GCSE Maths – Algebra

Factorising Linear and Quadratic Expressions

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to factorise linear and quadratic expressions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Factorise the expression $3y^2 + 9y$

Step 1: Write out the expression and work out the highest common numerical factor for the terms in the expression. Factorise the HCF outside the brackets by dividing out each term.

For the expression $3y^2 + 9y$ the highest common factor that goes into both terms is 3:

$$3y^2 + 9y = 3(y^2 + 3y)$$

Step 2: Work out the highest power of y that will go into both terms. Factorise this power of y outside the brackets by dividing it out of each term.

For $3(y^2 + 3y)$, the highest power of y that will go into both terms is y:

$$3y^2 + 9y = 3(y^2 + 3y) = 3y(y + 3)$$

So, the final factorised answer is $\mathbf{3y(y+3)}$*.*

Step 3: Quick check. Multiply out the brackets to check that you successfully obtain the original expression.

$$3y(y + 3) = (3y \times y) + (3y \times 3) = 3y^2 + 9y$$

Guided Example

Factorise the expression $8a^3 - 4a^2$

Step 1: Write out the expression and work out the highest common numerical factor for each of the terms in the expression. Factorise the HCF outside the brackets by dividing out each term.

4 is the HCF

$4(2a^3 - a^2)$

Step 2: Work out highest power of a that will go into both terms. Factorise this power of a outside the brackets by dividing it out of each term.

highest power of a is a^2

$4a^2(2a - 1)$

Step 3: Quick check. Multiply out the brackets to check that you successfully obtain the original expression.

$4a^2(2a - 1) = 8a^3 - 4a^2$

$4a^2(2a-1)$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

1. Factorise the following expressions:

a) $2x + 6$

2 is the HCF $\quad 2(x+3)$

b) $5 - 10z$

5 is the HCF $\quad 5(1-2z)$

c) $-8p + 16p$

8 is the HCF $\quad 8(-p+2p)$

p is the highest power $\quad 8p(-1+2) = 8p$

d) $-6e^3 - 18e^2$

-6 is the HCF $\quad -6(e^3+3e^2)$

e^2 is the highest power $\quad -6e^2(e+3)$

e) $-7z + 21z^2$

7 is the HCF $\quad 7(-z+3z^2)$

z is the highest power $\quad 7z(-1+3z) = 7z(3z-1)$

f) $2x^2 - xy + 714y$

No HCF

No highest power

Hence, no common factors. $2x^2-xy+714y$

g) $-120x^2yz + 3xyz^2 + 9x^2y^2z$

3 is the HCF $\quad 3(-40x^2yz + xyz^2 + 3x^2y^2z)$

The highest power of xyz is xyz $\quad 3xyz(-40x+z+3xy)$

h) $15xy + 5xyz - 125x^2y$

5 is the HCF

xy is the highest common power of xyz

$5(3xy + xyz - 25x^2y)$

$5xy(3 + z - 25x)$

i) $3p^3q - 9pq^3 - 3pq^2$

3 is the HCF

pq is the highest common power of pq

$3(p^3q - 3pq^3 - pq^2)$

$3pq(p^2 - 3q^2 - q)$

j) $-5xyz + 7xy + 10xy^2z$

No HCF

xy is the highest common power of xyz

$xy(-5z + 7 + 10yz)$

k) $6ef + 18efg - 3efg^2$

3 is the HCF

ef is the highest common power of efg

$3(2ef + 6efg - efg^2)$

$3ef(2 + 6g - g^2)$

l) $-48ab^2c^3 + 12b^2c^3 - 18a^2b^3c^2$

-6 is the HCF

b^2c^2 is the highest common power of abc

$-6(8ab^2c^3 - 2b^2c^3 + 3a^2b^3c^2)$

$-6b^2c^2(8ac - 2c + 3a^2b)$

m) $2abc^2 + 26a^2bc - 12ac^2$

2 is the HCF

ac is the highest common power of abc

$2(abc^2 + 13a^2bc - 6ac^2)$

$2ac(bc + 13ab - 6c)$

Section B

Worked Example

Factorise the expression $4x^2 - 64$

Step 1: Check if you can factorise the expression use the standard method. Check if there are any common number factors that can be taken out and if there are any common letter factors that are present in both terms.

For $4x^2 - 64$ there are common factors which can be taken out since 4 can be taken out as a factor:

$$4x^2 - 64 = 4(x^2 - 16)$$

Step 2: Check if the expression is in the form of one squared term subtracted from another squared term. If so, factorise using the difference of two squares (DOTS) general formula:

$$\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$$

For the expression $4(x^2 - 16)$, we have $4(x^2 - 16) = 4\,[(x)^2 - (4)^2]$

Using the DOTS general formula with $a = x$, $b = 4$ we get

$$4(x^2 - 16) \;=\; 4\,[(x)^2 - (4)^2] = 4\,[(x + 4)\,(x - 4)] = 4(x + 4)(x - 4)$$

So, the final factorised answer is $\boldsymbol{4\,(x + 4)(x - 4)}$.

Guided Example

Factorise the expression $9b^2 - 16c^2$

Step 1: Check if you can factorise the expression use the standard method. Check if there are any common number factors that can be taken out and if there are any common letter factors that are present in both terms.

Cannot do this

Step 2: Check if the expression is in the form of one squared term subtracted from another squared term. If so, factorise using the difference of two squares (DOTS) general formula:

$$\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$$

$\sqrt{9b^2} = \pm 3b$

$\sqrt{16c^2} = \pm 4c$

$(3b + 4c)(3b - 4c)$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. Factorise the following expressions:

a) $z^2 - 1$

DOTS $\sqrt{1} = \pm 1$

$(z+1)(z-1)$

b) $4p^2 - 25q^2$

DOTS

$\sqrt{4p^2} = \pm 2p$

$\sqrt{25q^2} = \pm 5q$

$(2p+5q)(2p-5q)$

c) $16s^2 - 49t^2$

DOTS

$\sqrt{16s^2} = \pm 4s$

$\sqrt{49t^2} = \pm 7t$

$(4s+7t)(4s-7t)$

d) $5x^2 - 125y^2$

Factorise 5 : $5(x^2 - 25y^2)$

DOTS :

$\sqrt{x^2} = x$

$\sqrt{25y^2} = 5y$

$5(x+5y)(x-5y)$

e) $2p^2 - 200q^2$

Factorise 2 : $2(p^2 - 100q^2)$

DOTS :

$\sqrt{p^2} = p$

$\sqrt{100q^2} = 10q$

$2(p-10q)(p+10q)$

Section C

Worked Example

Factorise the expression $\boldsymbol{x^2 + 7x + 12}$

Step 1: If necessary, rearrange the expression into the standard form of $ax^2 + bx + c$.

This is not necessary here as it is already in the right form.
Note, in this case $a = 1,\ b = 7$ and $c = 12$.

Step 2: Equate the expression to a pair of open brackets.

$$x^2 + 7x + 12 = (x \quad)(x \quad)$$

Step 3: Identify the factors that multiply to give the $'c'$ value (the constant term) but also add to give the $'b'$ value (the coefficient of x).

$$x^2 + 7x + 12$$

$1 \times 12 = 12$ and $1 + 12 = 13$
$-1 \times -12 = 12$ and $-1 + -12 = -13$
$2 \times 6 = 12$ and $2 + 6 = 8$
$-2 \times -6 = 12$ and $-2 + -6 = -8$
$3 \times 4 = 12$ and $3 + 4 = 7$
$-3 \times -4 = 12$ and $-3 + -4 = -7$

The pair of numbers 3 and 4 give the product of $c = 12$ and sum of $b = 7$.

Step 4: Write these values in the spaces in the brackets.

$$x^2 + 7x + 12 = (x + \mathbf{3})(x + \mathbf{4})$$

Step 5: Quick check. Expanding the brackets should obtain the original quadratic expression.

$$(x + \mathbf{3})(x + \mathbf{4}) = x^2 + 3x + 4x + 12 = x^2 + 7x + 12$$

It is indeed correct, so the final answer is

$$\boldsymbol{x^2 + 7x + 12 = (x + 3)(x + 4)}$$

Guided Example

Factorise the following expression: $x^2 + 7x + 10$

Step 1: If necessary, rearrange the expression into the standard form.

✓ $x^2 + 7x + 10$

Step 2: Equate the expression to a pair of open brackets.

$x^2 + 7x + 10 = (x + \quad)(x + \quad)$

Step 3: Identify the factors that multiply to give the $'c'$ value (the constant term) but also add to give the $'b'$ value (the coefficient of x).

10:	7:	
1×10	$1 + 10 = 11$	✗
2×5	$2 + 5 = 7$	✓

Step 4: Write these values in the spaces in the brackets.

$(x + 2)(x + 5)$

Step 5: Quick check. Expanding the brackets should obtain the original quadratic expression.

$(x + 2)(x + 5)$

$= x^2 + 5x + 2x + 10$

$= x^2 + 7x + 10$ ✓

$(x + 2)(x + 5)$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

3. Factorise the following expressions:

a) $x^2 + 4x + 4$

+ to 4 $2 \times 2 = 4$ 2 and 2
× to 4 $2 + 2 = 4$

$(x+2)(x+2) = (x+2)^2$

b) $x^2 + 3x + 2$

+ to 3 $1 + 2 = 3$ 1 and 2
× to 2 $1 \times 2 = 2$

$(x+1)(x+2)$

c) $x^2 - 10x + 21$

+ to −10 $-7 - 3 = -10$ −7 and −3
× to 21 $-7 \times -3 = -21$

$(x-7)(x-3)$

d) $-5x + x^2 - 24$

$x^2 - 5x - 24$

+ to −5 $-8 + 3 = -5$ −8 and 3
× to −24 $-8 \times 3 = -24$

$(x-8)(x+3)$

e) $x^2 + 30 - 11x$

$x^2 - 11x + 30$

+ to −11 $-5 - 6 = -11$ −5 and −6
× to 30 $-5 \times -6 = 30$

$(x-5)(x-6)$

Section D

Worked Example

Factorise the expression $\mathbf{3x^2 + 7x + 2}$

Step 1: If necessary, rearrange the expression into the standard form of $ax^2 + bx + c$.

This is not necessary here as it is already in the right form.
Note, in this case $a = 3,\ b = 7$ and $c = 2$.

Step 2: Multiply $'a'$with $'c'$ and call this value ac.

$$ac = 3 \times 2 = 6$$

Step 3: Write the factors of ac that will also add to give the b value.

$$1 \times 6 = 6 \text{ and } 1 + 6 = 7$$
$$-1 \times -6 = -6 \text{ and } -1 + -6 = -7$$
$$2 \times 3 = 6 \text{ and } 2 + 3 = 5$$
$$-2 \times -3 = 6 \text{ and } -2 + -3 = -5$$

The pair of numbers 1 and 6 will add to give the value of 7 which is the $'b'$ value.

Step 4: Write these values as the coefficients of separate x terms.

So, we will write 1 and 6 as x and $6x$.

Step 5: Write the original expression but with these x values as shown above replacing (splitting up) the bx term.

$$3x^2 + 7x + 2 = 3x^2 + x + 6x + 2$$

Step 6: Split the above quadratic into two different parts by grouping one of the bx terms with the first quadratic term and the other with the last constant term.

$$3x^2 + 7x + 2 = \mathbf{3x^2} + \boldsymbol{x} + \mathbf{6x} + \mathbf{2}$$

Step 7: Factorise each half of the expression separately.

$3x^2 + x = x(3x + 1)$
$6x + 2 = 2(3x + 1)$

So, $3x^2 + x + 6x + 2 = \mathbf{3x^2 + x + 6x + 2 = x(3x + 1) + 2(3x + 1)}$

Each bracket formed above is identical which indicates we have done it right.

Step 8: Write the expression as the product of two brackets. The first bracket will be equal to the common bracket in both factorisations. The second bracket will be made up of the coefficient terms of each of these identical brackets.

$$3x^2 + 1x + 6x + 2 = \boldsymbol{x}\mathbf{(3x + 1)} + \mathbf{2(3x + 1)} = \boldsymbol{x}(3x + 1) + \mathbf{2}(3x + 1) = (\boldsymbol{x} + \mathbf{2})(3x + 1)$$

Step 9: Quick check. Expanding the brackets should obtain the original quadratic expression.

$$(x + 2)(3x + 1) = 3x^2 + x + 6x + 2 = 3x^2 + 7x + 2$$

It is indeed correct, so the final answer is $\mathbf{3x^2 + 7x + 12 = (x + 2)(3x + 1)}$

Guided Example

Factorise the following expression: $4x^2 + 16x + 12$

Step 1: If necessary, rearrange the expression into the standard form of $ax^2 + bx + c$.

$$\checkmark \quad 4x^2 + 16x + 12$$

Step 2: Multiply $'a'$ with $'c'$ and call this value ac.

$$4 \times 12 = 48$$

Step 3: Write the factors of ac that will also multiply to give the b value.

$\times$ to 48 $\quad 12 \times 4 = 48$

$+$ to 16 $\quad 12 + 4 = 16$

12 and 4

Step 4: Write these values as the coefficients of separate x terms.

$4x$ and $12x$

Step 5: Write the original expression but with these x values as shown above replacing (splitting up) the bx term.

$$4x^2 + 4x + 12x + 12$$

Step 6: We will split the above into two different expressions.

$4x^2 + 4x$ and $12x + 12$

Step 7: Factorise each half of the expression separately.

$$4x^2 + 4x = 4x(x+1)$$
$$12x + 12 = 12(x+1)$$

Step 8: Write the expression as the product of two brackets. The first bracket will be equal to the common bracket in both factorisations. The second bracket will be made up of the coefficient terms of each of these identical brackets.

$$(x+1)(4x+12)$$

Step 9: Quick check. Expanding the brackets should obtain the original quadratic expression.

$$(4x+12)(x+1)$$

$$(x+1)(4x+12)$$

$$4x^2 + 12x + 4x + 12 = 4x^2 + 16x + 12$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

4. Factorise the following expressions:

a) $2x^2 + 3x + 1$

$2 \times 1 = 2$

$+$ to 3 $1 + 2 = 3$ 1 and 2

$\times$ to 2 $1 \times 2 = 2$

$2x^2 + 2x \mid + 1x + 1$

$2x(x+1) \mid 1(x+1)$

$(2x+1)(x+1)$

b) $7x^2 - 39x + 20$

$7 \times 20 = 140$

$+$ to -39 $-35 - 4 = -39$ -35 and -4

$\times$ to 140 $-35 \times -4 = 140$

$7x^2 - 35x \mid -4x + 20$

$7x(x-5) \mid -4(x-5)$

$(7x-4)(x-5)$

c) $18x^2 + 42x + 12$

6 is a common factor

$6(3x^2 + 7x + 2)$

$3 \times 2 = 6$

$+$ to 7 $\quad 1 + 6 = 7 \quad$ 1 and 6

$\times$ to 6 $\quad 1 \times 6 = 6$

$$6 \times \left[\begin{array}{c|c} 3x^2 + 6x & + 1x + 2 \\ 3x(x+2) & 1(x+2) \end{array} \right]$$

$6 \times (3x+1)(x+2)$

$6(3x+1)(x+2)$

d) $40x^2 + 38x - 12$

2 is a common factor

$2(20x^2 + 19x - 6)$

$20 \times -6 = -120$

$+$ to 19 $\quad 24 - 5 = 19 \quad$ 24 and -5

$\times$ to -120 $\quad 24 \times -5 = -120$

$$2 \times \left[\begin{array}{c|c} 20x^2 - 5x & + 24x - 6 \\ 5x(4x-1) & 6(4x-1) \end{array} \right]$$

$2 \times (5x+6)(4x-1)$

$2(5x+6)(4x-1)$