GCSE Maths – Algebra

Forming and Solving Equations

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions involving forming and solving equations. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Clara, Marty and Karen each win a share of a raffle prize. Clara wins twice as much as Marty, and Karen wins £50 less than Clara.

If the total prize money was £400, how much money did Marty win?

Step 1: Identify the unknown value which we are trying to find.

We are looking for the amount of money won by Marty.

$$Let\ x = the\ amount\ of\ money\ won\ by\ Marty$$

Step 2: Use the information in the question to express Clara and Karen's winning in an expression involving x.

Clara wins twice as much as Marty. So, Clara wins £$2x$.

Karen wins £50 *less than Clara, so Karen wins* £$(2x - 50)$.

Step 3: Create an equation using the amount won by each winner and other information we are given.

The total prize money is £400, so the sum of Marty, Clara and Karen's winnings must be £400.

$$x + 2x + (2x - 50) = 400$$

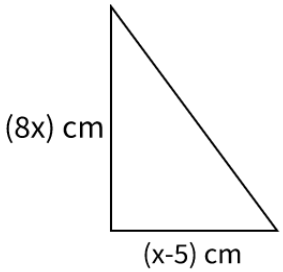
Step 4: Solve the linear equation to find x.

$$\begin{aligned} x + 2x + (2x - 50) &= 400 \\ 5x - 50 &= 400 \\ 5x &= 450 \\ x &= \frac{450}{5} \\ x &= 90 \end{aligned}$$

Marty won £90.

Guided Example

Consider the triangle shown on the right.
The area of the right-angled triangle is $56\ \text{cm}^2$.
Find the exact length of the hypotenuse.



Step 1: Create an equation for the area of the triangle in terms of x.

We know the formula for area of triangle:

$$Area = Base \times Height \times \frac{1}{2}$$

Area = 56
base = $(x-5)$
height = $8x$

substitute known values into equation

$56 = (x-5) \times (8x) \times \frac{1}{2}$

$56 = (x-5) \times 4x$

$56 = 4x^2 - 20x$

$\div 4$ ($4x^2 - 20x = 56$

$x^2 - 5x = 14$

-14 ($x^2 - 5x - 14 = 0$

Step 2: Solve the quadratic equation you have created to find x.

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x - 7 = 0$ or $x + 2 = 0$

$x = 7$ $\quad x = -2$

take the positive value of x because length cannot be negative

Hence, $x = 7$

Step 3: Substitute the value for x into the base and height lengths.

base $= (x-5)$
$= (7-5)$
$= 2$

height $= 8x$
$= 8(7)$
$= 56$

Step 4: Use Pythagoras' Theorem to find the length of the hypotenuse.

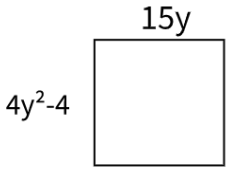
Pythagoras' Theorem : $a^2 + b^2 = c^2$ ← hypotenuse

$a^2 + b^2 = c^2$
$(2)^2 + (56)^2 = c^2$
$4 + 3136 = c^2$

$3140 = c^2$
$\sqrt{3140} = c$
$c = 56.036 \approx 56\ \text{cm}$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Consider the square below. Calculate the length of the side.



Since this is a square, the length should be the same.

$4y^2 - 4 = 15y$

$-15y$: $4y^2 - 15y - 4 = 0$

factorise: $(4y + 1)(y - 4) = 0$

$4y + 1 = 0$
$4y = -1$
$y = -1/4$

or

$y - 4 = 0$
$y = 4$

↖ take the positive integer

length of square $= 15y$
$= 15(4)$
$= 60$

2. Rose, David and Rani work for an advertising company, and are tasked with selling festival tickets. Rose sells 100 more than David and Rani sells three times as much as David. All together they sell 800 tickets. How many tickets did David sell?

Tickets sold by David = x
Tickets sold by Rose = $x + 100$
Tickets sold by Rani = $3x$
Total tickets sold = 800

$800 = x + x + 100 + 3x$
$= x + x + 3x + 100$

$800 = 5x + 100$

-100: $700 = 5x$

$\div 5$: $140 = x$

Tickets sold by David = 140

3. I am thinking of a number. I add four to the number, and then multiply the result by 8. I then divide the result by 3. The final result is 48. Form an equation to show the process the starting number has gone through and use it to calculate what number I started with.

Starting number = x
Add 4 to the number = $x + 4$
Multiply the result by 8 = $(x + 4) \times 8$
Divide the result by 3 = $(x + 4) \times 8 \div 3$
Final result = 48

$48 = \frac{(x + 4) \times 8}{3}$

$\times 3$: $144 = (x + 4) \times 8$

$\div 8$: $18 = x + 4$

-4: $x = 14$

4. Michelle is saving up for a handbag costing £108. She works for one week at £6.50 per hour, and at the end of the week, her grandmother gives her £17. If by this time she has the exact amount to purchase the handbag, how many hours did she work that week?

Michelle's salary = £6.50 per hour
Hours that Michelle worked = x
Michelle's salary by the end of the week: $6.50 \times x$
Money her grandmother has given: 17
Cost of the handbag = 108

$108 = 6.5x + 17$

-17: $91 = 6.5x$

$\div 6.5$: $14 = x$

Michelle worked for 14 hours that week.

Section B

Worked Example

Marge thinks of two positive integers. One number is four times the other, and the product of the two numbers is 64.

Step 1: Identify the unknown values we are trying to find as letters.

$$Let\ the\ smaller\ integer\ =\ a$$

$$Let\ the\ bigger\ integer\ =\ b$$

Step 2: Express the relationships between a and b as equations.

EQUATION 1
We know one number is four times the other:

$$4a = b$$

EQUATION 2
We know the product of the numbers is 64:

$$ab = 64$$

Step 3: Solve the simultaneous equations.

Substitute Equation 1 into Equation 2 to find a:

$$ab = 64$$
$$a(4a) = 64$$
$$4a^2 = 64$$
$$a^2 = 16$$
$$a = \pm 4$$

We know the values are positive integers, so $\boldsymbol{a = 4}$.

Substitute $a = 4$ into Equation 1:

$$4a = b$$
$$4(4) = b$$
$$\boldsymbol{b = 16}$$

Marge thought of the numbers 4 and 16.

Guided Example

A lawn is a rectangular shape and has a length twice as long as its width. The area of the lawn is $98\ \text{m}^2$. What is the perimeter of the lawn?

Step 1: Label the unknown length and width as letters.

width = x

length = $2x$

Step 2: Create an equation using the information in the question about the size and area of the lawn.

Area = 98

Area = width × length

$98 = x \times 2x$

$\div 2$ ↓ $98 = 2x^2$

$49 = x^2$

square root ↓ $\pm 7 = x$ ← only take the positive integer as the answer

$x = 7$

Step 3: Solve the equation and find the value of x. Substitute x into the equations in Step 1 to find the length and width. Then, find the perimeter of the lawn.

width = x

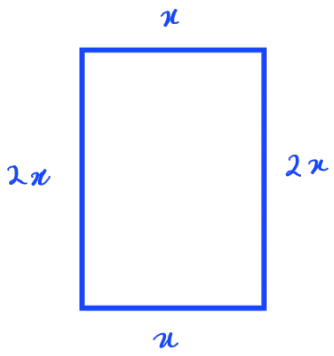
length = $2x$

Perimeter = $x + 2x + x + 2x$

$= 6x$

$= 6(7)$ ← substitute value of x

$= 42\ \text{m}$



Alternatively, we can substitute x to find the width and length first, then, we calculate the perimeter.

width = 7 length = 14 Perimeter = $7 + 7 + 14 + 14 = 42\ \text{m}$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Martin and Jason have a 39-year age gap between them. If Marten is four times Jason's age, how old is Jason?

Jason's age = x

Martin's age = $4x$

Age gap = 39

substitute known age gap → Age gap = $4x - x$

$39 = 4x - x$

$\div 3$ ↓

$39 = 3x$

$13 = x$

Jason's age is 13 years old.

6. Amelia and Peter are preparing their children for school. Amelia purchases four blazers and four skirts and spends £300. Peter needs to buy one blazer and three skirts and spends £121. Use the information to calculate the price of one blazer and one skirt.

Blazer = x

Skirt = y

Equation from Amelia:

$4x + 4y = 300$ — (1)

Equation from Peter:

$x + 3y = 121$

$-3y$ ↓

$x = 121 - 3y$ — (2)

Substitute Peter's equation into Amelia's equation:

$4(121 - 3y) + 4y = 300$

$484 - 12y + 4y = 300$

$484 - 8y = 300$

-300 ↓

$484 - 300 = 8y$

$184 = 8y$

$\div 8$ ↓

$23 = y$

Substitute value of y to find x

$x = 121 - 3y$

$= 121 - 3(23)$

$= 121 - 69 = 52$

One blazer costs £52 and one skirt costs £23

7. Polly thinks of a number. She doubles the number and adds six.

   Queenie thinks of a different number. She multiplies the number by eight and subtracts six.

   Polly and Queenie both end on the same number. The number Polly starts with is twice Queenie's.

   Use the information to find what number Polly and Queenie started with.

starting number for Polly = x

starting number for Queenie = y

Equation 1 → both end up having the same number at the end

$2x + 6 = y \times 8 - 6$

-6: $2x + 6 = 8y - 6$

$2x = 8y - 12$

$\div 2$: $x = 4y - 6$

Equation 2

$x = 2y$ ← starting number for Polly is twice Queenie's

Substitute one equation into the other:

$x = 4y - 6$

$2y = 4y - 6$

$0 = 4y - 2y - 6$

$6 = 2y$

$3 = y$

Find x

$x = 2(3) = 6$

Polly's starting number is 6 while Queenie's is 3.

8. Kelly and Sam take their 2 children to play crazy golf. The total cost is £33. Billy takes his 3 children to play crazy golf and the total cost is £27.50. Calculate the price of a child's ticket and an adult's ticket.

Price for 1 adult ticket = y

Price for 1 child ticket = x

Equation from Kelly and Sam

$2y + 2x = 33$

$2x = 33 - 2y$

$x = \frac{33}{2} - y$

Equation from Billy

$y + 3x = 27.50$

$y = 27.50 - 3x$

Substitute Billy's equation into Kelly's

$x = \frac{33}{2} - y$

$x = \frac{33}{2} - (27.5 - 3x)$

$x = \frac{33}{2} - 27.5 + 3x$

$-2x = \frac{33}{2} - 27.5$

$-2x = -11$

$x = \frac{11}{2} = 5.5$

Find y

$y = 27.5 - 3x$

$y = 27.5 - 3(5.5)$

$= 27.5 - 16.5$

$= 11$

The price for 1 adult's ticket is £11 and the price for 1 child's ticket is £5.50