GCSE Maths – Algebra

Equation of a Circle and its Tangent

(Higher Only)

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions involving circles and their tangents. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

A circle centred at the origin has a radius of 4. What is the equation of this circle?

Step 1: Use the basic equation of a circle centred at the origin $x^2 + y^2 = r^2$.

We know that the radius is 4, so we can square this to get the value of r^2.

$$r^2 = (4)^2 = 16$$

Therefore, the equation of this circle is:

$$\mathbf{x^2 + y^2 = 16}$$

Guided Example

A circle centred at the origin has the equation $x^2 + y^2 = 89$.

What is the radius of this circle?

Step 1: Use the general equation of a circle centred at the origin $x^2 + y^2 = r^2$.

$x^2 + y^2 = 89$

→ using equation of circle : $x^2 + y^2 = r^2$

$r^2 = 89$

$r = \sqrt{89}$

$= 9.43$ 2d.p. → we can leave it as $\sqrt{89}$ as well.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. What is the equation of a circle centred at the origin with radius of 9?

Using : $x^2+y^2=r^2$
$r^2=(9)^2=81$
$\therefore$ equation : $x^2+y^2=81$

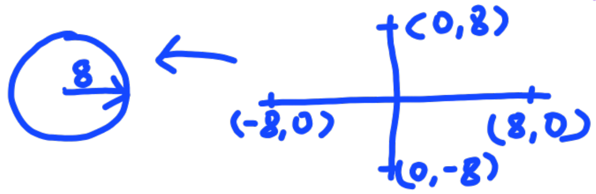
2. What is the radius of a circle that has the equation $x^2+y^2=56$?

$x^2+y^2=56$
$r^2=56$
$r=\sqrt{56}$
$=2\sqrt{14}$ or 7.48 2d.p

3. A circle passes through the points $(8,0)$, $(0,-8)$, $(-8,0)$ and $(0,8)$. What is the equation of this circle?

equation of circle : $x^2+y^2=r^2$

Hint : visualise
↳ plot a graph



circle with
radius : 8
centre : (0,0)
$r^2=(8)^2=64$

equation : $x^2+y^2=64$

4. A circle passes through the point $(3,7)$ with its centre at the origin $(0,0)$. What is the equation of this circle?

equation of circle : $x^2+y^2=r^2$
centre : (0,0)
Point : (3,7) → when $x=3$ $y=7$

$\therefore$ Subbing the point into the equation

$(3)^2+(7)^2=r^2$
$9+49=r^2$ → $r^2=58$
equation of circle : $x^2+y^2=58$

Section B

Worked Example

A circle has the equation $x^2 + y^2 = 98$.
What is the equation of the tangent that touches the circle at $(7, 7)$?

Step 1: Work out the gradient of the radius from the origin $(0, 0)$ to the point that touches the tangent.

To work out the gradient of the radius, we need to calculate the difference in y-coordinates divided by the difference in x-coordinates:

$$Gradient\ of\ radius\ connecting\ (0,0)\ to\ (7,7) = m_r = \frac{7-0}{7-0} = \frac{7}{7} = 1$$

Step 2: Take the negative reciprocal of the radius gradient to find the gradient of the tangent.

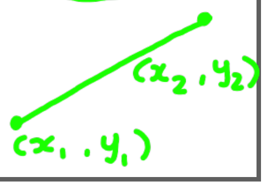
$$Gradient\ of\ tangent = m = -\frac{1}{Gradient\ of\ radius} = -\frac{1}{m_r} = -\frac{1}{1} = -1$$

Step 3: Calculate the equation of the tangent by substituting in the values of x and y that the tangent is known to pass through.

A straight line has the general form $y = mx + c$. For the tangent we know $m = -1$, so the tangent has equation $y = -x + c$. The tangent passes through the point $(7, 7)$, so use these values in to obtain the value of c:

$$7 = -1(7) + c$$
$$7 = -7 + c$$
$$c = 14$$

gradient $\to \dfrac{y_2 - y_1}{x_2 - x_1}$



The equation of the tangent is $\boldsymbol{y = -x + 14}$.

Guided Example

A circle has the equation $x^2 + y^2 = 40$. Find the equation of the tangent that touches the circle at $(-6, -2)$.

Step 1: Work out the gradient of the radius from the origin (0, 0) to the point that touches the tangent.

Grdient of radius: $(0,0)$ to $(-6,-2)$ $= m_r = \dfrac{-2-0}{-6-0} = \dfrac{1}{3}$

Step 2: Take the negative reciprocal of the radius gradient to find the gradient of the tangent.

Gradient of tangent: $= m = \dfrac{-1}{\text{gradient of radius}} = \dfrac{-1}{m_r} = \dfrac{-1}{1/3} = -3$

Step 3: Calculate the equation of the tangent by substituting in the values of *x* and *y* that the tangent is known to pass through.

equation of tangent: $y = mx + c \to [m = -3]$

Point $(-6,-2) \to$ sub into

when $x = -6$, $y = -2$

$$-2 = -3(-6) + c$$
$$-2 = 18 + c$$
$$-20 = c$$

$y = -3x - 20$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. A circle has the equation $x^2 + y^2 = 85$. Find the equation of the tangent that touches the circle at $(-2, 9)$.

$x^2 + y^2 = 85$ $\therefore$ centre $= (0,0)$

gradient of radius : $(0,0)$ to $(-2,9)$ $= \frac{9-0}{-2-0} = -\frac{9}{2}$

gradient of tangent : $m = \frac{-1}{\text{gradient of radius}} = \frac{-1}{-9/2} = \frac{2}{9}$

equation of tangent : $y = mx + c$

$m = \frac{2}{9}$ Point $= (-2, 9)$ Sub into equation

$9 = \frac{2}{9}(-2) + c$

$9 = -\frac{4}{9} + c \rightarrow c = 9 + \frac{4}{9} = \frac{85}{9}$ $\therefore y = \frac{2}{9}x + \frac{85}{9}$

6. A circle has equation $\frac{x^2+y^2}{2} = 40$. Find the equation of the tangent that touches the circle at $(8, 4)$.

$\frac{x^2+y^2}{2} = 40 \xrightarrow[\times 2]{} x^2 + y^2 = 80$ $\therefore$ centre $= (0,0)$

gradient of radius : $(0,0)$ to $(8,4)$ $= \frac{4-0}{8-0} = \frac{4}{8} = \frac{1}{2}$

gradient of tangent : $m = \frac{-1}{\text{gradient of radius}} = \frac{-1}{1/2} = -2$

equation of tangent : $y = mx + c$

$m = -2$ Point $= (8, 4)$ Sub into equation

$4 = -2(8) + c$

$4 = -16 + c \rightarrow c = 20$ $\therefore y = -2x + 20$

7. A circle has the equation $x^2 + y^2 = \frac{37}{2}$. Calculate the gradient of the tangent that touches the circle at $\left(\frac{5}{2}, \frac{7}{2}\right)$.

$x^2 + y^2 = \frac{37}{2}$ $\therefore$ centre $= (0,0)$

gradient of radius : $(0,0)$ to $\left(\frac{5}{2}, \frac{7}{2}\right)$ $= \frac{\frac{7}{2} - 0}{5/2 - 0} = \frac{\frac{7}{2}}{\frac{5}{2}} = \frac{7}{5}$

gradient of tangent : $m = \frac{-1}{\text{gradient of radius}} = \frac{-1}{7/5} = -\frac{5}{7}$

equation of tangent : $y = mx + c$

$m = -\frac{5}{7}$ Point $= \left(\frac{5}{2}, \frac{7}{2}\right)$ Sub into equation

$\frac{7}{2} = -\frac{5}{7}\left(\frac{5}{2}\right) + c$

$\frac{7}{2} = \frac{-25}{14} + c \rightarrow c = \frac{7}{2} + \frac{25}{14} = \frac{37}{7}$ $\therefore y = -\frac{5}{7}x + \frac{37}{7}$