

# **GCSE Maths – Algebra**

# **Factorising Linear and Quadratic Expressions**

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to factorise linear and quadratic expressions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

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## Section A

### Worked Example

Factorise the expression  $3y^2 + 9y$ 

Step 1: Write out the expression and work out the highest common numerical factor for the terms in the expression. Factorise the HCF outside the brackets by dividing out each term.

For the expression  $3y^2 + 9y$  the highest common factor that goes into both terms is 3:

$$3y^2 + 9y = 3(y^2 + 3y)$$

**Step 2:** Work out the highest power of y that will go into both terms. Factorise this power of y outside the brackets by dividing it out of each term.

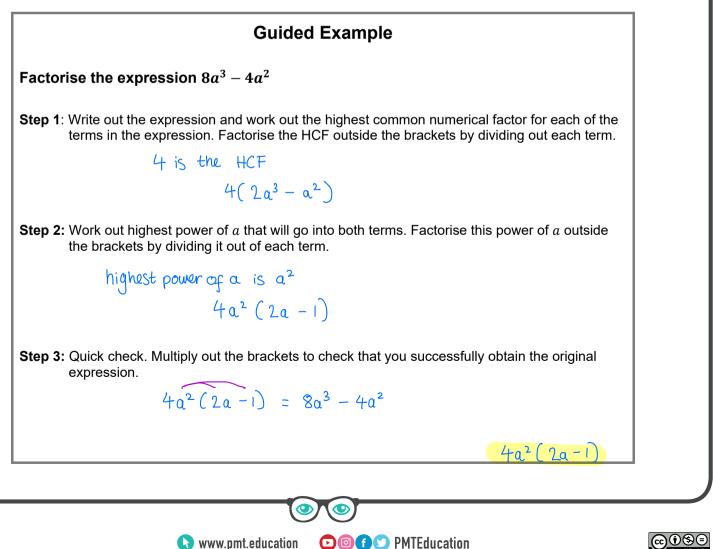
For  $3(y^2 + 3y)$ , the highest power of y that will go into both terms is y:

$$3y^2 + 9y = 3(y^2 + 3y) = 3y(y + 3)$$

So, the final factorised answer is 3y(y+3).

Step 3: Quick check. Multiply out the brackets to check that you successfully obtain the original expression.

 $3y(y + 3) = (3y \times y) + (3y \times 3) = 3y^2 + 9y$ 



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### Now it's your turn!

If you get stuck, look back at the worked and guided examples.

- 1. Factorise the following expressions:
- a) 2x + 6
  - 2 is the HCF 2(x+3)
- b) 5 10*z*

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5 is the HCF 5(1-2z)
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- c) -8p + 16p
  - 8 is the HCF 8(-p+2p)
    - P is the highest power 8p(-1+2) = 8p
- d)  $-6e^3 18e^2$ 
  - -6 is the HCF  $-6(e^3 + 3e^2)$
  - $e^2$  is the highest power  $-6e^2(e+3)$
- e)  $-7z + 21z^2$ 
  - 7 is the HCF 7(-z+3z)
  - Z is the highest power 7z(-1+3z) = 7z(3z-1)
- f)  $2x^2 xy + 714y$ 
  - No HCF
    - No highest power
      - Hence, no common factors. 2x2-xy+714y
- g)  $-120x^2yz + 3xyz^2 + 9x^2y^2z$ 
  - 3 is the HCF  $3(-40x^2yz + xyz^2 + 3x^2y^2z)$
  - The highest power of xyz is  $xyz = \frac{3xyz(-40x + z + 3xy)}{3xyz(-40x + z + 3xy)}$





- h)  $15xy + 5xyz 125x^2y$ 5 is the HCF  $5(3xy + xy^2 - 25x^2y)$  xy is the highest common power of xyz 5xy(3 + z - 25x)i)  $3p^3q - 9pq^3 - 3pq^2$ 3 is the HCF  $3(p^3q - 3pq^3 - pq^2)$  pq is the highest common power of pq  $3pq(p^2 - 3q^2 - q)$ j)  $-5xyz + 7xy + 10xy^2z$ No HCF xy is the highest common power of xyz xy(-5z + 7 + 10y 2)
- k)  $6ef + 18efg 3efg^2$ 3 is the HCF 3 (2ef + 6efg - efg<sup>2</sup>) ef is the hignest common power of efg  $3ef(2 + 6g - g^2)$

I)

$$-48ab^{2}c^{3} + 12b^{2}c^{3} - 18a^{2}b^{3}c^{2}$$

$$-6 \text{ is the HCF} - 6(8ab^{2}c^{3} - 2b^{2}c^{3} + 3a^{2}b^{3}c^{2})$$

$$b^{2}c^{2}\text{ is the hignest}$$

$$-6b^{2}c^{2}(8ac - 2c + 3a^{2}b)$$

m)  $2abc^{2} + 26a^{2}bc - 12ac^{2}$ 2 is the HCF ac is the hignest common power of abc 2 (abc<sup>2</sup> + 13a<sup>2</sup>bc - 6ac<sup>2</sup>) 2 ac (bc + 13ab - 6c)

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# **Section B**

### Worked Example

#### Factorise the expression $4x^2 - 64$

**Step 1**: Check if you can factorise the expression use the standard method. Check if there are any common number factors that can be taken out and if there are any common letter factors that are present in both terms.

For  $4x^2 - 64$  there are common factors which can be taken out since 4 can be taken out as a factor:

$$4x^2 - 64 = 4(x^2 - 16)$$

**Step 2:** Check if the expression is in the form of one squared term subtracted from another squared term. If so, factorise using the difference of two squares (DOTS) general formula:

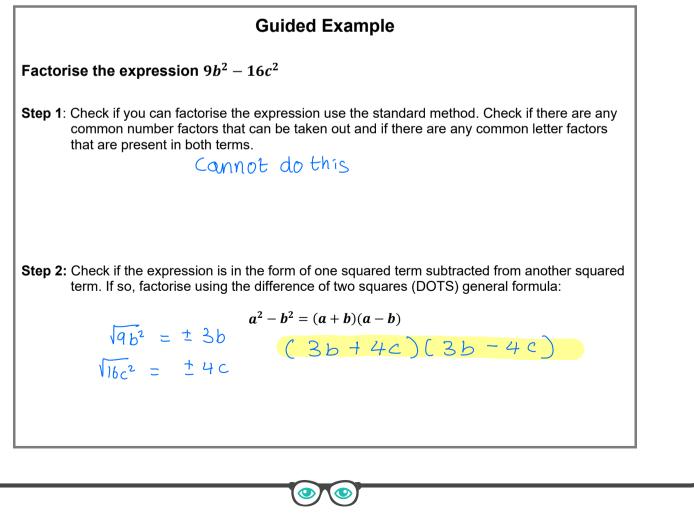
$$a^2 - b^2 = (a + b)(a - b)$$

For the expression  $4(x^2 - 16)$ , we have  $4(x^2 - 16) = 4[(x)^2 - (4)^2]$ 

Using the DOTS general formula with a = x, b = 4 we get

$$4(x^{2} - 16) = 4[(x)^{2} - (4)^{2}] = 4[(x + 4)(x - 4)] = 4(x + 4)(x - 4)$$

So, the final factorised answer is 4(x+4)(x-4).



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#### Now it's your turn!

If you get stuck, look back at the worked and guided examples.

- 2. Factorise the following expressions:
- a)  $z^2 1$  DOTS  $\sqrt{1} = \pm 1$ (Z + 1)(Z - 1)
- b)  $4p^2 25q^2$  DOTS  $\sqrt{4p^2} = \pm 2p$  (2p + 5q) (2p - 5q)  $\sqrt{25q^2} = \pm 5q$
- c)  $16s^2 49t^2$  DOTS  $\sqrt{16s^2} = \pm 4s$  (4s + 7t) (4s - 7t)  $\sqrt{49t^2} = \pm 7t$
- d)  $5x^2 125y^2$ Factorise 5: 5 (  $2c^2 - 25y^2$ ) DOTS:  $\sqrt{x^2} = 2$  $\sqrt{25y^2} = 5y$
- e)  $2p^2 200q^2$ Factorise 2: 2 ( $p^2 - 100q^2$ ) DOTS:  $\sqrt{p^2} = P$  $\sqrt{100q^2} = 10q$





# **Section C**

#### **Worked Example**

Factorise the expression  $x^2 + 7x + 12$ 

**Step 1**: If necessary, rearrange the expression into the standard form of  $ax^2 + bx + c$ .

This is not necessary here as it is already in the right form. Note, in this case a = 1, b = 7 and c = 12.

Step 2: Equate the expression to a pair of open brackets.

 $x^2 + 7x + 12 = (x)(x)$ 

**Step 3**: Identify the factors that multiply to give the 'c' value (the constant term) but also add to give the 'b' value (the coefficient of x).

 $x^2 + 7x + 12$ 

 $1 \times 12 = 12$  and 1 + 12 = 13  $-1 \times -12 = 12$  and -1 + -12 = -13  $2 \times 6 = 12$  and 2 + 6 = 8  $-2 \times -6 = 12$  and -2 + -6 = -8  $3 \times 4 = 12$  and 3 + 4 = 7 $-3 \times -4 = 12$  and -3 + -4 = -7

The pair of numbers 3 and 4 give the product of c = 12 and sum of b = 7.

Step 4: Write these values in the spaces in the brackets.

$$x^{2} + 7x + 12 = (x + 3)(x + 4)$$

**Step 5**: Quick check. Expanding the brackets should obtain the original quadratic expression.

$$(x + 3)(x + 4) = x^{2} + 3x + 4x + 12 = x^{2} + 7x + 12$$

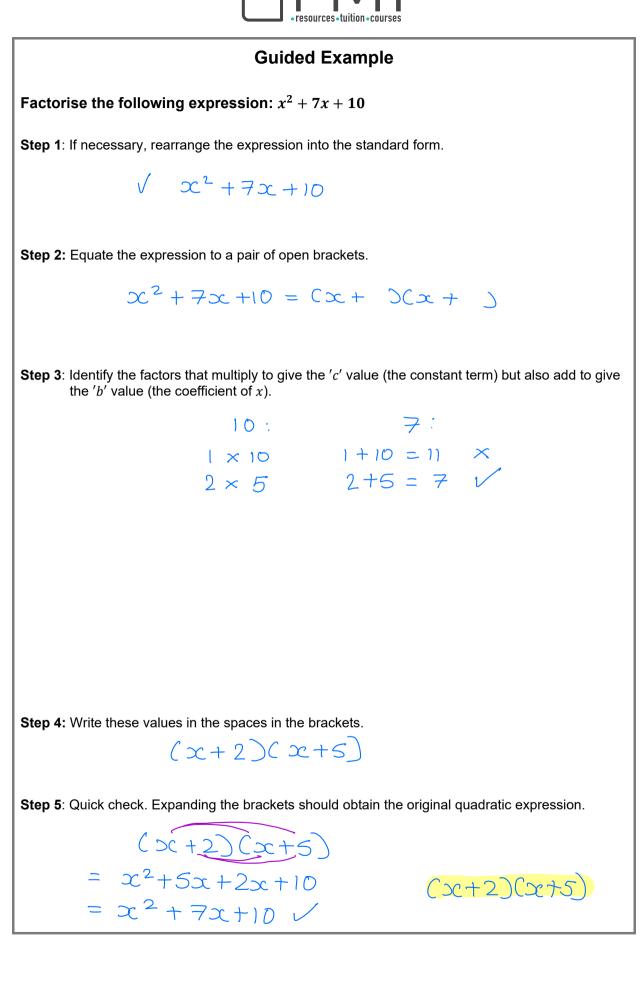
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It is indeed correct, so the final answer is

$$x^{2} + 7x + 12 = (x + 3)(x + 4)$$







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**Now it's your turn!** If you get stuck, look back at the worked and guided examples.

3. Factorise the following expressions:

a) 
$$x^{2} + 4x + 4$$
  
 $x \text{ to } 4$   $2x^{2} = 4$   $2 \text{ and } 2$   
 $x \text{ to } 4$   $2+2 = 4$   
 $(x+2)(x+2) = (x+2)^{2}$   
b)  $x^{2} + 3x + 2$   
 $+ 4 \text{ to } 3$   $1 + 2 = 3$   $1 \text{ and } 2$   
 $x \text{ to } 2$   $1 \times 2 = 2$   
 $(x + 1)(x + 2)$   
c)  $x^{2} - 10x + 21$   
 $+ 4 \text{ to } -10$   $-7 - 3 = -10$   $-7 \text{ and } -3$   
 $x \text{ to } 21$   $-7x - 3 = -21$   
 $(x - 7)(x - 3)$   
d)  $-5x + x^{2} - 24$   
 $x^{2} - 6x - 24$   
 $+ 50 - 5$   $-8 + 3 = -5$   $-8 \text{ and } 3$   
 $x \text{ to } -24$   $-8 \times 3 = -24$   
 $(x - 8)(x + 3)$   
e)  $x^{2} + 30 - 11x$   
 $x^{2} - 11x + 30$   
 $+ 50 - 11$   $-5 - 6 = -11$   $-5 \text{ and } -6$   
 $x \text{ to } 30 - 5x - 6 = 30$   
 $(x - 6)(x - 6)$ 

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# **Section D**

### Worked Example

#### Factorise the expression $3x^2 + 7x + 2$

**Step 1**: If necessary, rearrange the expression into the standard form of  $ax^2 + bx + c$ .

This is not necessary here as it is already in the right form. Note, in this case a = 3, b = 7 and c = 2.

**Step 2:** Multiply 'a' with 'c' and call this value ac.

 $ac = 3 \times 2 = 6$ 

**Step 3**: Write the factors of *ac* that will also add to give the *b* value.

 $1 \times 6 = 6 \text{ and } 1 + 6 = 7$ -1 \times -6 = -6 and -1 + -6 = -7 2 \times 3 = 6 and 2 + 3 = 5 -2 \times -3 = 6 and -2 + -3 = -5

The pair of numbers 1 and 6 will add to give the value of 7 which is the 'b' value.

**Step 4:** Write these values as the coefficients of separate *x* terms.

So, we will write 1 and 6 as x and 6x.

**Step 5**: Write the original expression but with these x values as shown above replacing (splitting up) the bx term.

$$3x^2 + 7x + 2 = 3x^2 + x + 6x + 2$$

**Step 6**: Split the above quadratic into two different parts by grouping one of the bx terms with the first quadratic term and the other with the last constant term.

 $3x^2 + 7x + 2 = 3x^2 + x + 6x + 2$ 

Step 7: Factorise each half of the expression separately.

 $3x^{2} + x = x(3x + 1)$ 6x + 2 = 2(3x + 1)

So,  $3x^2 + x + 6x + 2 = 3x^2 + x + 6x + 2 = x(3x + 1) + 2(3x + 1)$ 

Each bracket formed above is identical which indicates we have done it right.

**Step 8**: Write the expression as the product of two brackets. The first bracket will be equal to the common bracket in both factorisations. The second bracket will be made up of the coefficient terms of each of these identical brackets.

 $3x^2 + 1x + 6x + 2 = x(3x + 1) + 2(3x + 1) = x(3x + 1) + 2(3x + 1) = (x + 2)(3x + 1)$ 

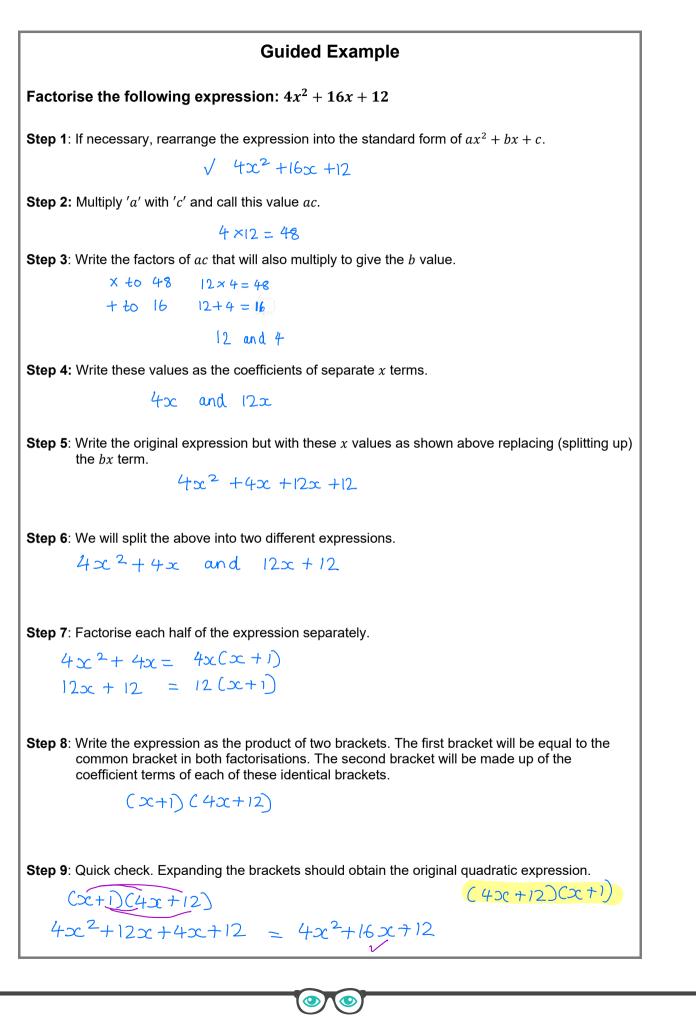
**Step 9**: Quick check. Expanding the brackets should obtain the original quadratic expression.

$$(x+2)(3x+1) = 3x^2 + x + 6x + 2 = 3x^2 + 7x + 2$$

It is indeed correct, so the final answer is  $3x^2 + 7x + 12 = (x + 2)(3x + 1)$ 











**Now it's your turn!** If you get stuck, look back at the worked and guided examples.

4. Factorise the following expressions:

a) 
$$2x^{2} + 3x + 1$$
  
 $2 \times 1 = 2$   
 $+ to 3 \quad 1 + 2 = 3$   
 $\times to 2 \quad 1 \times 2 = 2$   
 $2x^{2} + 2x \mid + 1x + 1$   
 $2x (x + 1) \mid 1 (x + 1)$   
 $(2x + 1)(x + 1)$ 

b) 
$$7x^2 - 39x + 20$$
  
 $7 \times 20 = 140$   
 $+ to -39$   $-35 - 4 = -39$   $-35$  and  $-4$   
 $x to 140$   $-35x - 4 = 140$   
 $7x^2 - 35x$   $-4x + 20$   
 $7x(x - 5)$   $-4(x - 5)$   
 $(7x - 4)(x - 5)$ 

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c)  $18x^{2} + 42x + 12$  6 is a common factor  $6(3x^{2} + 7x + 2)$  3x2 = 6  $+ \text{ to } 7 \quad 1 + 6 = 7$  *i* and 6  $x \text{ to } 6 \quad 1 \times 6 = 6$   $6 \times \left( 3x^{2} + 6x + 1x + 2 \\ 3x(x+2) + 1(x+2) \right)$  $6 \times (3x + 1)(x+2)$ 

6(3x+i)(x+2)

d) 
$$40x^2 + 38x - 12$$
  
2 is a common factor  
2 (  $20x^2 + 19x - 6$ )  
 $20x - 6 = -120$ 

+ to 
$$19$$
 24-5=19 24 and  $-5$   
x to -120 24x-5=-120 24 and  $-5$ 

$$2 \times \begin{bmatrix} 20x^2 - 5x + 24x - 6 \\ 5x(4x - 1) & 6(4x - 1) \end{bmatrix}$$

$$2 \times (5x+6)(4x-1)$$

$$2(5x+6)(4x-1)$$

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