GCSE Maths – Algebra

Solving Linear Inequalities

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions involving linear inequalities. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

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Section A

Worked Example

Solve the inequality $3x - 4 < 18 + x$. Present your answer in a number line.

Step 1: Rearrange the inequality to move all terms containing the unknown to one side of the equation.

$$3x - 4 < 18 + x$$

Subtract x from each side of the equation:

$$3x - 4 - x < 18 + x - x$$
$$2x - 4 < 18$$

Step 2: Solve for x. If you multiply or divide by -1 remember that the sign of the inequality flips direction.

$$2x - 4 < 18$$

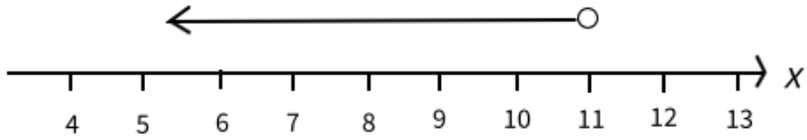
Add 4 to both sides of the equation to eliminate the constant term from the left-hand side:

$$2x < 22$$

Divide both sides of the equation by 2 to remove the coefficient of x:

$$x < 11$$

Step 3: Draw a number line to illustrate the answer. If the sign used is $\leq$ or $\geq$, a solid circle should be used. Otherwise, an open circle should be used.



Guided Example

Solve the inequality $5 \geq 13 - 2x$. Present your answer in a number line.

Step 1: Ensure only the unknown is present on one side of the inequality.

$5 \geqslant 13 - 2x$ (+2x, +2x)
$2x + 5 \geqslant 13$ (−5, −5)
$2x \geqslant 8$

Step 2: Solve for x.

$2x \geqslant 8$
$x \geqslant 4$

Step 3: Draw a number line to illustrate the answer.

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Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Solve the following inequalities and present your answer in a number line:

a) $2x + 1 \geq 5 + x$

$-x$

$x + 1 \geq 5$

-1

$x \geq 4$

b) $2(x + 2) < -14 - x$

$2x + 4 < -14 - x$

$+x$

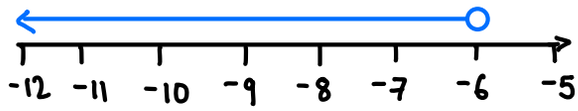
$3x + 4 < -14$

-4

$3x < -18$

$\div 3$

$x < -6$



c) $x - 6 \geq 4x + 3$

$-x$

$-6 \geq 3x + 3$

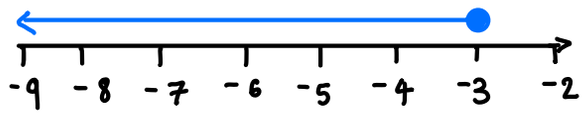
-3

$-9 \geq 3x$

$\div 3$

$-3 \geq x$

$x \leq -3$



d) $-4(x - 5) \leq -3(2x - 7)$

$-4x + 20 \leq -6x + 21$

$+6x$

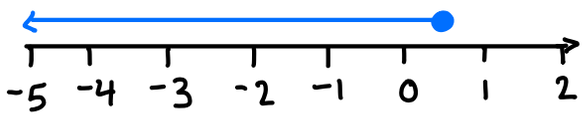
$2x + 20 \leq 21$

-20

$2x \leq 1$

$\div 2$

$x \leq 1/2$



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Section B

Worked Example

Solve the inequality $3x - 8 < 2x - 1 \geq 9$.

Present your answer in a number line and list the integer solutions.

Step 1: Split the inequality into two separate inequality parts.

a) $3x - 8 < 2x - 1$

b) $2x - 1 \geq 9$

Step 2: Solve each inequality separately. Put the answers together to find the values which x can take.

a) $3x - 8 < 2x - 1$

Subtract $2x$ from both sides of the equation: $x - 8 < -1$
Add 8 to both sides of the equation: $\boldsymbol{x < 7}$

b) $2x - 1 \geq 9$

Add 1 to both sides of the equation: $2x \geq 10$
Divide both sides of the equation by 2: $\boldsymbol{x \geq 5}$

Putting the inequalities together: $\mathbf{5 \leq x < 7}$

Step 3: Draw a number line to illustrate the inequality region.

The number line has a filled circle at 5 since x can take value 5. There is a non-filled circle at 7 since x cannot take the value 7.

Step 4: List the set of integers which satisfy the number line.

The integers which satisfy $\mathbf{5 \leq x < 7}$ are $\boldsymbol{x = 5}$ and $\boldsymbol{x = 6}$.

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Guided Example

Solve the inequality $x + 2 < 3(x + 2) \geq 4x$.

Present your answer in a number line and list down the integer solutions.

Step 1: Split the inequality into two separate inequality parts.

a) $x + 2 < 3(x + 2)$

b) $4x \leq 3(x + 2)$

Step 2: Solve each inequality separately. Put the answers together to find the values which x can take.

a) $x + 2 < 3(x + 2)$

$x + 2 < 3x + 6$

$-6 \quad -x$

$-4 < 2x$

$\div 2$

$-2 < x$

b) $4x \leq 3x + 6$

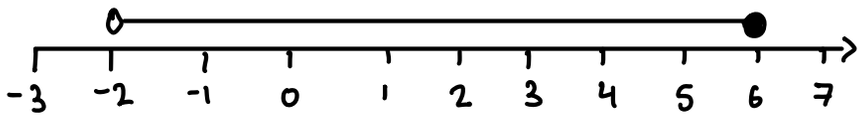
$-3x$

$x \leq 6$

Putting inequalities together

$-2 < x \leq 6$

Step 3: Draw a number line to illustrate the inequality region.



Step 4: List the set of integers which satisfy the number line.

$-1, 0, 1, 2, 3, 4, 5, 6$

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Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. Solve the following inequalities. List the integers in each solution set.

a) $1 \leq 2y - 1 \leq 6$

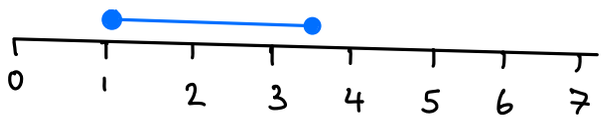
$1 \leq 2y - 1$ | $2y - 1 \leq 6$
$+1$ | $+1$
$2 \leq 2y$ | $2y \leq 7$
$\div 2$ | $\div 2$
$1 \leq y$ | $y \leq 7/2$

$1 \leq y \leq 7/2$

Alt Method:

$+1$ $\quad\quad$ $+1$
$2 \leq 2y \leq 7$
$\div 2$ $\quad\quad$ $\div 2$
$1 \leq y \leq 7/2$

between and including 1 and $7/2$



Integer Value: 1, 2, 3

b) $-6 < 3(p - 1) \geq 4p - 9$

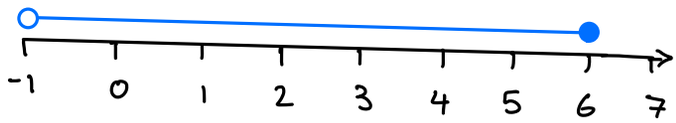
$-6 < 3p - 3 \geq 4p - 9$

split

$-6 < 3p - 3$
$+3$
$-3 < 3p$
$\div 3$
$-1 < p$

$3p - 3 \geq 4p - 9$
$-3p$
$-3 \geq p - 9$
$+9$
$6 \geq p$ ... $p \leq 6$

$-1 < p \leq 6$



Interger Values: 0, 1, 2, 3, 4, 5, 6

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Section C – Higher only

Worked Example

Solve the inequality $2x < 5 + 4x - y$.

Present your answer on a graph.

Step 1: Rearrange the inequality to ensure only y is present on the left-hand side and x is on the right-hand side. The sign for y should be positive and its coefficient should be 1.

$$2x < 5 + 4x - y$$

Add y to both sides of the inequality: $y + 2x < 5 + 4x$
Subtract $2x$ from both sides of the inequality: $y < 5 + 2x$

Step 2: Replace the inequality sign by '=' to help plot the required graph. Find the x –intercept and the y –intercept to find coordinates on the line.

Here, we are interested in the graph $y = 5 + 2x$.

- *To find the x –intercept, set $y = 0$:*

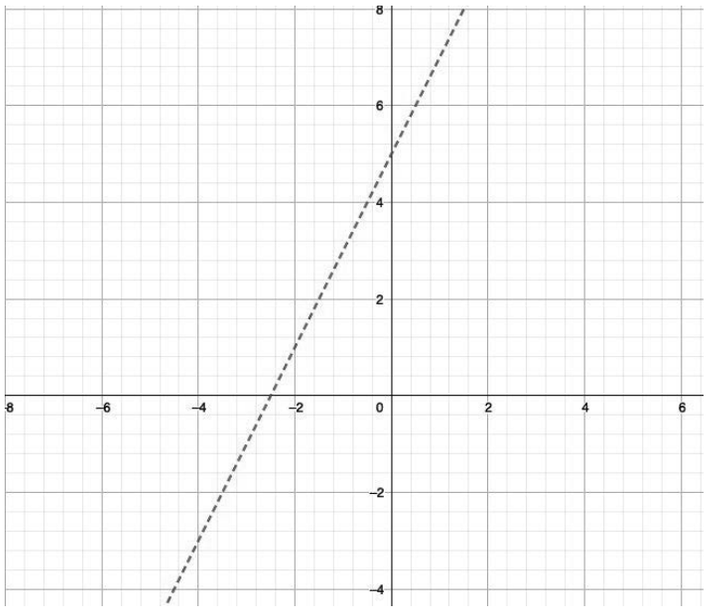
$$y = 5 + 2x$$
$$0 = 5 + 2x$$
$$x = -2.5$$

- *To find the y –intercept, set $x = 0$:*

$$y = 5 + 2\,(0)$$
$$y = 5$$

Step 3: Plot both the x –intercept and the y –intercept and draw the line equivalent to the inequality. If $<$ or $>$ sign is used, the line drawn should be a dotted line. However, if the sign used is $\leq$ or $\geq$, a solid line should be drawn.

From above, we can plot the points $(-2.5,0)$ and $(0,5)$.



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Step 4: Shade the region which satisfies the inequality.

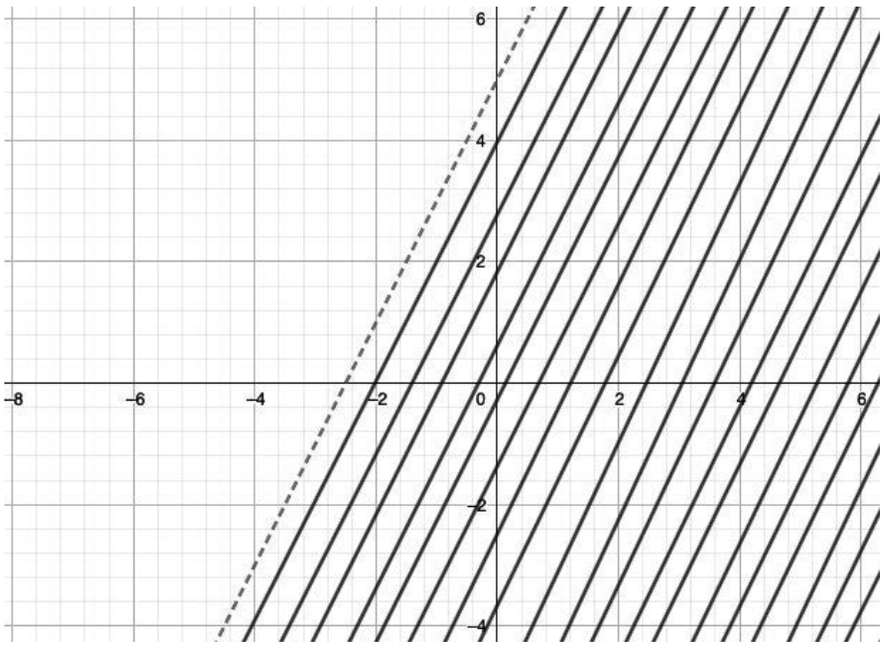
To help identify the correct region, choose a coordinate and substitute it into the inequality. If the coordinate satisfies the inequality, the region where the coordinate lies should be shaded.

For $\boldsymbol{y < 5 + 2x}$, the value of y should always be lower than the dotted line. Since y is lower in the lower region of the graph, that region should be shaded.

Alternatively, we can also choose a point in the lower region to check our answer. For instance, if we choose $(2, -2)$ and substitute it in the inequality (as shown below), we will get a correct statement.

$$y < 5 + 2x$$
$$-2 < 5 + 2(2)$$
$$-2 < 9$$

This means that the section containing $(2, -2)$ should be shaded.



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Guided Example

Solve the inequality $-y - 4 \geq 3(x + 1)$.

Present your answer in a graph.

Step 1: Rearrange the inequality to ensure only y is present on the left-hand side and x is on the right-hand side. The sign for y should be positive and its coefficient should be 1.

$-y - 4 \geq 3x + 3$

$+y$

$-4 \geq 3x + 3 + y$

$-3x \quad -3$

$-3x - 7 \geq y$

$y \leq -3x - 7$

Step 2: Replace the inequality sign by '=' to help plot the required graph. Find the x –intercept and the y –intercept to find coordinates on the line.

$y = -3x - 7$

When $x = 0$:

$y = -3(0) - 7$

$y = -7$

$(0, -7)$

When $y = 0$

$0 = -3x - 7$

$+3x$

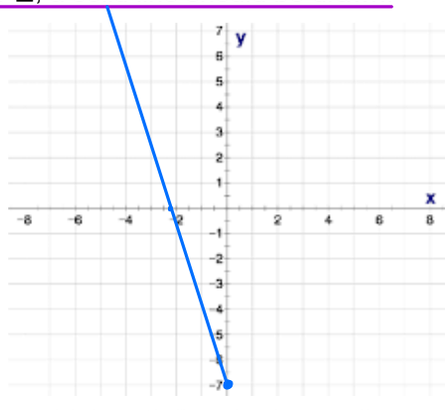
$3x = -7$

$\div 3$

$x = -7/3$

$(-7/3, 0)$

Step 3: Plot both the x –intercept and the y –intercept and draw the line equivalent to the inequality. If $<$ or $>$ sign is used, the line drawn should be a dotted line. However, if the sign used is $\leq$ or $\geq$, a solid line should be drawn.



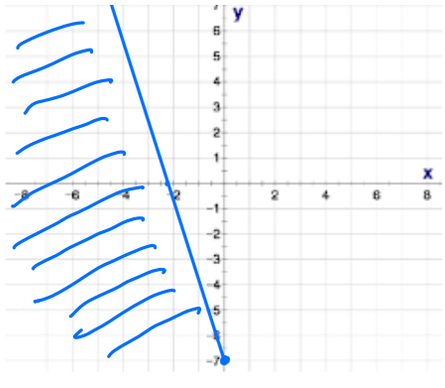
Step 4: Shade the region which satisfies the inequality.
To help identify the correct region, choose a coordinate and substitute it into the inequality. If the coordinate satisfies the inequality, the region where the coordinate lies should be shaded.

choose $(0,0)$ $-3(0) - 7 = -7$

$0 > -7$

$y = 0$

$(0,0)$ should not be shaded as $y \geq -3x - 7$ not $y \leq -3x - 7$



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Now it's your turn!
If you get stuck, look back at the worked and guided examples.

3. Solve the following inequalities and present your answers in a graph.

a) $3x - y < 8y + 2$

-2

$3x - 2 - y < 8y$

$+y$

$3x - 2 < 9y$

$\div 9$

$\frac{1}{3}x - \frac{2}{9} < y$

$y > \frac{1}{3}x - \frac{2}{9}$

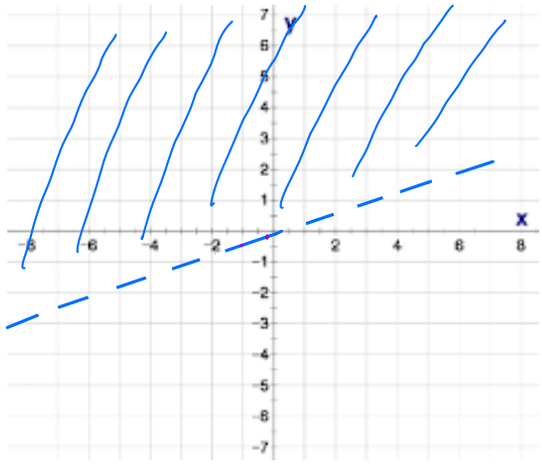
When $x = 0$,
$y = \frac{1}{3}(0) - \frac{2}{9}$
$y = -\frac{2}{9}$
$(0, -2/9)$

When $y = 0$
$0 = \frac{1}{3}x - \frac{2}{9}$
$+\frac{2}{9}$
$\frac{2}{9} = \frac{1}{3}x$
$\times 3$
$\frac{2}{3} = x$
$(2/3, 0)$

As it is $<$, draw a dotted line.

$(0,0)$ $\frac{1}{3}(0) > -\frac{2}{9}$

$0 > -\frac{2}{9}$, so $(0,0)$ is shaded



b) $2g + 2m \geq 7g - 10$

Treat m as y and g as x

$-2g$

$2m \geq 5g - 10$

$\div 2$

$m \geq \frac{5}{2}g - 5$

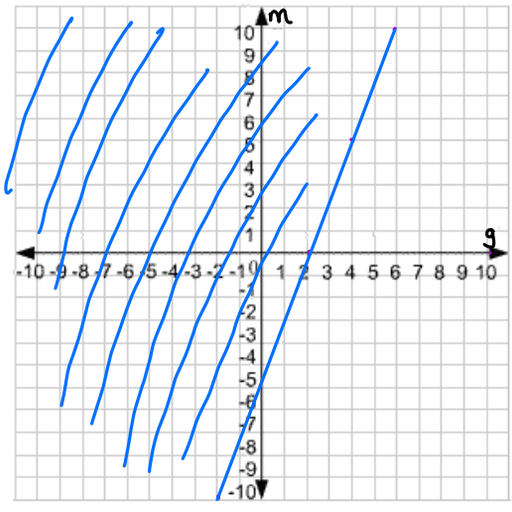
When $g = 0$,
$m = \frac{5}{2}(0) - 5$
$m = -5$
$(0, -5)$

When $m = 0$,
$0 = \frac{5}{2}g - 5$
$\frac{5}{2}g = 5$
$\div \frac{5}{2}$
$g = 2$
$(2, 0)$

As it is $\geq$, solid line.

$(0,0)$: $\frac{5}{2} \times 0 - 5 = -5$

$0 > -5$ so $(0,0)$ is shaded in



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c) $2a + 4b \geq 3a - 4$ Treat a as y and b as x

$-2a$

$4b \geqslant a - 4$

$+4$

$4b + 4 \geqslant a$

$a \leqslant 4b + 4$

As $\leqslant$, solid line

When $b = 0$

$a = 4(0) + 4$

$a = 4$

$(0, 4)$

When $a = 0$

$0 = 4b + 4$

$4b = -4$

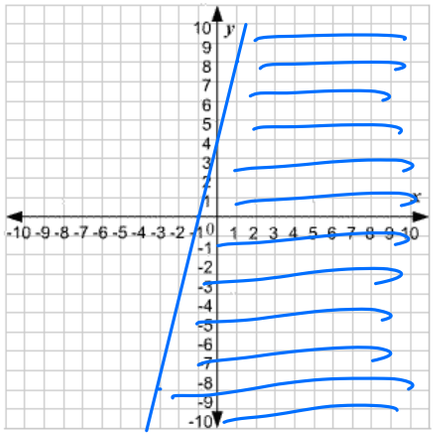
$b = -1$

$(-1, 0)$

$(0,0)$

$4(0) + 4 = 4$

$0 < 4$ so $(0,0)$ is shaded in



d) $-4x - 3y < -12$

$+3y$

$-4x < -12 + 3y$

$+12$

$-4x + 12 < 3y$

$\div 3$

$\frac{-4}{3}x + 4 < y$

$y > \frac{-4}{3}x + 4$

As $>$, dotted line.

$(0,0)$

$\frac{-4}{3}(0) + 4 = 4$

$0 < 4$ so $(0,0)$ is not shaded in

When $x = 0$

$y = \frac{-4}{3}(0) + 4$

$y = 4$

$(0, 4)$

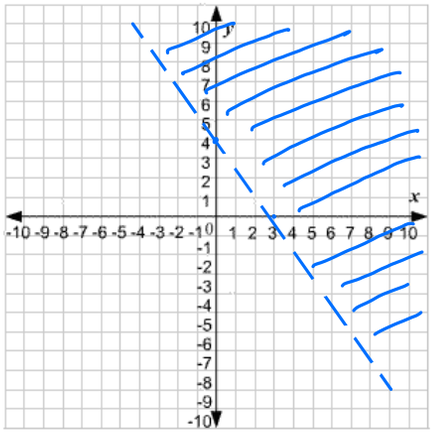
When $y = 0$

$0 = \frac{-4}{3}x + 4$

$\frac{4}{3}x = 4$

$x = 3$

$(3, 0)$



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