.

$$a^2 + b^2 = c^2 \qquad (7.5)^2 + (10)^2 = c^2$$

Step 2: Calculate the length of the hypotenuse (the value of c) to 3 significant figures.

$$c^2 = (7.5)^2 + (10)^2$$
$$= 56.25 + 100$$
$$= 156.25$$

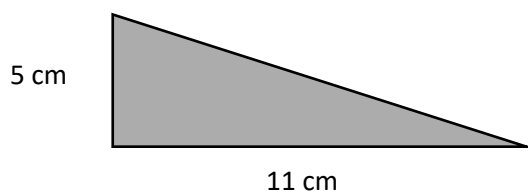$$c = \sqrt{156.25}$$
$$= 12.5$$

Hypotenuse : 12.5 cm

Now it's your turn!
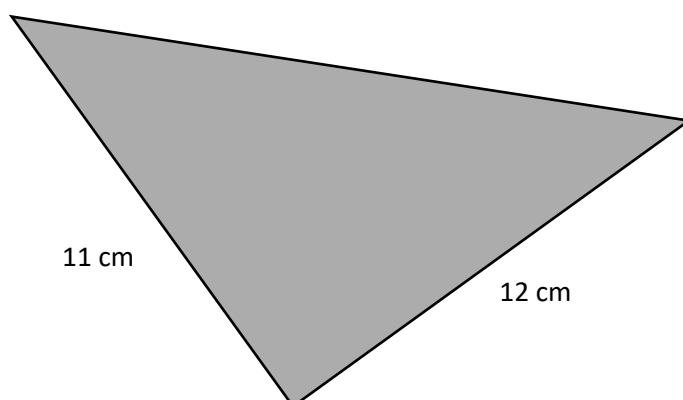If you get stuck, look back at the worked and guided examples.

2. For each of the following triangles, calculate the length of the hypotenuse. Give your answer to 3 significant figures.

a)

5 cm

11 cm

$$a^2 + b^2 = c^2$$
$$(5)^2 + (11)^2 = c^2$$
$$25 + 121 = c^2$$
$$146 = c^2$$
$$c = \sqrt{146}$$
$$= 12.08 \text{ cm}$$
$$\approx 12.1 \text{ cm}$$

↗ 3 significant figures

b)

11 cm

12 cm

$$a^2 + b^2 = c^2$$
$$(11)^2 + (12)^2 = c^2$$
$$121 + 144 = c^2$$
$$265 = c^2$$
$$c = \sqrt{265} = 16.3 \text{ cm}$$

c)

10.5 cm

11.4 cm
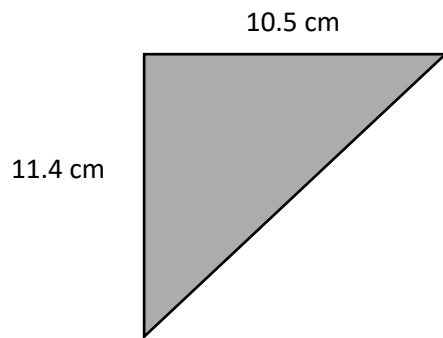
$$a^2 + b^2 = c^2$$
$$(10.5)^2 + (11.4)^2 = c^2$$
$$110.25 + 129.96 = c^2$$
$$240.21 = c^2$$
$$c = \sqrt{240.21}$$
$$= 15.4987$$
$$\approx 15.5 \text{ cm}$$

d)

10.8 cm

3.3 cm
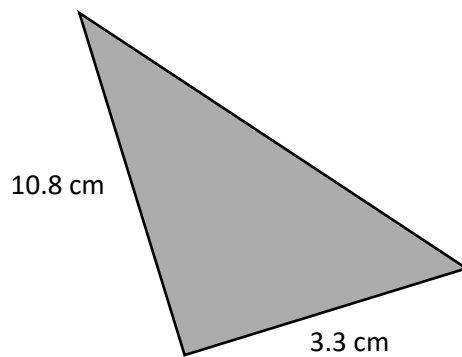
$$a^2 + b^2 = c^2$$
$$(3.3)^2 + (10.8)^2 = c^2$$
$$10.89 + 116.64 = c^2$$
$$127.53 = c^2$$
$$c = \sqrt{127.53}$$
$$= 11.293$$
$$\approx 11.3 \text{ cm}$$

Section C

Worked Example

Calculate the length x. Give your answer to 3 significant figures.
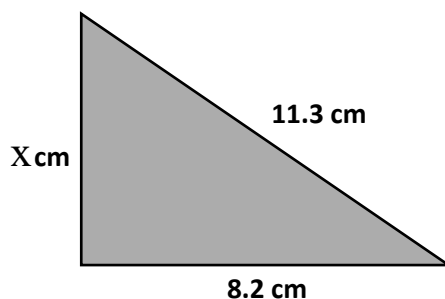
Step 1: Rearrange the formula to isolate the shorter side.

$$a^2 + b^2 = c^2$$

$$a^2 = c^2 - b^2$$

Step 2: Calculate the length of the missing side to 3 significant figures.

$$a^2 = c^2 - b^2$$

Here, $a = x$, $b = 8.2$ and $c = 11.3$:

$$x^2 = c^2 - b^2$$

$$x^2 = 11.3^2 - 8.2^2$$

$$x^2 = 127.69 - 67.24$$

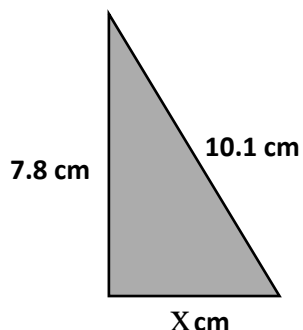$$x^2 = 60.45$$

$$x = \sqrt{60.45} = 7.7749 \dots$$

$$x = 7.77 \text{ cm}$$

The length of x is $7.77\ cm$.

Guided Example

Calculate the length x. Give your answer to 3 significant figures.

10.1 cm

7.8 cm

X cm

Step 1: Rearrange the formula to isolate the shorter side.

$$a^2 + b^2 = c^2$$
$$a^2 = c^2 - b^2$$

Step 2: Calculate the length of the missing side to 3 significant figures.

$$a^2 = c^2 - b^2$$
$$= (10.1)^2 - (7.8)^2$$
$$= 102.01 - 60.84$$
$$= 41.17$$
$$a = \sqrt{41.17}$$
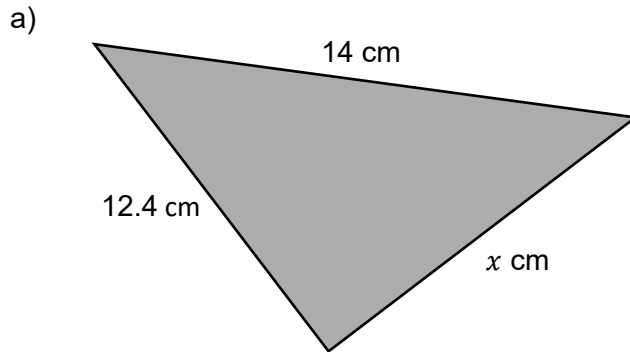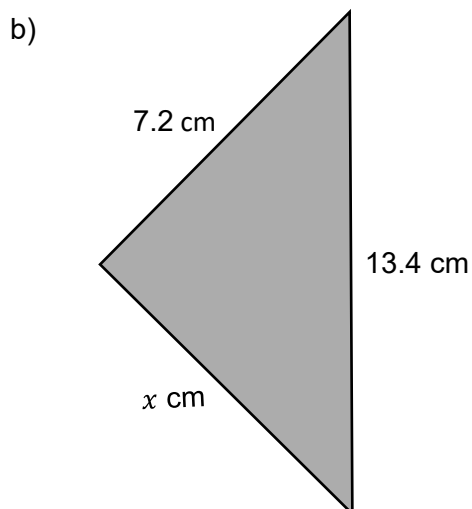$$= 6.416$$
$$\approx 6.42 \text{ cm}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

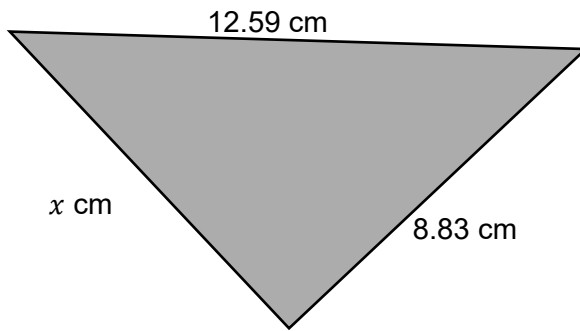3. For each of the following triangles, calculate the length x. Give your answer to 3 significant figures.

a)

14 cm

12.4 cm

x cm

$$a^2 + b^2 = c^2$$
$$a^2 = c^2 - b^2$$
$$(x)^2 = (14)^2 - (12.4)^2$$
$$x^2 = 196 - 153.76$$
$$= 42.24$$
$$x = \sqrt{42.24}$$
$$= 6.4992$$
$$= 6.50 \text{ cm}$$

b)

7.2 cm

13.4 cm

x cm

$$a^2 + b^2 = c^2$$
$$a^2 = c^2 - b^2$$
$$(x)^2 = (13.4)^2 - (7.2)^2$$
$$x^2 = 179.56 - 51.84$$
$$= 127.72$$
$$x = \sqrt{127.72}$$
$$= 11.301$$
$$= 11.3 \text{ cm}$$

c)

12.59 cm

x cm

8.83 cm

$a^2 + b^2 = c^2$

$a^2 = c^2 - b^2$

$(x)^2 = (12.59)^2 - (8.83)^2$

$x^2 = 158.5081 - 77.9689$

$\quad = 80.5392$

$x = \sqrt{80.5392}$

$\quad = 8.974$

$\quad = 8.97 \text{ cm}$

d)

x cm

4.32 cm

1.56 cm
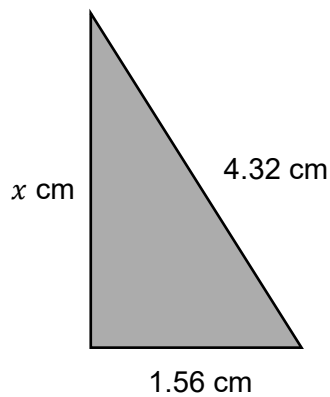
$a^2 + b^2 = c^2$

$a^2 = c^2 - b^2$

$(x)^2 = (4.32)^2 - (1.56)^2$

$x^2 = 18.6624 - 2.4336$

$\quad = 16.2288$

$x = \sqrt{16.2288}$

$\quad = 4.0285$

$\quad = 4.03 \text{ cm}$

Worked Example

Consider the following square based pyramid with height $H = 10$.
Calculate length x, giving your answer to 3 significant figures.
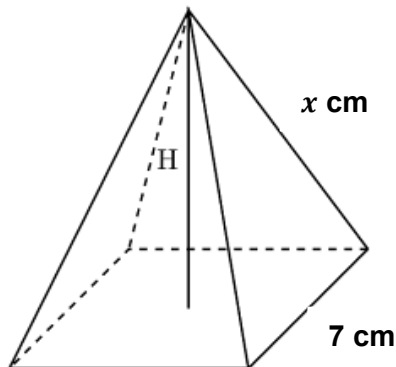
Step 1: Calculate the diagonal length of the base using Pythagoras' theorem.
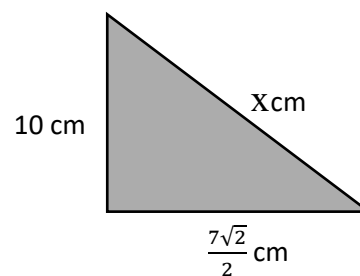
$$a^2 + b^2 = c^2$$

$$c^2 = a^2 + b^2$$

$$c^2 = 7^2 + 7^2$$

$$c^2 = 98$$

$$c = \sqrt{98} = 7\sqrt{2}$$

Step 2: Identify a triangle within the pyramid which will allow you to find the length x.

We are given the height so we will expect the height to form one of the sides of the triangle. The height meets the diagonal at its midpoint and at a right angle. Therefore, the base of the new triangle is half of the diagonal. The height pf the pyramid forms the height of the new triangle, and the length x forms the hypotenuse.

Step 3: Use Pythagoras' theorem to calculate x.

$$c^2 = a^2 + b^2$$

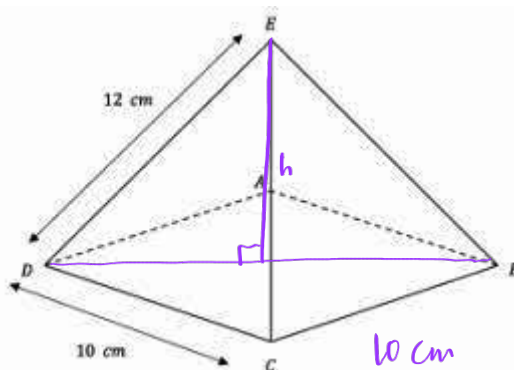$$x^2 = 10^2 + \left(\frac{7\sqrt{2}}{2}\right)^2$$

$$x^2 = 100 + 24.5 = 124.5$$

$$x = \sqrt{124.5} = 11.157\ldots = 11.2 \text{ cm}$$

The length of x is $11.2\ cm$.

Guided Example

Calculate the height of the pyramid, giving your answer to 3 significant figures.
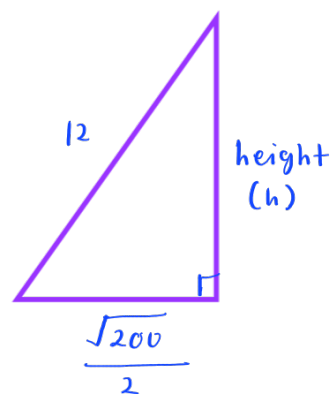

Step 1: Calculate the diagonal length of the base.

$$c^2 = a^2 + b^2$$
$$= 10^2 + 10^2$$
$$= 100 + 100$$

$$c^2 = 200$$
$$c = \sqrt{200}$$

Step 2: Identify a triangle within the pyramid which will allow you to find the height of the pyramid.

The base length of the new triangle is half the diagonal length of the base since the height meets the diagonal at its midpoint.

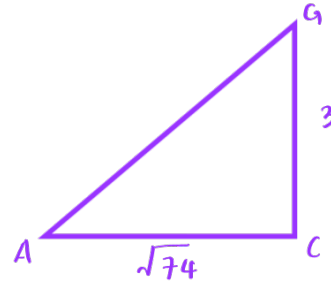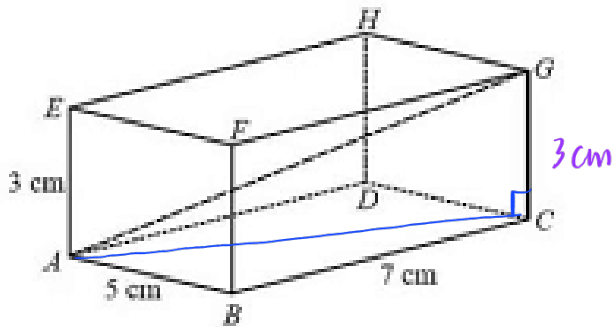**Step 3:** Use Pythagoras' theorem to calculate the height.

$$c^2 = a^2 + b^2$$
$$(12)^2 = \left(\frac{\sqrt{200}}{2}\right)^2 + (h)^2$$
$$144 = 50 + h^2$$
$$h^2 = 144 - 50$$

$$h^2 = 94$$
$$h = \sqrt{94}$$
$$= 9.695$$
$$= 9.70 \text{ cm}$$

4. Calculate length AG. Give your answer to one decimal place.

① Find length AC

$$c^2 = a^2 + b^2$$
$$(AC)^2 = (5)^2 + (7)^2$$
$$= 25 + 49$$
$$= 74$$
$$AC = \sqrt{74}$$

② Find length AG

$$c^2 = a^2 + b^2$$
$$(AG)^2 = (3)^2 + (\sqrt{74})^2$$
$$= 9 + 74$$
$$= 83$$
$$AG = \sqrt{83} = 9.1 \text{ cm}$$

5. Calculate length AY. Give your answer to one decimal place.
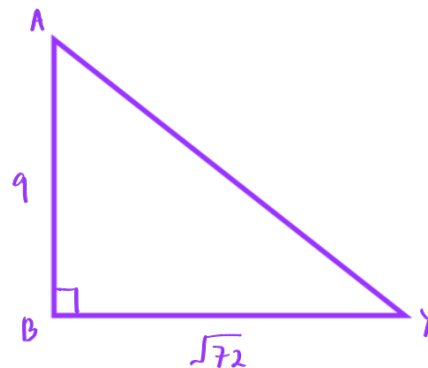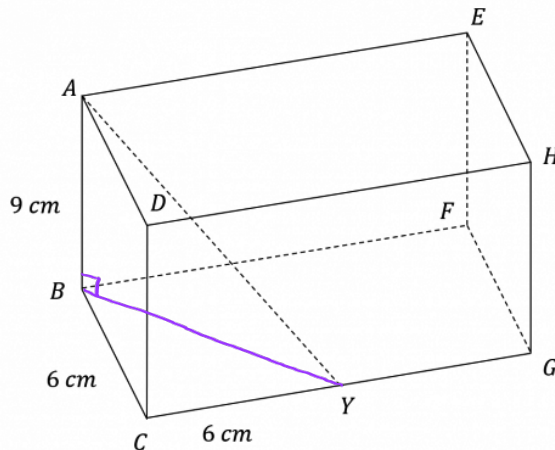
① Calculate length BY
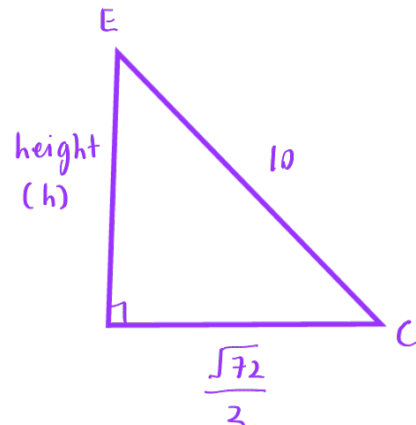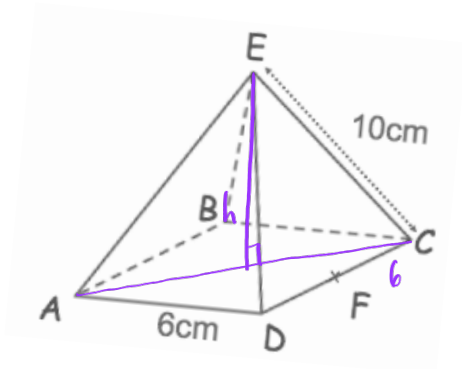
$$c^2 = a^2 + b^2$$
$$(BY)^2 = 6^2 + 6^2$$
$$= 36 + 36$$
$$= 72$$
$$BY = \sqrt{72}$$

$$c^2 = a^2 + b^2$$
$$(AY)^2 = (9)^2 + (\sqrt{72})^2$$
$$= 81 + 72$$
$$= 153$$
$$AY = \sqrt{153}$$
$$= 12.4 \text{ cm}$$

6. Calculate the height of this square-based pyramid. Give your answer to one decimal place.

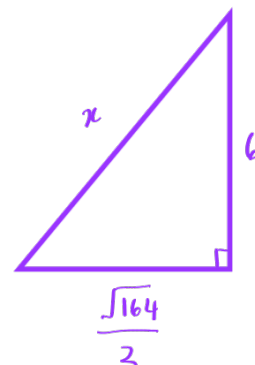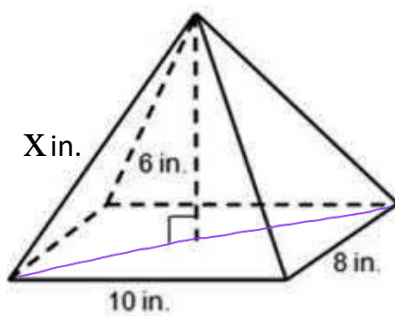① Calculate the diagonal length of the base

$$c^2 = a^2 + b^2$$
$$= (6)^2 + (6)^2$$
$$= 36 + 36$$
$$= 72$$
$$c = \sqrt{72}$$

$$c^2 = a^2 + b^2$$
$$(10)^2 = (h)^2 + \left(\frac{\sqrt{72}}{2}\right)^2$$
$$100 = h^2 + 18$$
$$h^2 = 100 - 18$$
$$= 82$$
$$h = \sqrt{82} = 9.1 \text{ cm}$$

7. Calculate length x. Give your answer to one decimal place.

① Calculate the diagonal length of the base

$$c^2 = a^2 + b^2$$
$$= (8)^2 + (10)^2$$
$$= 64 + 100$$
$$= 164$$
$$c = \sqrt{164}$$

$$c^2 = a^2 + b^2$$
$$(x)^2 = (6)^2 + \left(\frac{\sqrt{164}}{2}\right)^2$$
$$= 36 + 41$$
$$= 77$$
$$x = \sqrt{77} = 8.8 \text{ inches}$$