GCSE Maths – Algebra

Functions

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of function questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Find the value of $g(x) = x^3 + 2x^2 - 9x + 1$ when $x = 3$.

Step 1: Substitute $x = 3$ into the function $g(x)$. Replace every x in the function with the numerical value 3.

$$g(3) = (3)^3 + 2(3)^2 - 9(3) + 1$$

Step 2: Simplify where possible by multiplying out the brackets and applying powers.

$$27 + 2(9) - 27 + 1 = 27 + 18 - 27 + 1$$

Step 3: Complete the final sum.

$$27 + 18 - 27 + 1 = 18 + 1 = 19$$

Answer: $\mathbf{g(3) = 19}$

Guided Example

If $f(x) = (x + 7)(6 - x^2)(4 + x)$ what is the value of $f(4)$?

Step 1: Substitute $x = 4$ into the function $f(x)$.

$$f(4) = (4 + 7)(6 - 4^2)(4 + 4)$$

Step 2: Simplify each term where possible.

$$f(4) = (11) \times (6 - \overset{4^2}{16}) \times (8)$$

$$= 11 \times -10 \times 8$$

Step 3: Compute the final sum.

$$f(4) = \underset{11 \times 8}{88} \times -10 = -880$$

$$f(4) = -880$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Complete the following:

a) Find the value of $f(5)$ when $f(x) = 3x - 2$.

$f(5) = 3(5) - 2$
$= 15 - 2$
$= 13$

$f(5) = 13$

b) Find the value of $g(7)$ when $g(x) = 4x - x^2$.

$g(7) = 4(7) - (7)^2$ ← $7 \times 7 = 49$
$= 28 - 49$
$= -21$

$g(7) = -21$

c) Find the value of $h(9)$ when $h(a) = \sqrt{a} - 2a^3 + 1$.

$h(9) = \sqrt{9} - 2(9)^3 + 1$
$= 3 - (2 \times 729) + 1$
$= 4 - 1458 = -1454$

$9^3 = 9 \times 9 \times 9 = 81 \times 9$
$= 729$

$h(9) = -1454$

d) Find the value of $f(-8)$ when $f(x) = 5x + 3(\sqrt[3]{x}) - 2x^2$.

$f(-8) = 5(-8) + 3(\sqrt[3]{-8}) - 2(-8)^2$ ← $-8 \times -8 = 64$

$5 \times -8 = -40$

$-2^3 = -8$

$= -40 + 3(-2) - 2(64)$
$= -40 - 6 - 128$
$= -46 - 128 = -174$

$f(-8) = -174$

2. Solve the following for x:

a) $h(x) = x^2 - 5x + 2,\ h(x) = -4$.

$h(x) = x^2 - 5x + 2$
$x^2 - 5x + 2 = -4$
$+4 \quad +4$
$x^2 - 5x + 6 = 0$
then factorise to find x

$(x-3)(x-2) = x^2 - 5x + 6$
$x^2 \quad -3x \quad -2x \quad +6$

$x = 3$
$x = 2$

b) $f(x) = 18x - 2,\ f(x) = 6$.

$f(x) = 18x - 2$
$18x - 2 = 6$
$+2 \quad +2$
$18x = 8$

$\frac{18x}{18} = \frac{8}{18}$

$x = \frac{4}{9}$

3. If $g(x) = x^3 - 2(\sqrt[4]{x}) + 1$ what is an expression for $g(16x)$?

$g(x) = x^3 - 2(\sqrt[4]{x}) + 1$

$g(16x) = (16x)^3 - 2(\sqrt[4]{16x}) + 1$

$16 \times 16 \times 16 = 4096$

$16 = 2^4$ and -2^4

$g(16x) = 4096x^3 - 2 \times \pm 2(\sqrt[4]{x}) + 1$

$g(16x) = 4096x^3 \pm 4(\sqrt[4]{x}) + 1$

Section B – Higher Only

Worked Example

Find the inverse function of $f(x) = 2x + 1$.

Step 1: Replace $f(x)$ with y.

$$y = 2x + 1$$

Step 2: Rearrange the terms to make x the subject of the equation.

$$y - 1 = 2x$$

$$\frac{y-1}{2} = x$$

Step 3: Replace x with y and y with $f^{-1}(x)$.

$$\frac{x-1}{2} = f^{-1}(x)$$

Answer: $f^{-1}(x) = \frac{x-1}{2}$

Guided Example

Find the inverse function of $h(x) = \frac{2-x}{x}$.

Step 1: Replace $h(x)$ with y.

$$h(x) = \frac{2-x}{x} \qquad y = \frac{2-x}{x}$$

Step 2: Make x the subject of the equation.

$$\times x \quad y = \frac{2-x}{x} \quad \times x$$

$$yx = 2 - x$$
$$+x \qquad +x$$

$$yx + x = 2$$

take x out as a factor

$$x(y+1) = 2$$
$$\div y+1 \qquad \div y+1$$

$$x = \frac{2}{y+1}$$

Step 3: Replace x with y and y with $h^{-1}(x)$.

$$x = \frac{2}{y+1}$$

becomes

$$h^{-1}(x) = \frac{2}{x+1}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

4. Find the inverse of the following functions:

a) $f(x) = 4x + 7$

$y = 4x + 7$
$-7 \quad -7$

$4x = y - 7$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{y-7}{4}$

$x = \frac{y-7}{4}$

$\boxed{f^{-1}(x) = \frac{x-7}{4}}$

b) $g(x) = 15x^2 + 3$

$y = 15x^2 + 3$
$-3 \quad -3$

$15x^2 = y - 3$

$\frac{\cancel{15}x}{\cancel{15}} = \frac{y-3}{15}$

$x^2 = \frac{y-3}{15}$

$x = \pm\sqrt{\frac{y-3}{15}}$

$\boxed{g^{-1}(x) = \pm\sqrt{\frac{x-3}{15}}}$

c) $f(x) = \frac{2x}{3+x}$

$y = \frac{2x}{3+x}$
$\times 3+x \quad \times 3+x$

$3y + yx = 2x$
$-3y \quad -3y$

$yx = 2x - 3y$
$-2x \quad -2x$

$yx - 2x = -3y$
take x out as a factor

$x(y-2) = -3y$
$\div y-2 \quad \div y-2$

$x = \frac{-3y}{y-2}$

$\boxed{f^{-1}(x) = \frac{-3x}{x-2}}$

d) $h(x) = \frac{4-3x}{x+3}$

$y = \frac{4-3x}{x+3}$
$\times x+3 \quad \times x+3$

$yx + 3y = 4 - 3x$
$-3y \quad -3y$

$yx = 4 - 3x - 3y$
$+3x \quad +3x$

$yx + 3x = 4 - 3y$
take x out as a factor

$x(y+3) = 4 - 3y$
$\div y+3 \quad \div y+3$

$x = \frac{4-3y}{y+3}$

$\boxed{h^{-1}(x) = \frac{4-3x}{x+3}}$

5. Find the value of $f(x) = x^2 - 3$ if $x = f^{-1}(6)$.

find $f^{-1}(x)$ → $y = x^2 - 3$
$\quad +3 \quad +3$

$x^2 = y + 3$

$x = \sqrt{y+3}$

$f^{-1}(x) = \sqrt{x+3}$

$f^{-1}(6) = \sqrt{6+3} = \sqrt{9}$

$x = 3$

$f(3) = (3)^2 - 3$

$= 9 - 3$

$\boxed{f(x) = 6}$

6. Find the value of $g(x) = \frac{9-9x^2}{x^2}$ if $x = g^{-1}(7)$.

find $g^{-1}(x)$ → $y = \frac{9-9x^2}{x^2}$
$\times x^2 \quad \times x^2$

$yx^2 = 9 - 9x^2$
$+9x^2 \quad +9x^2$

$yx^2 + 9x^2 = 9$

take x^2 out as a factor

$x^2(y+9) = 9$
$\div y+9 \quad \div y+9$

$x^2 = \frac{9}{y+9}$

$x = \sqrt{\frac{9}{y+9}}$

$g^{-1}(x) = \sqrt{\frac{9}{x+9}}$

$g^{-1}(7) = \sqrt{\frac{9}{7+9}}$

$= \frac{3}{4}$

expand and cancel out

$g\left(\frac{3}{4}\right) = \frac{9 - 9\left(\frac{3}{4}\right)^2}{\left(\frac{3}{4}\right)^2}$

$= \frac{63}{9}$

$= 7$

$\boxed{g(x) = 7}$

7. Solve $h(x) = 3x^2 + 6$ for x given $h^{-1}(x) = 2$.

find $h^{-1}(x)$ → $y = 3x^2 + 6$
$-6 \quad -6$

$3x^2 = y - 6$
$\div 3 \quad \div 3$

$x^2 = \frac{y-6}{3}$

$x = \sqrt{\frac{y-6}{3}}$

$h^{-1}(x) = \sqrt{\frac{x-6}{3}}$

$\sqrt{\frac{x-6}{3}} = 2$

square both sides

$\frac{x-6}{3} = 4$
$\times 3 \quad \times 3$

$x - 6 = 12$
$+6 \quad +6$

$x = 18$

$18 \times 18 = 324$

$h(x) = 3(18)^2 + 6$

$= (3 \times 324) + 6$

$= 972 + 6$

$\boxed{h(x) = 978}$

8. If $f^{-1}(x) = 5x^2 - 3$ what is $f(x)$?

inverse of $f^{-1}(x) = f(x)$

$y = 5x^2 - 3$
$+3 \quad +3$

$5x^2 = y + 3$
$\div 5 \quad \div 5$

$x^2 = \frac{y+3}{5}$

$x = \sqrt{\frac{y+3}{5}}$

$\boxed{f(x) = \sqrt{\frac{x+3}{5}}}$

Section C – Higher Only

Worked Example

Given $f(x) = 2x - 7$ and $g(x) = 4x + 5$ find $fg(x)$.

Step 1: Place the innermost function $g(x)$ into the outermost function $f(x)$ in place of x.

$$fg(x) = 2g(x) - 7 = 2(4x + 5) - 7$$

Step 2: Simplify by expanding the brackets and collecting like terms.

$$fg(x) = 2(4x + 5) - 7 = 8x + 10 - 7$$

$$fg(x) = 8x + 3$$

Answer: $\boldsymbol{fg(x) = 8x + 3}$

Worked Example

Given $f(x) = x^2 - 7x$ and $g(x) = 3x - 3$ find $fg(2)$.

Step 1: Place the innermost function $g(x)$ into the outermost function $f(x)$ in place of x.

$$fg(x) = (3x - 3)^2 - 7(3x - 3)$$

Step 2: Simplify by expanding the brackets and collecting like terms.

$$fg(x) = (3x - 3)(3x - 3) - 7(3x - 3)$$

$$fg(x) = (9x^2 - 9x - 9x + 9) - 7(3x - 3)$$

$$fg(x) = 9x^2 - 18x + 9 - 21x + 21 = 9x^2 - 39x + 30$$

Step 3: Substitute $x = 2$ into $fg(x)$ to find $fg(2)$.

$$fg(2) = 9(2)^2 - 39(2) + 30$$

$$fg(2) = -12$$

Answer: $\boldsymbol{fg(2) = -12}$

Guided Example

Given $f(x) = 5x + 3$ and $g(x) = 3x^2 + 12$, find $gf(x)$.

Step 1: Place the innermost function $f(x)$ into the outermost function $g(x)$ in place of x.

$$gf(x) = 3(5x+3)^2 + 12$$

put $f(x)$ in place of x in $g(x)$

Step 2: Simplify by expanding the brackets and collecting like terms.

$$gf(x) = 3\underbrace{(5x+3)^2}_{\text{expand}} + 12 = 3(25x^2 + 30x + 9) + 12$$

$(5x+3)(5x+3)$

$+25x^2 \quad +15x \quad +15x \quad +9$

$3 \times 25 = 75\ x^2$
$3 \times 30 = 90\ x$
$3 \times 9 = 27$

$$= 75x^2 + 90x + 27 + 12$$

$$gf(x) = 75x^2 + 90x + 39$$

Guided Example

Given $f(x) = \frac{x^2-3}{x}$ and $g(x) = 3x^2 + 12$, find $fg(6)$.

Step 1: Place the innermost function $g(x)$ into the outermost function $f(x)$ in place of x.

$$fg(x) = \frac{(3x^2+12)^2 - 3}{3x^2+12}$$

Substitute $g(x)$ as x in $f(x)$

Step 2: Simplify by expanding the brackets and collecting like terms.

$$fg(x) = \frac{(3x^2+12)^{\cancel{2}}}{\cancel{3x^2+12}} - \frac{\cancel{3}\,1}{\cancel{3}x^2+\cancel{12}\,4}$$

split equation into two

$$= 3x^2 + 12 - \frac{1}{x^2+4}$$

Step 3: Substitute $x = 6$ into $fg(x)$ to find $fg(6)$.

$$fg(6) = 3(6)^2 + 12 - \frac{1}{(6)^2+4}$$

$$= \underbrace{(3 \times 36)}_{108} + 12 - \frac{1}{\underbrace{36+4}_{40}}$$

$$= \underbrace{108 + 12}_{120} - \frac{1}{40}$$

$$= 120 - \frac{1}{40}$$

$$120 = \frac{120 \times 40}{40} = \frac{4800}{40}$$

$4800 - 1 = 4799$

$$fg(6) = \frac{4800}{40} - \frac{1}{40}$$

$$fg(6) = \frac{4799}{40}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

8. Find $fg(x)$, given $f(x) = 3(x-4)$ and $g(x) = \frac{x}{5} + 1$.

$fg(x) = 3\left(\left(\frac{x}{5}+1\right) - 4\right)$

g(x) as x in f(x)

$fg(x) = 3\left(\frac{x}{5}+1\right) - 3(4)$

expand out brackets

$= \frac{3x}{5} + 3 - 12$

$= \frac{3x}{5} - 9$

$fg(x) = \frac{3x}{5} - 9$

9. Find $gf(x)$, given $g(x) = 3x + 5$ and $f(x) = \frac{1}{3}x - \frac{5}{2}$.

$gf(x) = 3\left(\frac{1}{3}x - \frac{5}{2}\right) + 5$

f(x) as x in g(x)

$gf(x) = 3\left(\frac{1}{3}x\right) - 3\left(\frac{5}{2}\right) + 5$

expand brackets

$= x - \frac{15}{2} + 5$

$= x - \frac{15}{2} + \frac{10}{2} = x - \frac{5}{2}$

$-15 + 10 = 5$

$gf(x) = x - \frac{5}{2}$

10. Given $f(x) = 1 - 2x^3$ and $g(x) = \frac{3}{x} - 4$, show that $gf(x) = \frac{8x^3-1}{1-2x^3}$.

$gf(x) = \frac{3}{(1-2x^3)} - 4$

f(x) as x in g(x)

$= \frac{3 - 4(1-2x^3)}{1-2x^3}$

expand

$4 \times 1 = 4$
$4 \times 2x^3 = 8x^3$

$= \frac{8x^3 - 4 + 3}{1-2x^3}$

$gf(x) = \frac{8x^3 - 1}{1-2x^3}$

11. Given $f(x) = 4x + 6$ and $g(x) = x^2 - 9$, find the value of $fg(3)$.

$fg(x) = 4(x^2-9)+6$

g(x) as x in f(x)

expand

$4 \times x^2 = 4x^2$
$-9 \times 4 = -36$

$= 4x^2 - 36 + 6$

$= 4x^2 - 30$

so when $x = 3$

$fg(3) = 4(3)^2 - 30$

$= 4 \times 9 - 30$

$= 36 - 30$

$= 6$

$fg(3) = 6$

12. Given $f(x) = x^2 - 4$ and $g(x) = 4x - 1$, find the value of $fg(x-2)$.

$g(x-2) = 4(x-2) - 1$

use $x-2$ as x

$= 4x - 8 - 1$

$= 4x - 9$

expand

$(4x-9)(4x-9)$

$4x \times 4x = 16x^2$

$-9 \times 4x = -36x \leftarrow \times 2$

$-9 \times -9 = 81$

$fg(x-2) = (4x-9)^2 - 4$

$= 16x^2 - 36x - 36x + 81 - 4$

$fg(x-2) = 16x^2 - 72x + 77$

13. Given $g(x) = \frac{5}{4x}$ and $f(x) = 7x^2 + 3x - 2$, find the value of $gf(4)$.

$gf(x) = \dfrac{5}{4(7x^2+3x-2)}$ ← $f(x)$ as x

expand

$4 \times 7x^2 = 28x^2$

$4 \times 3x = 12x$

$4 \times -2 = -8$

$= \dfrac{5}{28x^2 + 12x - 8}$

$gf(4) = \dfrac{5}{28(4)^2 + 12(4) - 8}$

$28 \times 4 \times 4 = 448$ $\quad 12 \times 4 = 48$

$gf(4) = \dfrac{5}{448 + 48 - 8}$

$= \dfrac{5}{488}$

$gf(4) = \dfrac{5}{488}$

14. If $f(x) = x^2 + 1$, what is $ff(x)$?

$ff(x)$ is putting $f(x)$ inside $f(x)$

so...

expand

$(x^2+1)(x^2+1)$

$x^2 \times x^2 = x^4$

$x^2 \times 1 = x^2 \leftarrow \times 2$

$1 \times 1 = 1$

$ff(x) = (x^2+1)^2 + 1$

$= x^4 + x^2 + x^2 + 1 + 1$

$= x^4 + 2x^2 + 2$

$ff(x) = x^4 + 2x^2 + 2$

15. If $f(x) = 2 - 3x^2$ and $g(x) = 12x - 1$, which value is greater: $fg(3)$ or $gf(3)$?

$fg(x) = 2 - 3(12x-1)^2$

expand

$(12x-1)(12x-1)$

$12x \times 12x = 144x^2$

$-1 \times 12x = -12x \leftarrow \times 2$

$-1 \times -1 = 1$

$= 2 - 3(144x^2 - 24x + 1)$

$= 2 - 432x^2 + 72x - 3$

$= -432x^2 + 72x - 1$

$fg(3) = -432(3)^2 + 72(3) - 1$

$= -3888 + 216 - 1$

$= -3673$

$gf(x) = 12(2 - 3x^2) - 1$

expand

$12 \times 2 = 24$

$12 \times -3x^2 = -36x^2$

$= -36x^2 + 24 - 1$

$= -36x^2 + 23$

$gf(3) = -36(3)^2 + 23$

$= -324 + 23$

$= -301$

As $-301 > -3673$,

$gf(3)$ has the greater value.