GCSE Maths – Geometry and Measures

Surface Area of 3D Shapes

Worksheet

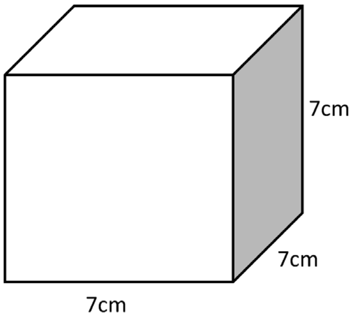
WORKED SOLUTIONS

This worksheet will show you how to work out different types of surface area of 3D shapes questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Find the surface area of the following cube:



Step 1: Find the area of one face.

$$Area\ of\ face = length \times width = 7 \times 7 = 49\ \text{cm}^2$$

Step 2: Multiply the area of one face by 6 to find the total surface area.

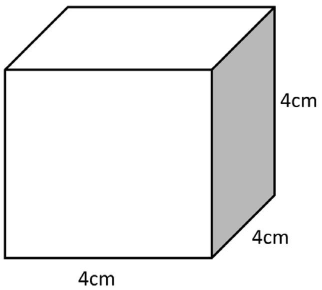
Since all faces of a cube are the same, we can simply multiply the area of one side by 6 to find the total surface area:

$$Surface\ area = 49 \times 6 = \mathbf{294\ cm^2}$$

Remember that the final answer is in units^2 because it is a measure of area.

Guided Example

Find the surface area of the cube.



Step 1: Find the area of one face.

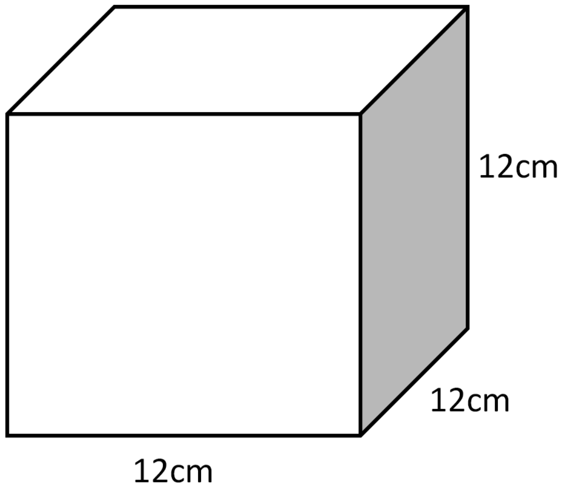
Area of face = length × width = 4 cm × 4 cm = 16 cm^2

Step 2: Multiply the area of one face by 6 to find the total surface area.

Total surface area = 16 cm^2 × 6 = 96 cm^2

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Calculate the surface area of the following cube:



Area of one face $= 12 \times 12$
$= 144 \text{ cm}^2$

Total surface area :

$144 \times 6 = 864 \text{ cm}^2$

a cube has 6 equal faces

2. Calculate the length of each side of a cube if its total surface area is 433.5 cm^2.

Total surface area = area of one face $\times$ 6
433.5 = area of one face $\times$ 6
area of one face $= \dfrac{433.5}{6}$
$= 72.25 \text{cm}^2$

area of one face = length $\times$ width
$72.25 = x^2$ equal length
$\sqrt{72.25} = x$
$x = 8.5 \text{ cm}$

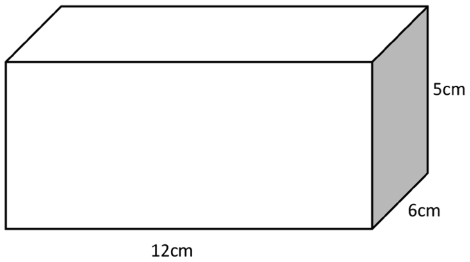
3. The length of a cube measures 5.5 cm. What is the surface area of the cube?

Area of one face $= 5.5 \times 5.5$
$= 30.25 \text{ cm}^2$
Total surface area $= 30.25 \times 6 = 181.5 \text{ cm}^2$

Section B

Worked Example

Find the surface area of the cuboid:



Step 1: Find the area of three different faces.

The opposite faces of a cuboid are identical. Therefore, we only need to find the area of one face of each pair. This means we need to calculate the area of three *different* *faces. We can find the area of the front-facing, right-facing and upwards-facing sides:*

$$Front\ face\ area = 12 \times 5 = 60\ \text{cm}^2$$
$$Right\ face\ area = 6 \times 5 = 30\ \text{cm}^2$$
$$Top\ face\ area = 12 \times 6 = 72\ \text{cm}^2$$

Step 2: Add together the areas of the three different faces, then double to find the total surface area.

$$Area\ of\ the\ three\ faces = 60 + 30 + 72 = 162\ \text{cm}^2$$
$$Total\ surface\ area = 162 \times 2 = \mathbf{324\ cm^2}$$

Guided Example

Find the surface area of the cuboid:

Step 1: Find the area of three different faces.

Front face area : 8 cm x 3 cm = 24 cm^2
Right face area : 3 cm x 5 cm = 15 cm^2
Top face area : 8 cm x 5 cm = 40 cm^2

Step 2: Add together the areas of the three different faces, then double to find the total surface area.

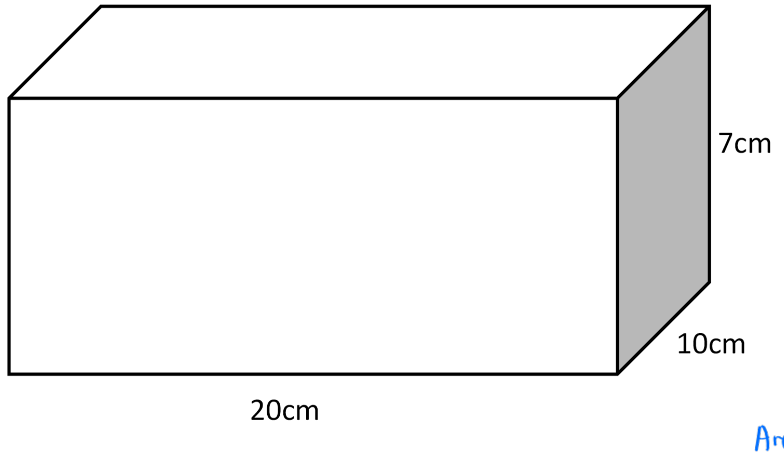
Area of three surfaces : 24 + 15 + 40 = 79 cm^2

Total surface area : 79 x 2 = 158 cm^2

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

4. Calculate the following:

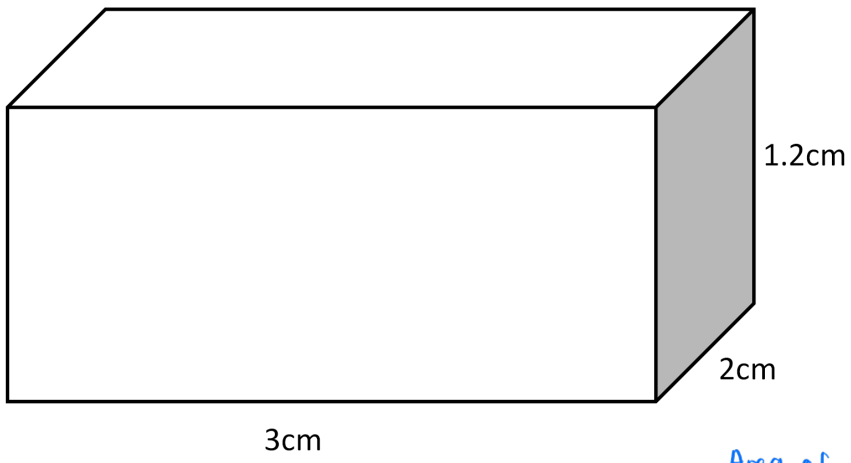
a) The surface area of this cuboid



Front face area : 20 cm x 7 cm = 140 cm^2
Right face area : 10 cm x 7 cm = 70 cm^2
Top face area : 20 cm x 10 cm = 200 cm^2

Area of 3 surfaces :
140 + 70 + 200 = 410 cm^2
Total surface area :
410 x 2 = 820 cm^2

b) The surface area of this cuboid



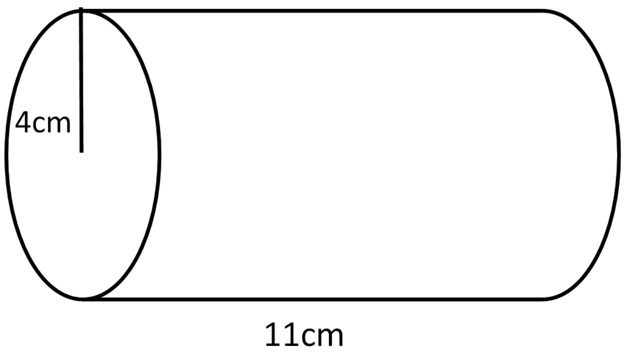
Front face area : 3 cm x 1.2 cm = 3.6 cm^2
Right face area : 2 cm x 1.2 cm = 2.4 cm^2
Top face area : 3 cm x 2 cm = 6 cm^2

Area of 3 surfaces :
3.6 + 2.4 + 6 = 12 cm^2
Total surface area :
12 x 2 = 24 cm^2

Section C

Worked Example

Find the surface area of the following cylinder:



Step 1: Find the area of the two circular faces using the formula for the area of a circle.

The formula for the area of a circle is

$$\boldsymbol{Area} = \boldsymbol{\pi} \times \boldsymbol{r^2},$$

where r is the radius of the circle.

As there are two circular faces, we double the area of one to find the total area of the two circular faces:

$$Area\ of\ one\ face = \pi \times 4^2 = 50.265\ \mathrm{cm}^2$$

$$Area\ of\ two\ faces = 2 \times 50.265\ \mathrm{cm}^2 = 100.531\ \mathrm{cm}^2$$

Step 2: Find the area of the curved face in the middle by multiplying the length of cylinder by the circumference of the circular face.

We can imagine the curved face in the middle as a rectangle wrapped around, with its length as the length of the cylinder and its width as the circumference of the circle.

$$Circumference\ of\ circular\ face = \pi \times diameter = \pi \times 8 = 25.133\ \mathrm{cm}$$

$$Curved\ face\ area = cylinder\ length \times face\ circumference = 11 \times 25.133 = 276.46\ \mathrm{cm}^2$$

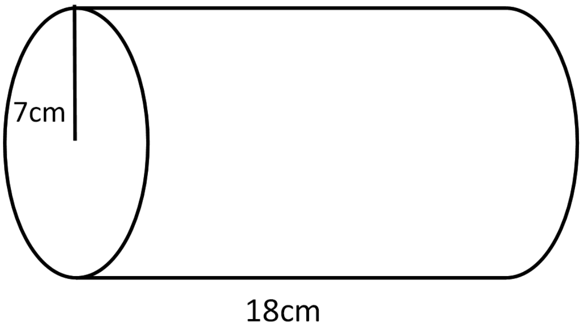
Step 3: Add together the areas of the circular faces and curved face to find the total surface area.

$$Surface\ area = Area\ of\ curved\ surface + Area\ of\ faces$$

$$= 276.46 + 100.531 = \mathbf{376.99\ cm^2}$$

Guided Example

Find the surface area of the following cylinder:



Step 1: Find the area of the two circular faces using the formula for the area of a circle.

Area of one circular face $= \pi \times r^2 = \pi \times 7^2 = 153.93804\ cm^2$

Area of two circular faces $: 2 \times 153.93804 = 307.876...$

$\approx 307.88\ cm^2$

Step 2: Find the area of the curved face in the middle by multiplying the length of cylinder by the circumference of the circular face.

Circumference of the circular face $: \pi \times d = \pi \times (7 \times 2) = \pi \times 14$

$= 43.98229...$

curved face area $: 43.98229 \times 18 = 791.6813...$

$\approx 791.68\ cm^2$

Step 3: Add together the areas of the circular faces and curved face to find the total surface area.

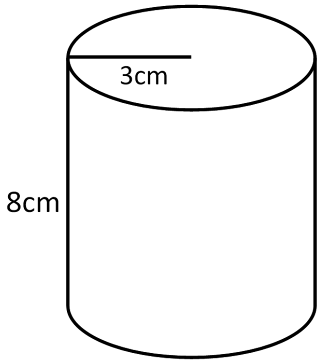
Total surface area $: 307.88 + 791.68$

$: 1099.56\ cm^2$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Calculate the following:

a) The surface area of this cylinder



Area of one circular face : $\pi \times (3)^2 = 28.274...$

Area of two circular faces : $28.274.. \times 2 = 56.548...$

$\approx 56.55\ cm^2$

3×2

Circumference : $\pi \times d = \pi \times 6 = 18.8495...$

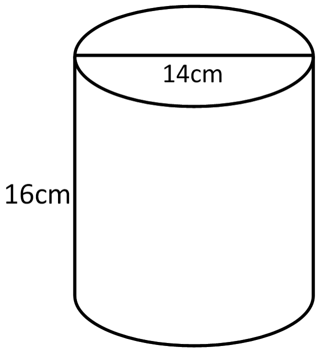
Curved face area $= 18.8495... \times 8 = 150.796...$

$\approx 150.80\ cm^2$

Total surface area : $56.55 + 150.80 = \boxed{207.35\ cm^2}$

b) The surface area of this cylinder

$14 \div 2$



Area of one circular face : $\pi \times (7)^2 = 153.938..$

Area of two circular faces : $153.938 \times 2 = 307.876$

$\approx 307.88\ cm^2$

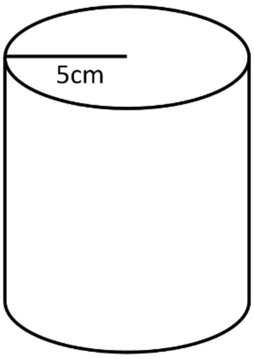
Circumference : $\pi \times d = \pi \times 14 = 43.982...$

Curved face area $= 43.982 \times 16 = 703.716...$

$\approx 703.72\ cm^2$

Total surface area : $307.88 + 703.72 = \boxed{1011.60\ cm^2}$

c) The height of this cylinder if its total surface area is 471.24 cm^2



Area of one circular face : $\pi \times (5)^2 = 78.5398$

Area of two circular faces : $78.5398 \times 2 = 157.079$

≈ 157.08

Total surface area : circular faces + curved face area

$471.24 = 157.08 +$ curved face area

Curved face area $= 471.24 - 157.08 = 314.16$

Circumference : $\pi \times d = \pi \times 10 = 31.4159$

Curved face area : $31.4159 \times h$

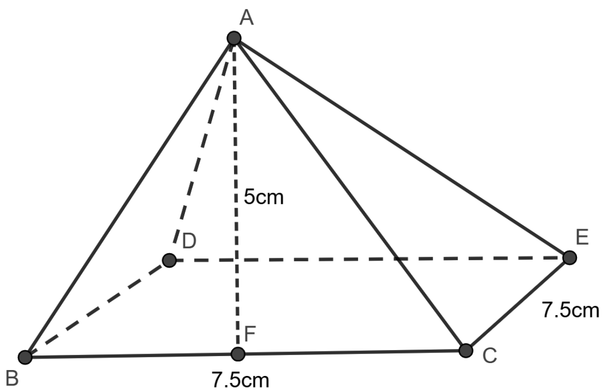
$314.16 = 31.4159 \times h$

$h = \dfrac{314.16}{31.4159} = \boxed{10\ cm}$

Section D

Worked Example

Find the surface area of the following pyramid:



Step 1: Find the area of the base of the pyramid.

The base for this pyramid is a square, so we find the area by multiplying the length by the width:

$$Area\ of\ base = 7.5 \times 7.5 = 56.25\ \text{cm}^2$$

Step 2: As this is a square-based pyramid, all triangular faces are the same. Find the area of one of the triangular faces, then multiply by the number of triangular faces.

All the triangular faces have the same base and height, so we simply find the area of one and multiply by 4 (as there are 4 triangular faces here).

$$Area\ of\ one\ triangular\ face = \frac{Base \times Height}{2} = \frac{7.5 \times 5}{2} = 18.75\ \text{cm}^2$$

$$Total\ area\ of\ triangular\ faces = 4 \times 18.75 = 75\ \text{cm}^2$$

Remember that the triangular faces may not always be identical – if we had a rectangle-based pyramid, or sometimes even a triangular-based pyramid, the bases of the triangular sides may not be the same. The heights will be the same, as each face has the same termination at the tip. In this case, we have to calculate the area of each triangular face individually and then add them together.

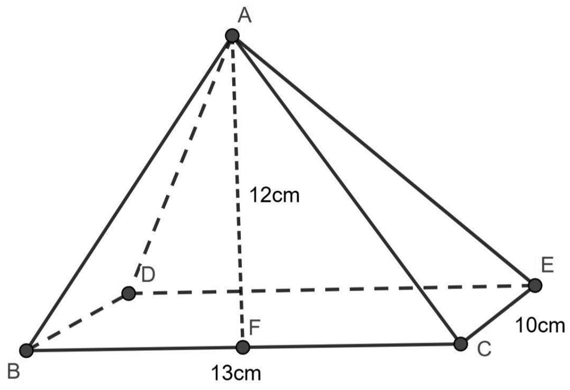
Step 3: Add together the areas of the base and triangular faces to find the total surface area.

$$Total\ surface\ area = Area\ of\ base + Area\ of\ triangular\ faces$$

$$= 56.25 + 75 = \mathbf{131.25\ cm^2}$$

Guided Example

Find the surface area of the following pyramid:



Step 1: Find the area of the base.

Area of base : 13 cm x 10 cm = 130 cm^2

Step 2: As this is a rectangle-based pyramid, the base lengths of each triangular face are not the same. Find the area of the triangular faces individually using the base lengths and height given.

Triangular face ABC = $\frac{1}{2} \times 13 \times 12 = 78$ cm^2

Triangular face ACE : $\frac{1}{2} \times 10 \times 12 = 60$ cm^2

Triangular face ADE = Triangular face ABC
Triangular face ADB = Triangular face ACE
} opposite faces are similar

Step 3: Add together the areas of the base and triangular faces to find the total surface area.

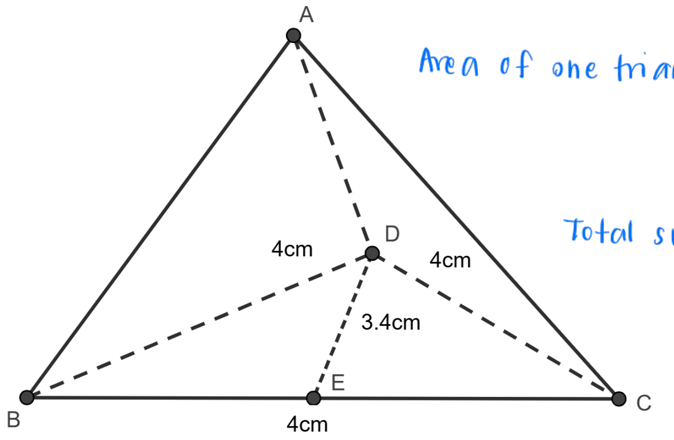
Triangular face areas : $(78 \times 2) + (60 \times 2)$
$= 156 + 120 = 276$ cm^2

Total surface area = $\underbrace{130}_{\text{base area}} + \underbrace{276}_{\text{triangular face areas}} = 406$ cm^2

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

6. Calculate the following:

a) The surface area of this pyramid where all faces are the same, including the base:



Area of one triangular face : $\frac{1}{2} \times 4 \times 3.4$

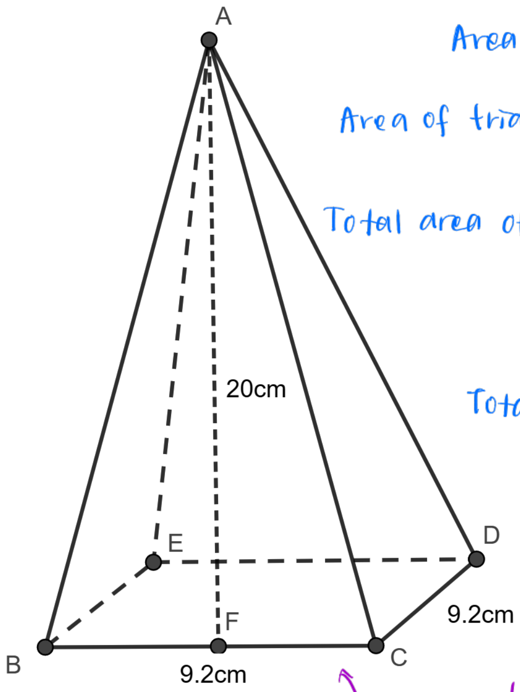
$= 6.8 \text{ cm}^2$

Total surface area : 6.8×4

this pyramid has 4 equal faces

Total surface area : $\boxed{27.2 \text{ cm}^2}$

b) The surface area of this pyramid



Area of base : $9.2 \times 9.2 = 84.64 \text{ cm}^2$

Area of triangular face : $\frac{1}{2} \times 9.2 \times 10 = 92 \text{ cm}^2$

Total area of triangular faces : $92 \times 4 = 368 \text{ cm}^2$

4 triangular faces

Total surface area : $84.64 + 368$

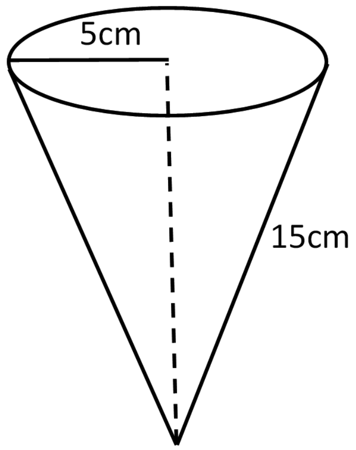
$= \boxed{452.64 \text{ cm}^2}$

square base pyramid. This means all triangular faces are similar.

Section E

Worked Example

Find the surface area of the following cone:



Step 1: Find the area of the circular face.

The formula for the area of a circle is

$$Area = \pi \times r^2,$$

where r is the radius of the circle.

$$Area\ of\ circular\ face = \pi \times 5^2 = 78.54\ \text{cm}^2$$

Step 2: Find the area of the curved face using the formula $\pi r l$.

Remember to use the sloped height of the cone, not the perpendicular height!

$$Area\ of\ curved\ face = \pi r l = \pi \times 5 \times 15 = 235.62\ \text{cm}^2$$

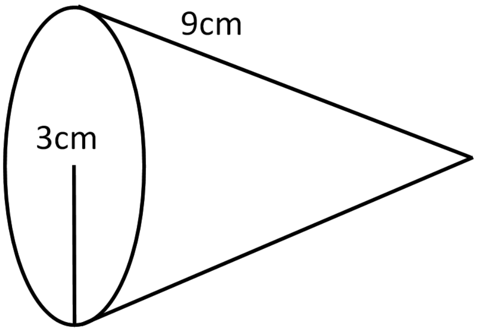
Step 3: Add together the areas of the circular face and curved face to find the total surface area.

$$Total\ surface\ area = Area\ of\ circular\ face + Area\ of\ curved\ face$$

$$= 78.54 + 235.62 = \mathbf{314.16\ cm^2}$$

Guided Example

Find the surface area of the following cone:



Step 1: Find the area of the circular face.

$$\text{Area of circular face} = \pi \times r^2$$
$$= \pi \times (3)^2$$
$$= 28.274 \text{ cm}^2$$

Step 2: Find the area of the curved face using the formula $\pi r l$.

$$\text{Area of curved face} = \pi r l$$
$$= \pi \times 3 \times 9$$
$$= 27\pi$$
$$= 84.823 \text{ cm}^2$$

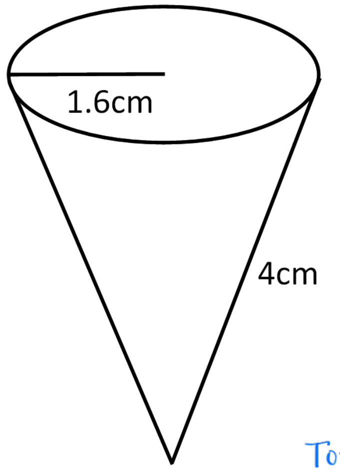
Step 3: Add together the areas of the circular face and curved face to find the total surface area.

$$\text{Total surface area} = \text{circular face} + \text{curved face}$$
$$= 28.274 + 84.823$$
$$= 113.097 \text{ cm}^2$$
$$\approx 113.10 \text{ cm}^2 \quad (2 \text{ dp})$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

7. Calculate the following:

a) The surface area of this cone

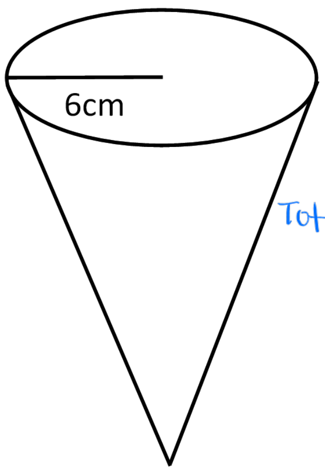


Area of circular face $= \pi \times (1.6)^2$
$= 8.0425 \text{ cm}^2$

Area of curved face $= \pi \times r \times l$
$= \pi \times 1.6 \times 4$
$= 6.4\pi$
$= 20.1062 \text{ cm}^2$

Total surface area $= 8.0425 + 20.1062$
$= 28.1487$
$\approx 28.15 \text{ cm}^2$ (2 dp)

b) The length of this cone if the total surface area is 339.29 cm^2



Area of circular face $= \pi \times (6)^2$
$= 36\pi$
$= 113.097 \text{ cm}^2$

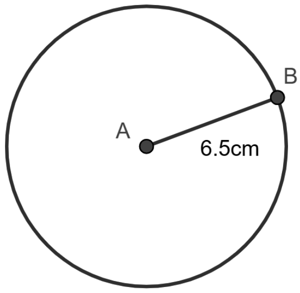
Total surface area = circular face + curved face
$339.29 = 113.097 +$ curved face
Curved face $= 339.29 - 113.097$
$= 226.193 \text{ cm}^2$

area of curved face $= \pi \times r \times l$
$= \pi \times 6 \times l$
$226.193 = 18.85 \times l$
$l = 226.193 \div 18.85 = 11.99 \approx 12 \text{ cm}$

Section F

Worked Example

Find the surface area of the sphere. The following diagram represents a cross-section of the sphere:



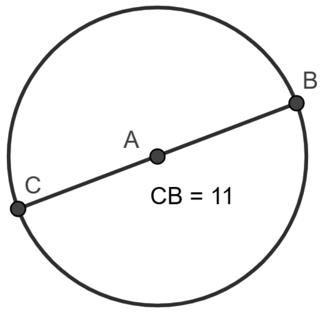
Step 1: Calculate the surface area using the formula for the surface area of a sphere.

$$Surface\ area = 4\pi r^2$$

$$Surface\ area = 4 \times \pi \times 6.5^2 = \mathbf{530.93\ cm^2}$$

Guided Example

Find the surface area of the sphere. The following diagram represents a cross-section of the sphere:



Step 1: Calculate the surface area using the formula for the surface area of a sphere.

$$Surface\ area = 4\pi r^2$$

$$Surface\ area = 4 \times \pi \times (11 \div 2)^2$$

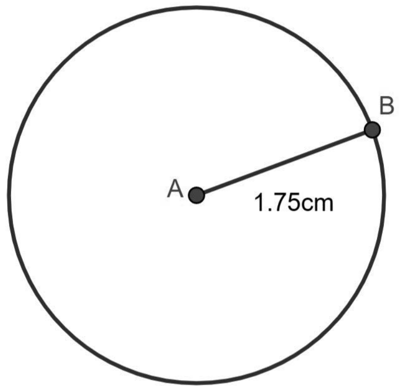
$$= 4 \times \pi \times (5.5)^2$$

$$= 380.13\ unit^2$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

8. Calculate the following:

a) The surface area of this sphere



Surface area of sphere $= 4\pi r^2$

Surface area $= 4 \times \pi \times (1.75)^2$

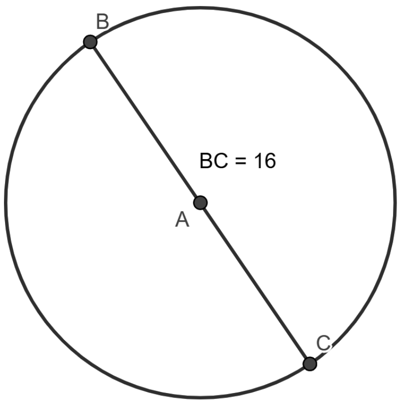
$= 4 \times \pi \times 3.0625$

$= 12.25 \times \pi$

$= 38.4845$

$\approx$ 38.48 cm^2 (2dp)

b) The surface area of this sphere



Surface area of sphere $= 4\pi r^2$

Surface area $= 4 \times \pi \times (8)^2$ — $16 \div 2$

$= 4 \times \pi \times 64$

$= 256 \times \pi$

$= 804.2477$

$\approx$ 804.25 $unit^2$ (2dp)

c) The radius of a sphere if its surface area is 1017.88 mm^2

Surface area of sphere $= 4\pi r^2$

$1017.88 = 4 \times \pi \times r^2$

$\frac{1017.88}{4\pi} = r^2$

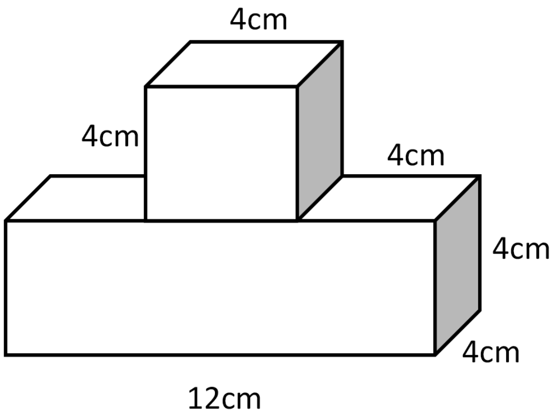
$81 = r^2$

$r = \sqrt{81} = 9$ mm

Section G

Worked Example

Find the surface area of the following composite shape:



Step 1: Work out which sides are facing outwards (these ones count as the total surface area).

For the cube at the top, all sides except the bottom side are facing outward, so 5 faces are counted.

For the cuboid, 5 full faces are facing outwards, and part of the top face. We can calculate how much of the top face is covered by the cube.

Step 2: Work out the surface area of the sides facing outwards.

For the cuboid:

$$Front\ face\ area = 12 \times 4 = 48\ \text{cm}^2$$
$$Front\ and\ back\ face\ area = 48 \times 2 = \mathbf{96\ cm^2}$$

$$Right\ face\ area = 4 \times 4 = 16\ \text{cm}^2$$
$$Right\ and\ left\ face\ area = 16 \times 2 = \mathbf{32\ cm^2}$$

$$Bottom\ face\ area = 12 \times 4 = \mathbf{48\ cm^2}$$

$$Top\ face\ area = (12 - length\ of\ cube) \times 4 = 8 \times 4 = \mathbf{32\ cm^2}$$

$$\boldsymbol{Total\ surface\ area} = 96 + 32 + 48 + 32 = 208\ \text{cm}^2$$

For the cube:

$$One\ face\ area = 4 \times 4 = 16\ \text{cm}^2$$
$$Area\ of\ 5\ faces = 16 \times 5 = \mathbf{80\ cm^2}$$

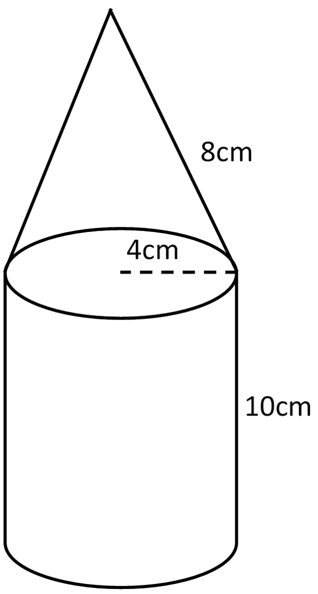
Step 3: Add together the surface area of each shape.

$$Surface\ area = Cube\ surface\ area\ contribution + cuboid\ surface\ area\ contribution$$

$$= 80 + 208 = \mathbf{288\ cm^2}$$

Guided Example

Find the surface area of the following composite shape:



Step 1: Work out which faces are facing outwards. These faces count towards the total surface area.

For the cone, only the curved face is facing outwards.

For the cylinder, one curved face and one circular bottom face is facing outwards.

Step 2: Work out the surface area of the sides facing outwards.

Cone : ① area of curved face $= \pi \times r \times l$

$= \pi \times 4 \times 8$

$= 100.53096 \text{ cm}^2$

Cylinder : ① area of curved face $= \pi \times d \times h$

$= \pi \times (4 \times 2) \times 10 = \pi \times 8 \times 10$

$= 251.327 \text{ cm}^2$

② area of circular face (bottom) $= \pi \times r^2$

$= \pi \times (4)^2 = 50.265 \text{ cm}^2$

Step 3: Find the total surface area.

Total surface area $= 100.53096 + 251.327 + 50.265$

$= 402.12296 \text{ cm}^2$

$\approx 402.12 \text{ cm}^2$ (2 dp)

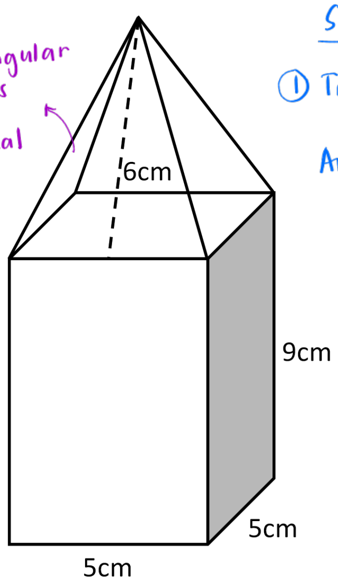
Now it's your turn!

If you get stuck, look back at the worked and guided examples.

9. Calculate the following:

a) The surface area of this composite shape

triangular faces are equal



Square based pyramid :

① Triangular faces = $\frac{1}{2} \times 5 \times 6 = 15 \text{ cm}^2$

Area of triangular faces = $4 \times 15 = 60 \text{ cm}^2$

4 similar faces facing outwards

cuboid :

① front face = $5 \times 9 = 45 \text{ cm}^2$

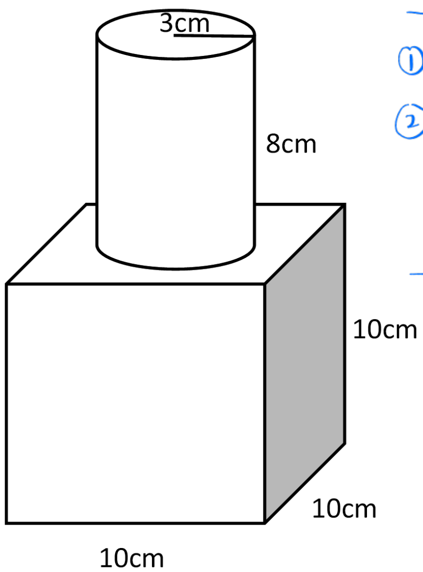
Total vertical faces : $45 \times 4 = 180 \text{ cm}^2$

4 equal faces

② base area = $5 \times 5 = 25 \text{ cm}^2$

Total surface area = $60 + 180 + 25 = 265 \text{ cm}^2$

b) The area of this composite shape



cylinder :

① circular face = $\pi \times (3)^2 = 28.274 \text{ cm}^2$

② curved face = $\pi \times d \times h = \pi \times 6 \times 8$

2 x 3

$= 150.796 \text{ cm}^2$

Cube :

① front face : $10 \times 10 = 100 \text{ cm}^2$

total faces : $100 \times 5 = 500 \text{ cm}^2$

5 full faces facing outwards

② top face area : face area − circular area

$= 100 - 28.274$

$= 71.726 \text{ cm}^2$

Total surface area : $28.274 + 150.796 + 500 + 71.726$

$= 750.796 \text{ cm}^2$

$\approx 750.80 \text{ cm}^2$ (2 dp)