GCSE Maths - Algebra

Solving Linear Equations

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions involving linear equations. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Solve the equation $4x + 9 = -3$

Step 1: Rearrange the equation to ensure only terms containing the unknown are present on the left-hand side.

We want to rearrange the equation so that only terms containing x are present on the left. This means any whole number present on the left-hand side should be eliminated.

Subtract 9 from both sides of the equation:

$$4x + 9 = -3$$
$$4x = -12$$

Step 2: Eliminate the coefficient of the unknown by dividing both sides of the equation by the coefficient of the unknown variable.

Divide both sides by 4:

$$4x = -12$$
$$x = -3$$

Step 3: Check the answer by substituting the value of the unknown back into the original equation. The value of the left-hand side should be equal to the value of the right-hand side.

Substitute $x = -3$:

$$4x + 9 = 4(-3) + 9 = -3$$
$$-3 = -3$$

Hence, the final answer is $x = -3$.

Guided Example

Solve the equation $5 - 2y = -11$

Step 1: Rearrange the equation to ensure only terms containing the unknown are present on the left-hand side.

$5 - 2y = -11$ (-5) (-5)
$-2y = -16$

Step 2: Eliminate the coefficient of the unknown by dividing both sides of the equation by the coefficient of the unknown variable.

The coefficient of the unknown is -2:

$-2y = -16$ (÷-2) (÷-2)
$y = 8$

Step 3: Check the answer by substituting the value of the unknown back into the original equation.

CHECK: $5 - 2(8) = 5 - 16 = -11$ ✓

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

1. Solve the following linear equations for x:

a) $3x - 4 = 11$

$3x - 4 = 11$ (+4) (+4)

$3x = 15$ (÷3) (÷3)

$x = 5$

CHECK: $3(5) - 4 = 15 - 4 = 11$ ✓

b) $4 - 8x = 20$

$4 - 8x = 20$ (-4) (-4)

$-8x = 16$ (÷-8) (÷-8)

$x = -2$

CHECK: $4 - 8(-2) = 4 + 16 = 20$ ✓

c) $10 - x = 4$

$10 - x = 4$ (-10) (-10)

$-x = -6$ (÷-1) (÷-1)

$x = 6$

CHECK: $10 - (6) = 4$ ✓

d) $2x + 9 = 7$

$2x + 9 = 7$ (-9) (-9)

$2x = -2$ (÷2) (÷2)

$x = -1$

CHECK: $2(-1) + 9 = -2 + 9 = 7$ ✓

Section B

Worked Example

Solve the equation $5x - 12 = 3x + 4$

Step 1: Rearrange the equation to ensure only terms containing the unknown are present on the left-hand side.

$$5x - 12 = 3x + 4$$

Subtract $3x$ from both sides of the equation:

$$5x - 12 - 3x = 3x + 4 - 3x$$

$$2x - 12 = 4$$

Eliminate the constant term on the same side as the unknown.

$$2x - 12 = 4$$

Add 12 to both sides of the equation:

$$2x - 12 + 12 = 4 + 12$$

$$2x = 16$$

Step 2: Eliminate the coefficient of the unknown by dividing both sides of the equation by the coefficient of the unknown variable.

$$2x = 16$$

Divide both sides of the equation by 2:

$$x = 8$$

Step 3: Check the answer by substituting the value of the unknown back into the original equation.

$$5x - 12 = 3x + 4$$

$$5(8) - 12 = 3(8) + 4$$

$$28 = 28$$

Hence, the final answer is $\boldsymbol{x = 8}$.

Guided Example

Solve the equation $3x - 15 = 7x + 5$

Step 1: Rearrange the equation to ensure only terms containing the unknown are present on the left-hand side.

$3x - 15 = 7x + 5$

$(-7x)$ $(-7x)$

$3x - 7x - 15 = 5$

$(+15)$ $(+15)$

$3x - 7x = 5 + 15$

Collect like terms ↓

$-4x = 20$

Step 2: Eliminate the coefficient of the unknown by dividing both sides of the equation by the coefficient of the unknown variable.

$-4x = 20$

$(\div -4)$ $(\div -4)$

$x = -5$

Step 3: Check the answer by substituting the value of the unknown back into the original equation.

CHECK: $3x - 15 = 7x + 5$

$3(-5) - 15 = 7(-5) + 5$

$-15 - 15 = -35 + 5$

$-30 = -30$ ✓

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

2. Solve the following linear equations for x:

a) $2 + 5x = 6 - 3x$

$2 + 5x = 6 - 3x$

$(+3x)$ $(+3x)$

$2 + 5x + 3x = 6$

(-2) (-2)

$5x + 3x = 6 - 2$

$8x = 4$

$(\div 8)$ $(\div 8)$

$x = \frac{4}{8} = \frac{1}{2}$

CHECK: $2 + 5(\frac{1}{2}) = 6 - 3(\frac{1}{2})$

$\frac{9}{2} = \frac{9}{2}$ ✓

b) $2x - 15 = 5x - 21$

$2x - 15 = 5x - 21$

$(-5x)$ $(-5x)$

$2x - 5x - 15 = -21$

$(+15)$ $(+15)$

$2x - 5x = 15 - 21$

$-3x = -6$

$(\div -3)$ $(\div -3)$

$x = 2$

CHECK: $2(2) - 15 = 5(2) - 21$

$-11 = -11$ ✓

c) $5x + 15 = 4x + 5$

$5x + 15 = 4x + 5$

$(-4x)$ $(-4x)$

$5x - 4x + 15 = 5$

(-15) (-15)

$5x - 4x = 5 - 15$

$x = -10$

CHECK: $5(-10) + 15 = 4(-10) + 5$

$-35 = -35$ ✓

d) $-8 - 9x = 28 - 3x$

$-8 - 9x = 28 - 3x$

$(+3x)$ $(+3x)$

$-8 - 9x + 3x = 28$

$(+8)$ $(+8)$

$-9x + 3x = 8 + 28$

$-6x = 36$

$(\div -6)$ $(\div -6)$

$x = -6$

CHECK: $-8 - 9(-6) = 28 - 3(-6)$

$46 = 46$ ✓

Section C

Worked Example

Solve the equation $x - 1 = \frac{1}{4}(2x + 14)$

Step 1: Expand any brackets present in the equation.

$$x - 1 = \frac{1}{4}(2x + 14)$$

$$x - 1 = \frac{2x + 14}{4}$$

Step 2: Eliminate any fractions by multiplying both sides of the equation by the greatest common denominator. Expand any brackets that appear.

Here, the common denominator is 4 so we multiply both sides by 4:

$$4(x - 1) = 4\left(\frac{2x + 14}{4}\right)$$
$$4x - 4 = 2x + 14$$

Step 3: Rearrange the equation to ensure only terms containing the unknown are present on the left-hand side.

$$4x - 4 = 2x + 14$$

Subtract $2x$ from both sides of the equation:

$$2x - 4 = 14$$

Add 4 to both sides of the equation to eliminate the constant term on the left:

$$2x = 18$$

Step 4: Eliminate the coefficient of the unknown by dividing both sides of the equation by the coefficient of the unknown variable.

Divide both sides of the equation by 2:

$$2x = 18$$
$$x = 9$$

Step 5: Check the answer by substituting the value of the unknown back into the original equation.

$$x - 1 = \frac{1}{4}(2x + 14)$$
$$9 - 1 = \frac{1}{4}(2(9) + 14)$$
$$8 = 8$$

Hence, the final answer is $\boldsymbol{x = 9}$.

Guided Example

Solve the equation $\frac{3}{2}(3x - 2) = 3x + 5$

Step 1: Expand any brackets present in the equation.

$$\frac{3}{2}(3x-2) = 3x+5$$

$$\frac{9x-6}{2} = 3x+5$$

Step 2: Eliminate any fractions by multiplying both sides of the equation by the greatest common denominator. Expand any brackets that appear.

$$9x-6 = 2(3x+5)$$

$$9x-6 = 6x+10$$

Step 3: Rearrange the equation to ensure only terms containing the unknown are present on the left-hand side.

$$9x-6 = 6x+10$$

$$9x-6x = 6+10$$

$$3x = 16$$

Step 4: Eliminate the coefficient of the unknown by dividing both sides of the equation by the coefficient of the unknown variable.

$$3x = 16$$

$$x = \frac{16}{3}$$

Step 5: Check the answer by substituting the value of the unknown back into the original equation.

$$\frac{3}{2}(3x-2) = 3x+5$$

$$\frac{3}{2}\left(3\left(\frac{16}{3}\right)-2\right) = 3\left(\frac{16}{3}\right)+5$$

$$\frac{3}{2}(16-2) = 16+5$$

$$21 = 21 \quad \checkmark$$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

3. Solve the following linear equations for x:

a) $\frac{1}{5}x = x - 2$

$\frac{1}{5}x = x - 2$

$x = 5(x-2)$

$x = 5x - 10$

$-4x = -10$

$x = \frac{-10}{-4}$

$x = \frac{5}{2}$

CHECK: $\frac{1}{5}\left(\frac{5}{2}\right) = \frac{5}{2} - 2$

$\frac{1}{2} = \frac{1}{2}$ ✓

b) $2x - 6 = \frac{2}{3}x + 4$

$2x - 6 = \frac{2}{3}x + 4$

$2x - 6 - 4 = \frac{2}{3}x$

$2x - 10 = \frac{2}{3}x$

$3(2x-10) = 2x$

$6x - 30 = 2x$

$6x - 2x = 30$

$4x = 30$

$x = \frac{30}{4}$

$x = \frac{15}{2}$

CHECK: $2\left(\frac{15}{2}\right) - 6 = \frac{2}{3}\left(\frac{15}{2}\right) + 4$

$15 - 6 = 5 + 4$

$9 = 9$ ✓

c) $\frac{(4+3x)}{5} - \frac{(x+4)}{2} = \frac{3}{2}$

$\frac{(4+3x)}{5} - \frac{(x+4)}{2} = \frac{3}{2}$

(x5) (x5)

$(4+3x) - 5\frac{(x+4)}{2} = \frac{3}{2} \times 5$

(x2) (x2)

Expand brackets ↓

$2(4+3x) - 5(x+4) = 3 \times 5$

$8 + 6x - 5x - 20 = 15$

$6x - 5x = 15 + 20 - 8$

$x = 27$

CHECK: $\frac{4+3(27)}{5} - \frac{(27+4)}{2} = \frac{3}{2}$

$\frac{4+81}{5} - \frac{31}{2} = \frac{3}{2}$

$\frac{85}{5} - \frac{31}{2} = \frac{3}{2}$

$17 - \frac{31}{2} = \frac{3}{2}$

$\frac{3}{2} = \frac{3}{2}$ ✓

d) $\frac{(x-3)}{3} + \frac{(2x+4)}{2} = 5$

$\frac{(x-3)}{3} + \frac{(2x+4)}{2} = 5$

$(x-3) + 3\frac{(2x+4)}{2} = 3 \times 5$

$2(x-3) + 3(2x+4) = 2 \times 3 \times 5$

$2x - 6 + 6x + 12 = 30$

$8x = 30 + 6 - 12$

$8x = 24$

$x = 3$

CHECK: $\frac{(3-3)}{3} + \frac{(2(3)+4)}{2} = 5$

$0 + \frac{6+4}{2} = 5$

$\frac{10}{2} = 5$

$5 = 5$ ✓

Section D

Worked Example

A graph has equation $y = 5x + 4$.
Using a graph, determine the approximate value of x when $y = 2$.

Step 1: If the graph is not given, we need to sketch the graph. We can do this by determining the x –intercept and the y –intercept.

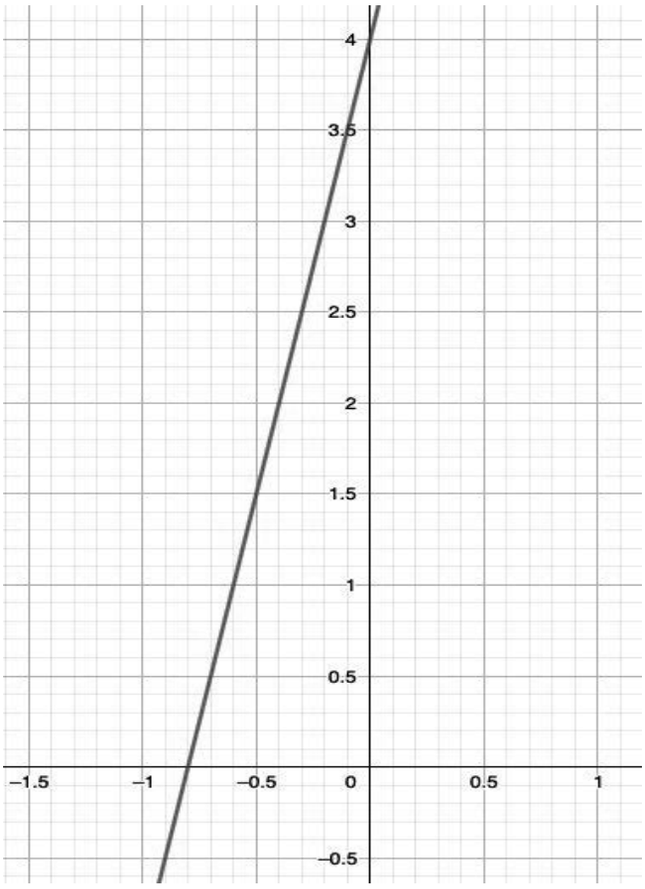
With the use of the normal form of a straight-line equation, $y = mx + c$, where m is the gradient and c is the y –intercept. We can determine both values from the given equation:

$$y = mx + c$$
$$y = 5x + 4$$

The gradient of the line is $m = 5$ and the y –intercept is $c = 4$.
To determine the x –intercept, substitute in $y = 0$:

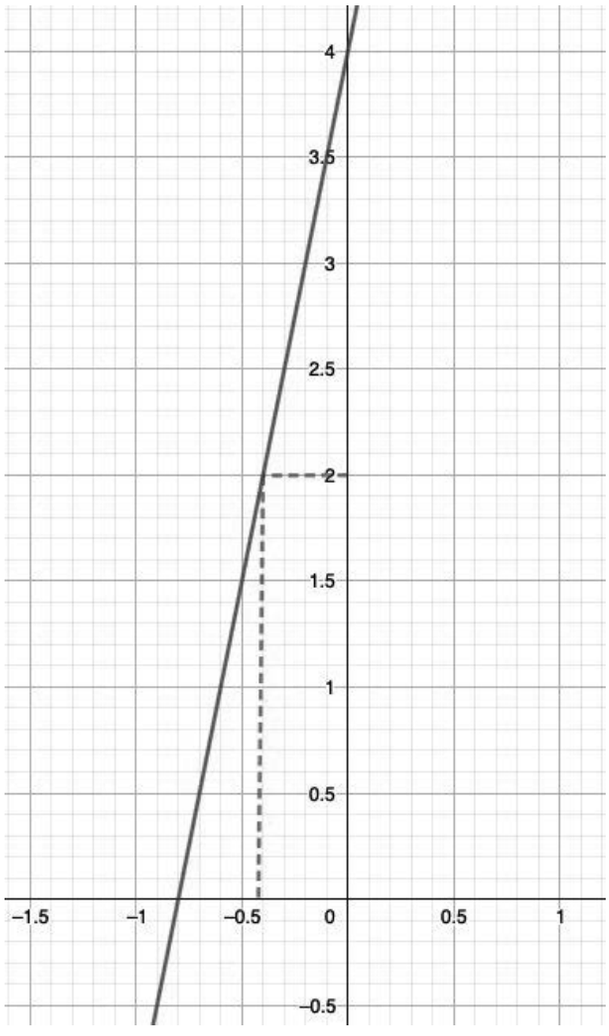
$$0 = 5x + 4$$
$$5x = -4$$
$$x = -\frac{4}{5}$$

Step 2: Using the coordinates of the y –intercept and the x –intercept, sketch the linear graph.



Step 3: Draw a dotted line on the graph from the value of y that is required by the question to find the approximate value of x.

In this case, since the question asks for the value of x when $y = 2$, we draw a dotted line from $y = 2$.



Step 4: Determine the approximate value of x from the dotted line.

From the graph, we identify where the red dotted line intersects the x-axis. This corresponds

$$x = -0.4$$

Hence, the approximate value of x when $y = 2$ is $\boldsymbol{x = -0.4}$.

Guided Example

A graph has equation $y = 3 - 2x$.
Using a graph, determine the approximate value of x when $y = 2$.

Step 1: If the graph is not given, we need to sketch the graph. We can do this by first determining the x –intercept and the y –intercept.

$y = mx + c$

$y = -2x + 3$

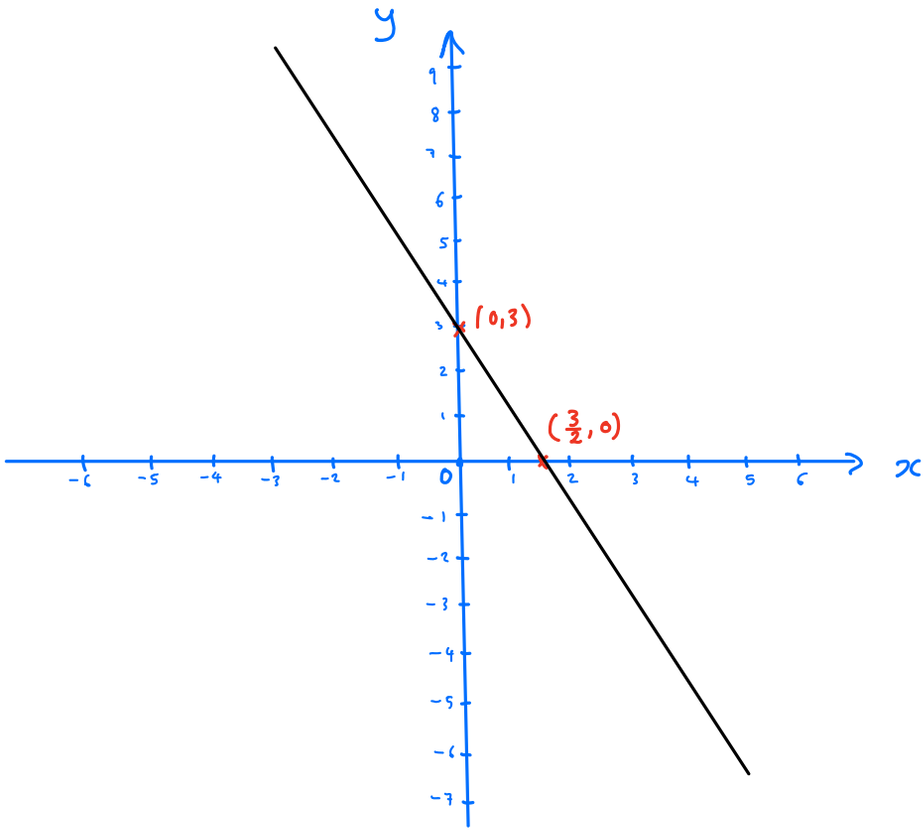
y-intercept: $(0, c) = (0, 3)$

x-intercept: when $y = 0$, $3 - 2x = 0$

$-2x = -3$

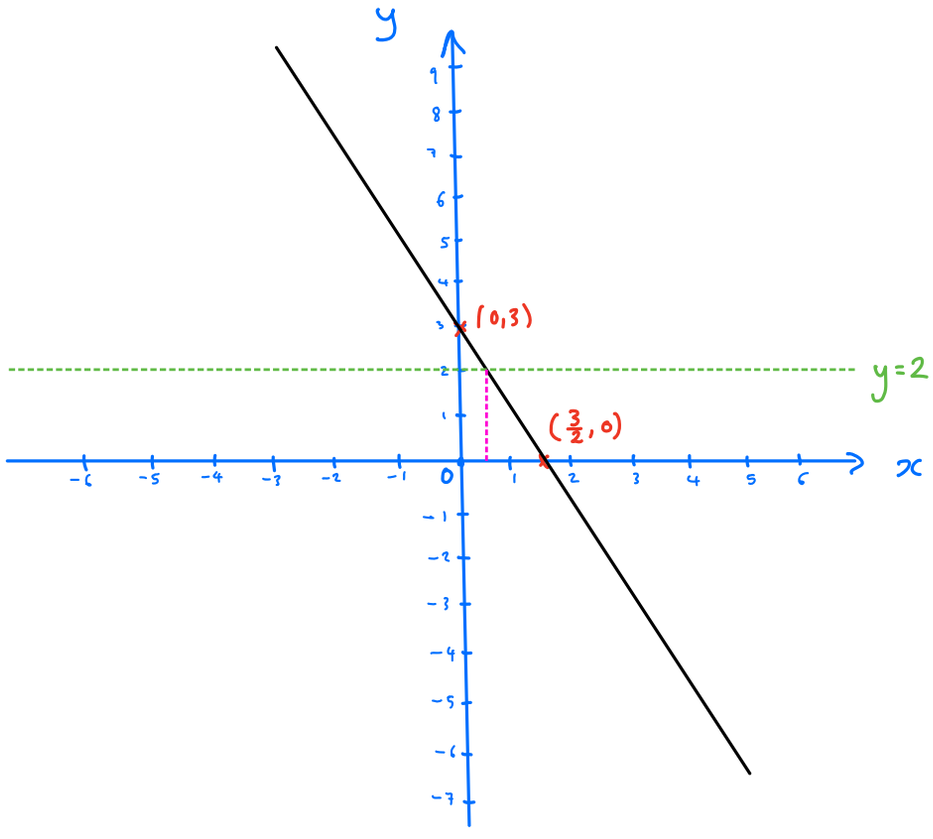
$x = \frac{3}{2}$ $\rightarrow$ $(\frac{3}{2}, 0)$

Step 2: Using the coordinates of the y –intercept and the x –intercept, sketch the linear graph.



Step 3: Draw a dotted line on the graph from the value of y that is required by the question to find the approximate value of x.

We are interested in the value of x when $y = 2$ so we draw a dotted line at $y = 2$.



Step 4: Determine the approximate value of x from the dotted line.

Drawing a dotted line from where the line $y = 3 - 2x$ intersects $y = 2$ down to the x-axis: (see line --------- on graph above).

We estimate $x = 0.5$ when $y = 2$.

Now it's your turn!

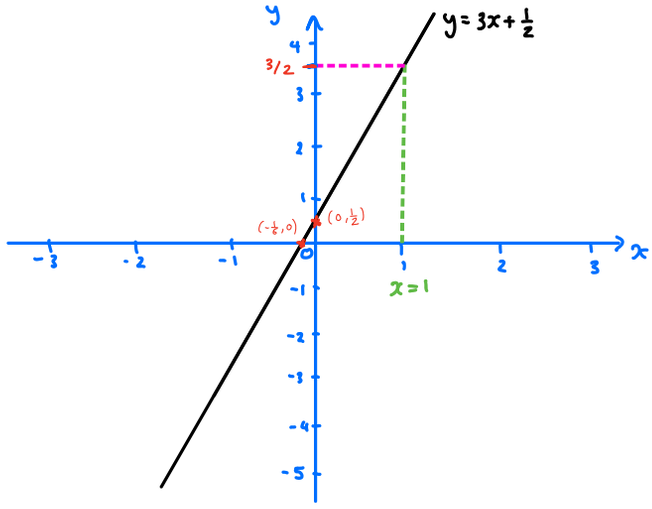
If you get stuck, look back at the worked and guided examples.

4. With the use of a graph, perform the following estimations:

a) The equation of a line is given by $y = 3x + \frac{1}{2}$.
Estimate the value of y when $x = 1$.

$y = mx + c$
$y = 3x + \frac{1}{2}$

y-intercept: $(0, 1/2)$
x-intercept: when $y = 0$, $3x + \frac{1}{2} = 0$
$x = -\frac{1}{6} \rightarrow (-\frac{1}{6}, 0)$

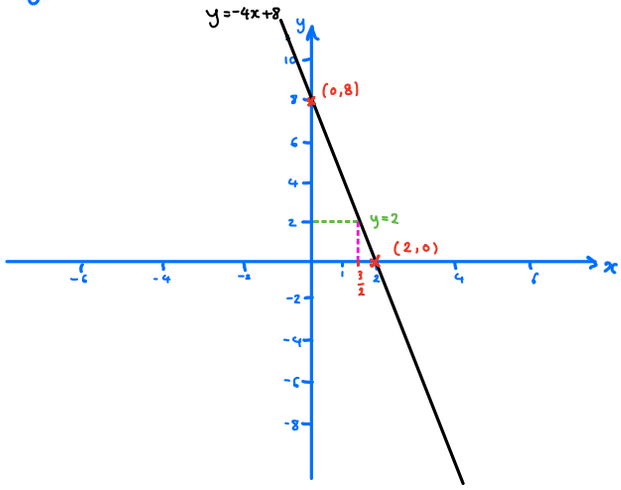


From the graph, we see that when $x = 1$, $y = \frac{3}{2}$.

b) The equation of the line is given by $y - 6 = 2 - 4x$.
Estimate the value of x when $y = 2$.

Rearrange equation of the line into the form $y = mx + c$:
$y - 6 = 2 - 4x$
$y = -4x + 2 + 6$
$y = -4x + 8$

y-intercept: $(0, c) = (0, 8)$
x-intercept: when $y = 0$, $-4x + 8 = 0$
$-4x = -8$
$x = 2 \rightarrow (2, 0)$



From the graph, we see that when $y = 2$, $x = \frac{3}{2}$.