GCSE Maths – Algebra

Completing the Square (Higher Only)

Worksheet

NOTES

SOLUTIONS

This worksheet will show you how to work out different types of completing the square questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Complete the square for the quadratic expression $3x^2 + 6x - 10$

Step 1: If necessary, factorise the equation by the coefficient of x^2.
This step can be skipped if the coefficient is 1.

For this equation we factorise out the 3:

$$3x^2 + 6x - 10 = 3\left(x^2 + 2x - \frac{10}{3}\right)$$

Step 2: Complete the square of the part of the expression which is in the form $x^2 + bx + c$.
Start by identifying the coefficients b and c.

$$x^2 + bx + c = x^2 + 2x - \frac{10}{3}$$

$$b = 2, \qquad c = -\frac{10}{3}$$

Step 3: Divide the b coefficient by 2. This new value will be the value added to the x in the square part of the completed expression, i.e. $\left(x + \frac{b}{2}\right)^2$.

$$\frac{2}{2} = 1$$

Step 4: Substitute in the values to the completed square form.

General formula: $\qquad x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

$$x^2 + 2x - \frac{10}{3} = (x+1)^2 - 1^2 - \frac{10}{3}$$

$$3\left(x^2 + 2x - \frac{10}{3}\right) = 3\left[(x+1)^2 - 1^2 - \frac{10}{3}\right]$$

Step 5: Simplify the expression.

$$3\left[(x+1)^2 - 1^2 - \frac{10}{3}\right] = 3\left[(x+1)^2 - \frac{13}{3}\right] = 3(x+1)^2 - 13$$

Hence, $3x^2 + 6x - 10 = 3(x+1)^2 - 13.$

Guided Example

Complete the square for the quadratic expression $x^2 + 12x + 9$

Step 1: If necessary, factorise the equation by the coefficient of x^2.
       This step can be skipped if the coefficient is 1.

Step 2: Complete the square of the part of the expression which is in the form $x^2 + bx + c$.
       Start by identifying the coefficients b and c.

Step 3: Divide the b coefficient by 2. This new value will be the value added to the x in the square
       part of the completed expression, i.e. $\left(x + \frac{b}{2}\right)^2$.

Step 4: Substitute in the values to the completed square form.

   General formula: $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

Step 5: Simplify the expression.

1. Complete the square for the following quadratic expressions:

a) $x^2 + 6x + 6$

b) $2x^2 + 12x + 18$

c) $3x^2 + 24x - 5$

d) $-2x^2 + 14x - 7$

Section B

Worked Example

Calculate the turning point and line of symmetry of $3x^2 + 6x - 10 = 0$

Step 1: Complete the square for this quadratic equation.

$$3(x + 1)^2 - 13$$

Step 2: For a quadratic equation in the form $a(x + p)^2 + q$, the turning point occurs at $(-p, q)$. Identify values for p and q and hence identify the turning point.

Here,

$$a(x + p)^2 + q = 3(x + 1)^2 - 13$$

$$p = 1, \quad q = -13$$

The turning point is $(-p, q)$
Therefore, the turning point of this equation is: $(-\mathbf{1}, -\mathbf{13})$

Step 3: The line of symmetry occurs at $x = -p$. Identify the line of symmetry for this quadratic.

The line of symmetry for this equation is at $x = -\mathbf{1}$.

Guided Example

Calculate the turning point and line of symmetry of $x^2 + 12x + 9 = 0$

Step 1: Complete the square for this quadratic equation.

Step 2: Identify values for p and q and hence identify the turning point.

Step 3: The line of symmetry occurs at $x = -p$. Identify the line of symmetry for this quadratic.

2. Find the turning point and line of symmetry of each of the following quadratic equations:

a) $x^2 + 4x + 9 = 0$

b) $4x^2 - 20x + 1 = 0$

c) $6x^2 + 3x - 9 = 0$

d) $-4x^2 + 12x - 1 = 0$

Section C

Worked Example

By completing the square, solve the quadratic equation $3x^2 + 6x - 10 = 0$

Step 1: Complete the square.

$$3x^2 + 6x - 10 = 3(x + 1)^2 - 13 = 0$$

Step 2: Solve the equation by rearranging to make x the subject.

$$3(x + 1)^2 - 13 = 0$$

$$3(x + 1)^2 = 13$$

$$(x + 1)^2 = \frac{13}{3}$$

$$x + 1 = \pm\sqrt{\frac{13}{3}}$$

$$x = -1 \pm \sqrt{\frac{13}{3}}$$

The solutions to this quadratic equation are

$$x = -1 + \sqrt{\frac{13}{3}}, \qquad x = -1 - \sqrt{\frac{13}{3}}$$

Guided Example

By completing the square, solve the quadratic equation $x^2 + 14x + 9 = 0$

Step 1: Complete the square.

Step 2: Solve the equation by rearranging to make x the subject.

3. By completing the square, solve the following quadratic equations:

a) $x^2 + 6x + 3 = 0$

b) $2x^2 - 4x - 10 = 0$

c) $2x^2 + 12x + 17 = 0$

d) $3x^2 + 9x - 14 = 0$