GCSE Maths – Algebra

Gradients and Areas of Graphs in Context
(Higher Only)

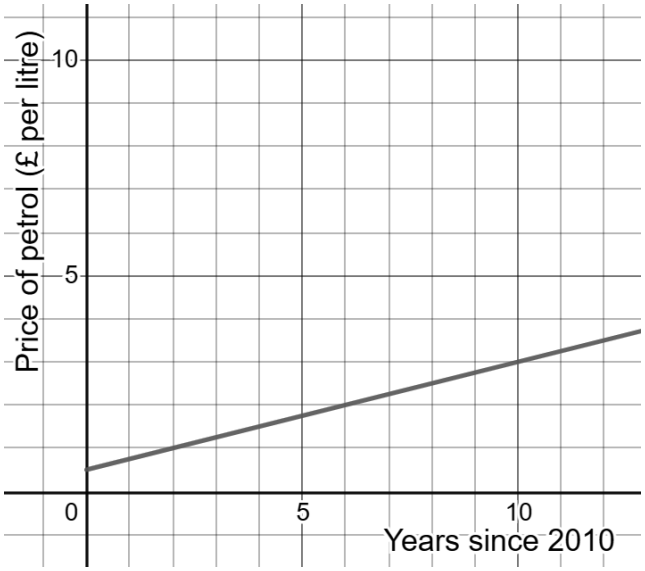
WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions relating to gradients and areas of graphs in. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Below is a graph of the change in the price of petrol over time. What rate is the price of petrol increasing at?



Step 1: Work out if the graph is linear or non-linear.

This graph is linear as the line is straight. This makes finding the gradient of the line (which represents the rate of change of the price of petrol) easier.

Step 2: Identify two points on the line, then calculate the gradient by calculating the difference in y divided by the difference in x.

We can use the points $(2, 1)$ *and* $(10, 3)$.

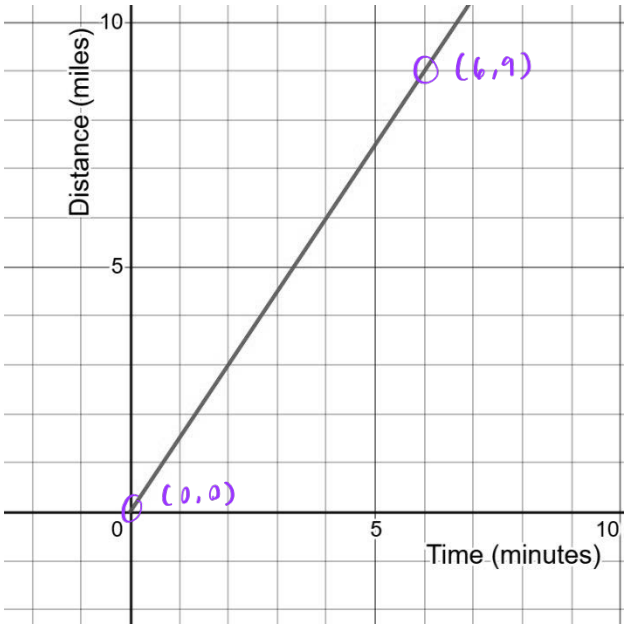
$$Gradient = \frac{Change\ in\ y}{Change\ in\ x} = \frac{3-1}{10-2} = \frac{2}{8} = \frac{1}{4}$$

Step 3: Interpret the gradient.

The gradient is $\frac{1}{4}$.
This means that each year, the price of petrol increases by $\frac{1}{4}$ *of* $£1$, *or* $25p$, *per year.*

Guided Example

Below is a distance-time graph of a car. What is the speed of this car?



Step 1: Work out if the graph is linear or non-linear.

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The graph is linear as the line is straight

Step 2: Identify two points on the line, then calculate the gradient by calculating the difference in y divided by the difference in x.

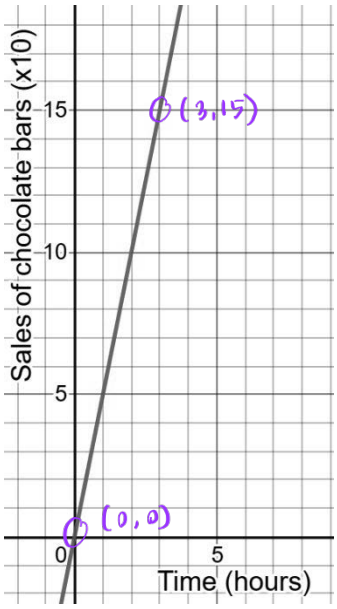
$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{9-0}{6-0} = \frac{9}{6} = \frac{3}{2} = 1.5$$

Step 3: Interpret the gradient.

The gradient is 1.5. This means the speed of the car is 1.5 miles per minute.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Calculate the rate of sales of chocolate bars.



① Choose 2 points along the line that is easy to read.

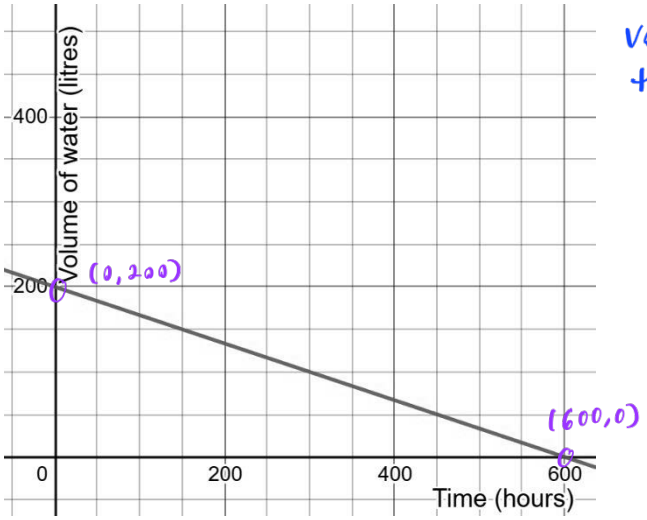
The coordinates are (0,0) and (3,15)

Rate of sales = gradient = $\frac{y_1 - y_2}{x_1 - x_2}$

gradient = $\frac{15-0}{3-0} = \frac{15}{3} = 5$

The rate of sales of chocolate bars is 5(x10) chocolate bars per hour.

2. Calculate the rate of water being emptied from a tank



Volume of water in the tank per hour

$= \frac{y_1 - y_2}{x_1 - x_2}$

$= \frac{200 - 0}{0 - 600}$

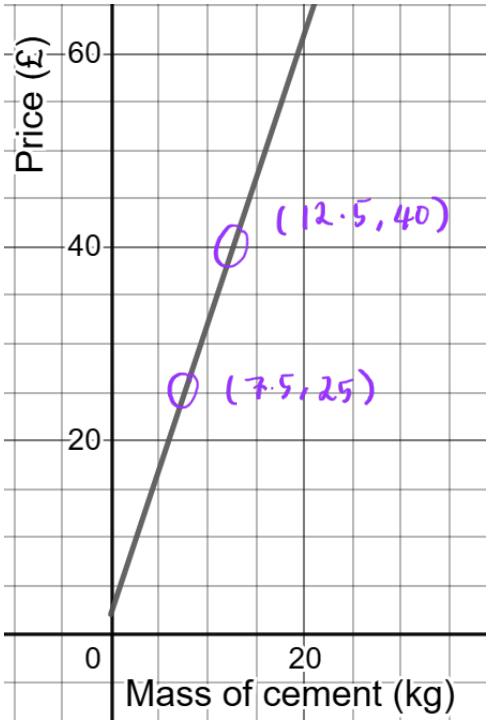
$= \frac{200}{-600}$

$= -\frac{1}{3}$

The rate of water being emptied from the tank is $\frac{1}{3}$ litres per hour.

don't need to include the negative sign because the sign only means the water is leaving the tank

3. Calculate the price per kg of cement



price per kg of cement $= \frac{y_1 - y_2}{x_1 - x_2}$

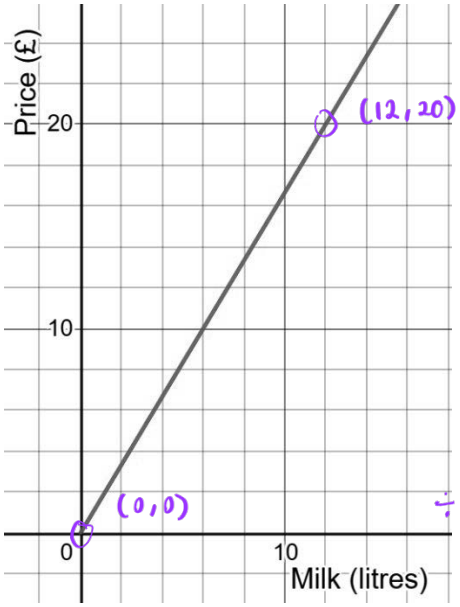
$= \frac{40 - 25}{12.5 - 7.5}$

$= \frac{15}{5}$

$= 3$

The price per kg of cement is £3.

4. Calculate the price per pint of milk.



Price per litre of milk $= \frac{y_1 - y_2}{x_1 - x_2}$

$= \frac{20 - 0}{12 - 0}$

$= \frac{20}{12} = \frac{5}{3}$

$= £1.67$

1 litre is equal to 1.76 pint

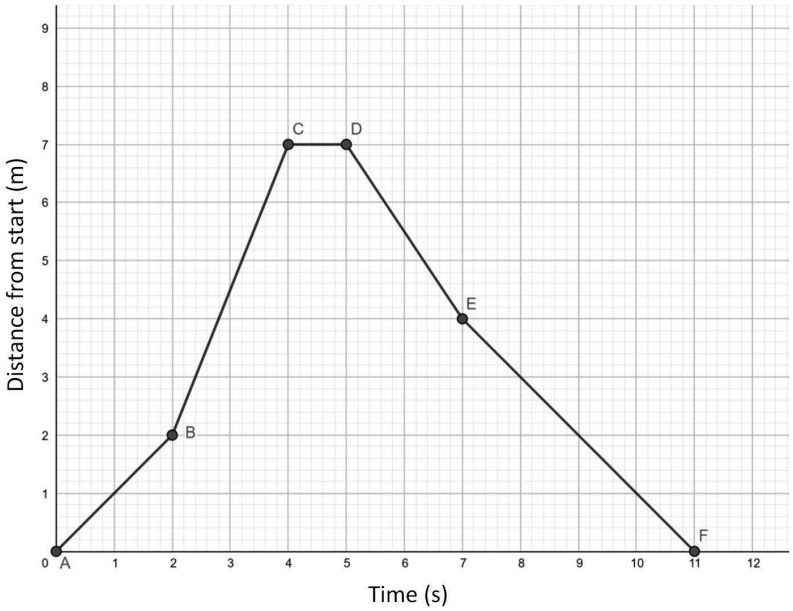
÷1.76 (1 litre = £1.67 ; 1 pint = £0.95) ÷1.76

The price per pint of milk is £0.95

Section B

Worked Example

Below is a displacement-time graph of an object.
Calculate the speed of the object between points B and C.



Step 1: Work out the coordinates of the points asked for.

Point B is at $(2, 2)$ *and point C is at* $(4, 7)$.

Step 2: Calculate the gradient of the line between the required coordinates.

To find speed, we need to divide the total distance by the time. Therefore, we divide the difference in y by the difference in x.

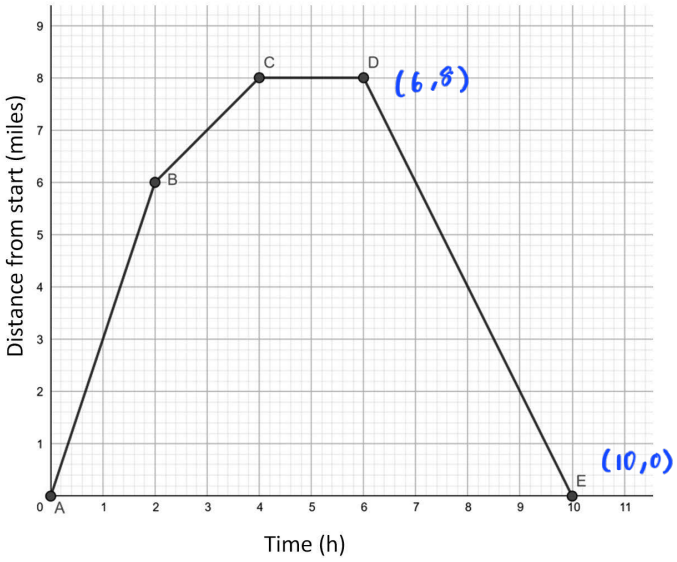
$$Gradient = \frac{Change\ in\ y}{Change\ in\ x} = \frac{7-2}{4-2} = \frac{5}{2}$$

Step 3: Interpret the gradient in terms of the question context.

The gradient tells us the speed of the object between points B and C. The speed is $2.5\ m/s$.

Guided Example

The displacement-time graph below shows the distance walked by a group. Calculate their speed between points D and E.



Step 1: Work out the coordinates of the points asked for.

Point D = (6,8) Point E = (10,0)

Step 2: Calculate the gradient of the line between the required coordinates.

$$\text{gradient} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{8-0}{6-10} = \frac{8}{-4} = -2$$

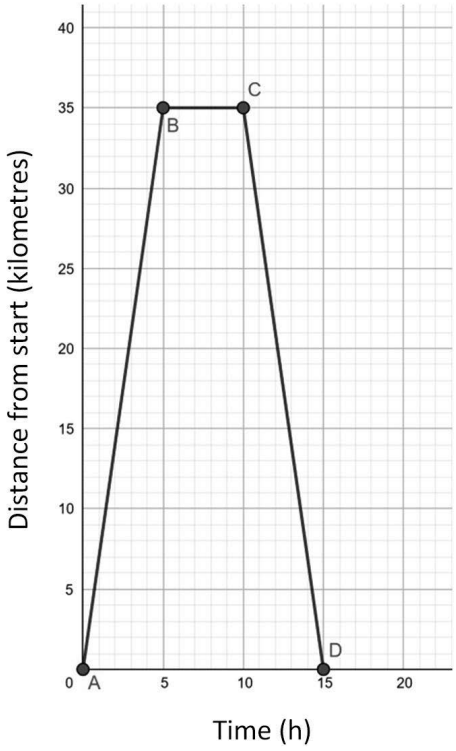
Step 3: Interpret the gradient in terms of the question context.

The gradient tells us the speed walked by the group from point D to E. Their speed is 2 miles/hour.

We did not include the negative sign because this only means that the distance walked by the group is decreasing

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Calculate the speed of this car between points A and B. What is happening between points B and C?



Point A = (0, 0)

Point B = (5, 35)

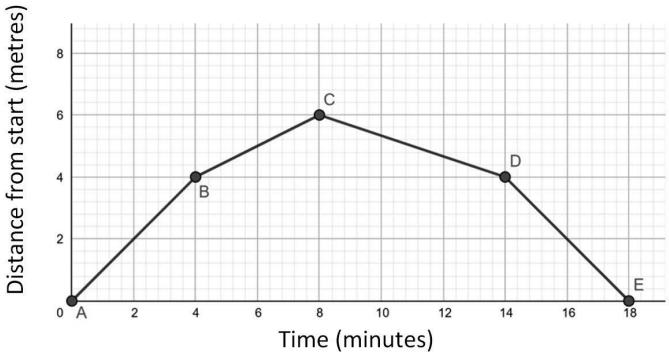
Speed from A to B $= \frac{35-0}{5-0}$

$= \frac{35}{5}$

= 7 kilometres/hour

From B to C, the distance remains at 35 km. The gradient is also 0 which means the speed is 0. In other words, the car is not moving from point B to C.

6. Calculate the speed between the points C and D.



Point C = (8, 6)

Point D = (14, 4)

Speed from C to D $= \frac{4-6}{14-8}$

$= \frac{-2}{6}$

$= -\frac{1}{3}$

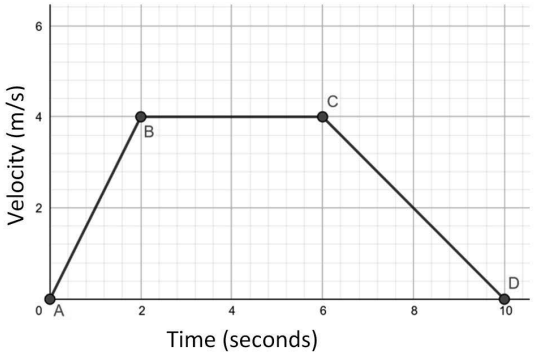
$= -0.333$

The speed from C to D is 0.33 metres/min

Section C

Worked Example

Below is a velocity-time graph of an object. Calculate the acceleration of the object between points A and B, and calculate the total distance travelled by the object.



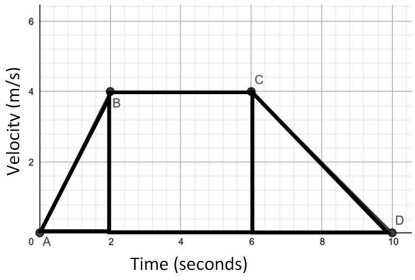
Step 1: Identify the points asked for in the question, then calculate the gradient of the line between these two points.

Point A is at $(0, 0)$ *and point B is at* $(2, 4)$*. The gradient of the line represents acceleration because it is the change in velocity divided by the time it takes.*

$$Acceleration = \frac{Change\ in\ velocity}{Change\ in\ time} = \frac{4-0}{2-0} = \frac{4}{2} = 2$$

The acceleration of the object between points A and B is $\mathbf{2\ m/s^2}$.

Step 2: For the second part of the question, divide the area under the curve into shapes, such as squares, triangles and rectangles.



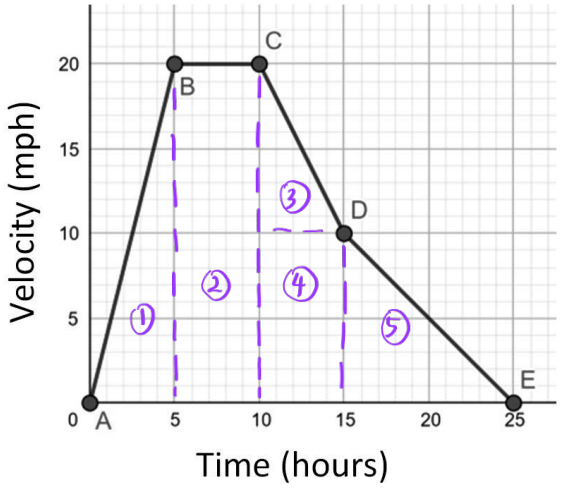
$$First\ Triangle\ Area: \frac{1}{2} \times 2 \times 4 = 4\ \mathrm{m}^2$$
$$Square\ Area: 4 \times 4 = 16\ \mathrm{m}^2$$
$$Second\ Triangle\ Area: \frac{1}{2} \times 4 \times 4 = 8\ \mathrm{m}^2$$

Step 3: Add together the areas of the shapes to find the total distance travelled.

$$Distance\ travelled = Area\ under\ graph = 4 + 16 + 8 = \mathbf{28\ m}$$

Guided Example

Below is a velocity-time graph for a car. Calculate the deceleration of the car between points C and D, and find the total distance travelled by the car.



Step 1: Identify the points asked for in the question, then calculate the gradient of the line between these two points.

Point C = (10, 20)

Point D = (15, 10)

gradient : $\frac{10-20}{15-10} = \frac{-10}{5} = -2$

deceleration of the car = 2 mph/hour

Step 2: For the second part of the question, divide the area under the curve into shapes, such as squares, triangles, and rectangles.

① Area of triangle : $\frac{1}{2} \times 5 \times 20 = 50$

② Area of rectangle : $5 \times 20 = 100$

③ Area of triangle : $\frac{1}{2} \times 5 \times 10 = 25$

④ Area of rectangle = $5 \times 10 = 50$

⑤ Area of triangle = $\frac{1}{2} \times 10 \times 10 = 50$

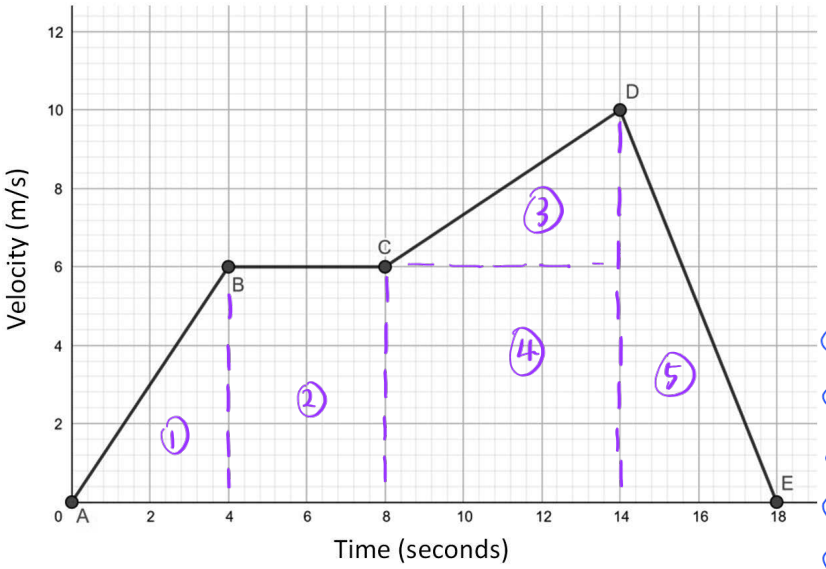
Step 3: Add together the areas of the shapes to find the total distance travelled.

Distance travelled : Areas under the graph =

$50 + 100 + 25 + 50 + 50 = 275$ miles

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

7. Find the deceleration between points D and E, and calculate the total distance travelled by this object.



Point D = (14, 10)

Point E = (18, 0)

gradient from point D to E :

$= \frac{10-0}{14-18} = \frac{10}{-4} = -2.5$

deceleration = $2.5\ ms^{-2}$

① Area : $\frac{1}{2} \times 4 \times 6 = 12$

② Area : $4 \times 6 = 24$

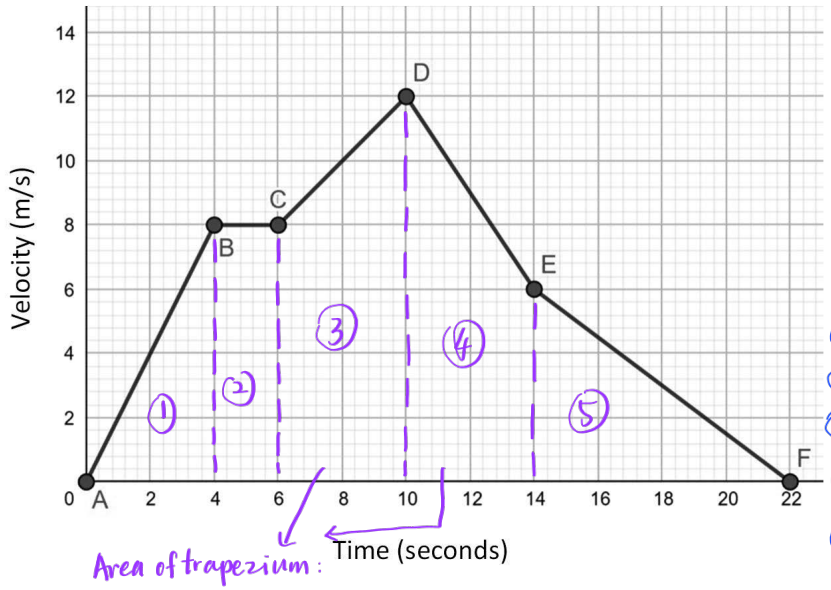
③ Area : $\frac{1}{2} \times 6 \times 4 = 12$

④ Area : $6 \times 6 = 36$

⑤ Area : $\frac{1}{2} \times 4 \times 10 = 20$

Total distance travelled by object : $12 + 24 + 12 + 36 + 20 = 104$ m

8. Calculate the acceleration between points A and B, and the total distance travelled.



Point A = (0, 0)

Point B = (4, 8)

acceleration = $\frac{8-0}{4-0}$

$= \frac{8}{4} = 2$

acceleration between point A to B is $2ms^{-2}$.

① Area : $\frac{1}{2} \times 4 \times 8 = 16$

② Area : $2 \times 8 = 16$

③ Area : $\frac{1}{2} \times (8+12) \times 4 = 40$

④ Area : $\frac{1}{2} \times (12+6) \times 4 = 36$

⑤ Area : $\frac{1}{2} \times 8 \times 6 = 24$

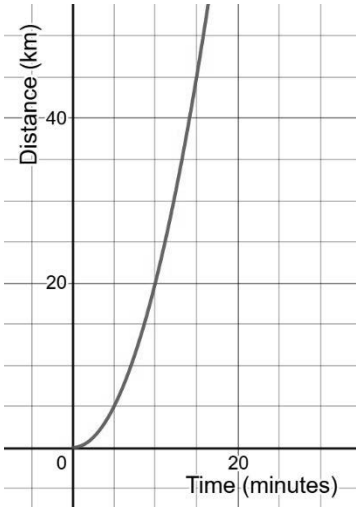
Area of trapezium : $\frac{1}{2}(a+b) \times h$

Total distance travelled : $16 + 16 + 40 + 36 + 24 = 132$ m

Section D

Worked Example

Below is a distance-time graph of a car. Calculate the speed at which the car is travelling at minute 15.

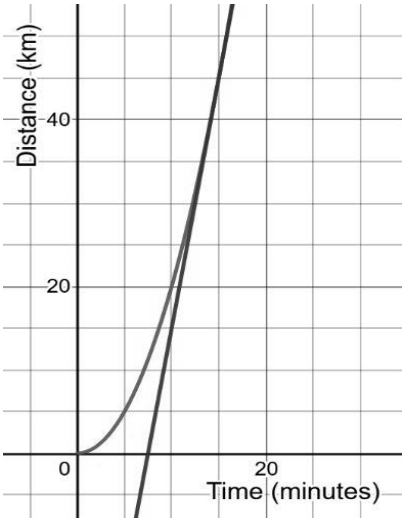


Step 1: Work out if the graph is linear or non-linear.

This graph is non-linear because it is not a straight line. This means we will have to use tangents to find the gradient.

Step 2: Draw a tangent to the line at the point asked about in the question and identify two coordinates on this tangent that could be used to calculate the gradient.

We are asked to find the gradient when $x = 15$. *The point on the line is* $(15, 45)$.



This tangent line passes through the points $(10, 15)$ *and* $(15, 45)$. *Use the difference between the y-coordinates and x-coordinates to find the gradient.*

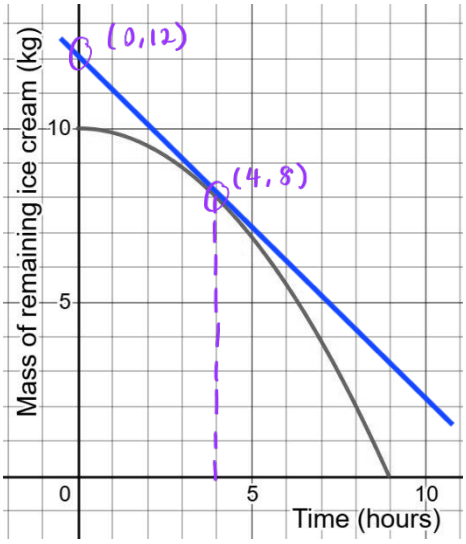
$$Gradient = \frac{Change\ in\ y}{Change\ in\ x} = \frac{45 - 15}{15 - 10} = \frac{30}{5} = 6$$

Step 3: Interpret the gradient in terms of the question context.

The gradient tells us the speed of the car at minute $\mathbf{15}$ *is* $\mathbf{6\ km/min}$.

Guided Example

Below is a graph showing the remaining amount of ice cream in a café over time. Find the rate at which ice cream is being sold at the 4th hour.



Step 1: Work out if the graph is linear or non-linear.

The graph is non-linear because it is not a straight line.

Step 2: Draw a tangent to the line at the point asked about in the question and identify two coordinates on this tangent that could be used to calculate the gradient.

2 points along the tangent are (0,12) and (4,8).

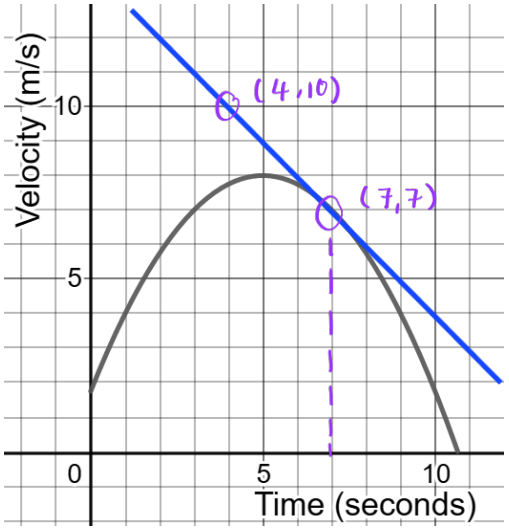
$$\text{gradient} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{12-8}{0-4} = \frac{4}{-4} = -1$$

Step 3: Interpret the gradient in terms of the question context.

The gradient tells us the rate at which ice cream is sold at the 4th hour is 1 kg/hour.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

9. The graph below is a velocity-time graph of an object. Find the deceleration of the object at 7 seconds.

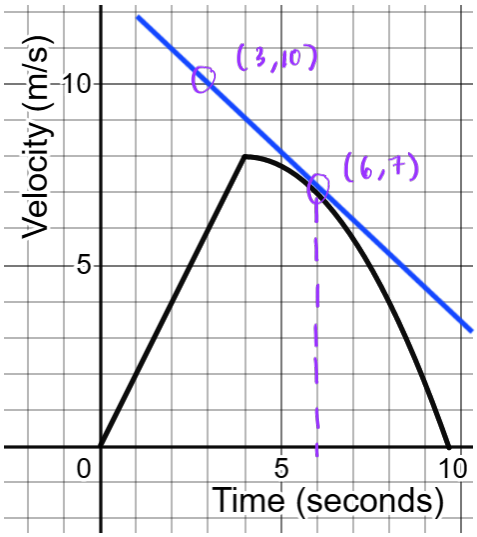


① Draw the tangent at point 7 seconds.
② Identify 2 coordinates along the tangent.
(4, 10) and (7, 7)
③ Calculate gradient

gradient : $\frac{10-7}{4-7} = \frac{3}{-3} = -1$

The deceleration of the object at 7 seconds is 1 ms^{-2}.

10. Below is a velocity-time graph of an object. Find the deceleration of the object at 6 seconds.



2 coordinates along the tangent are :
(3,10) and (6,7)

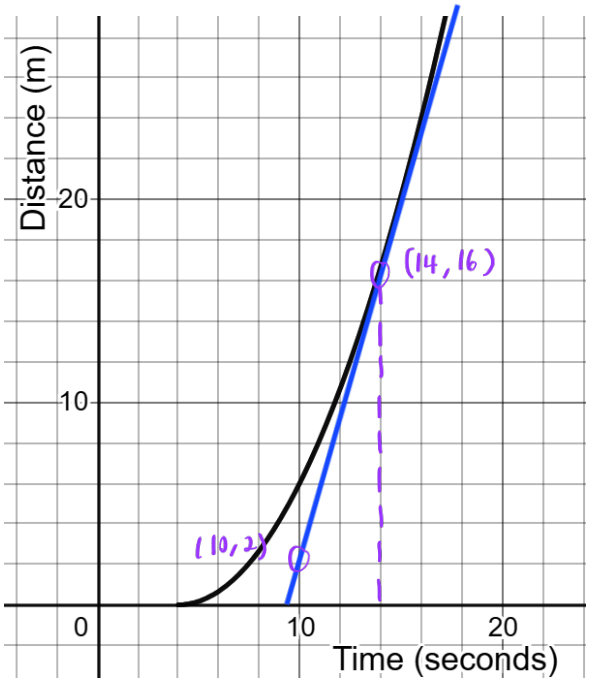
acceleration of object $= \frac{10-7}{3-6}$

$= -\frac{3}{3}$

$= -1$

The deceleration of the object at 6 seconds is 1 ms^{-2}.

11. Below is a distance-time graph of an object. Calculate the speed at the 14th second.



Two coordinates along the tangent are:

(14, 16) and (10, 2)

Speed of the object $= \frac{16-2}{14-10}$

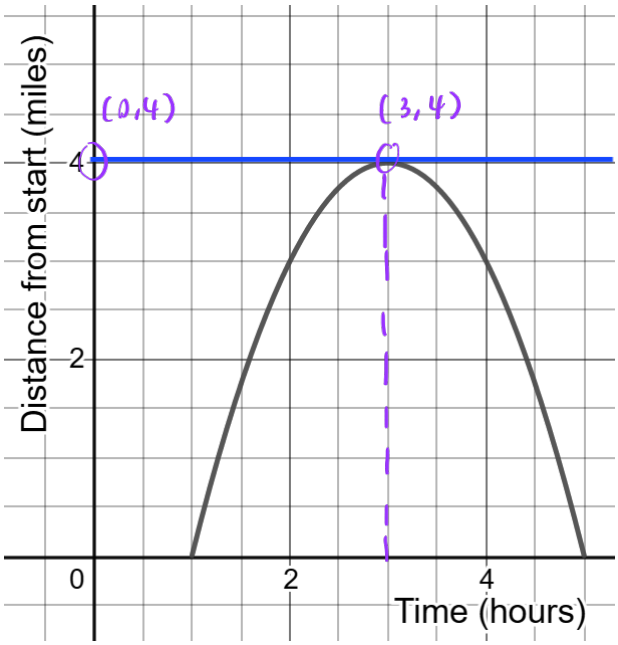
$= \frac{14}{4}$

$= \frac{7}{2}\ ms^{-1}$

$= 3.5\ ms^{-1}$

The speed of the object at 14 seconds is 3.5 ms^{-1}.

12. Below is a displacement-time graph. Calculate the speed at 3 hours.



2 coordinates along the tangent are

(0,4) and (3,4)

Speed of object $= \frac{4-4}{0-3}$

$= \frac{0}{-3}$

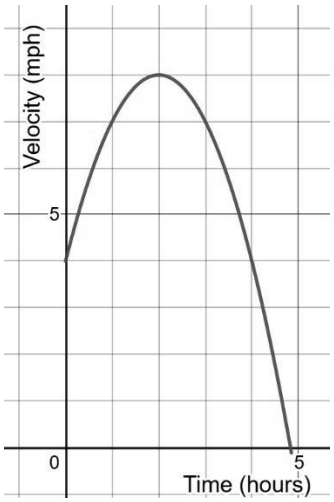
$= 0$

The speed of object at 3 hours is 0 miles/hour

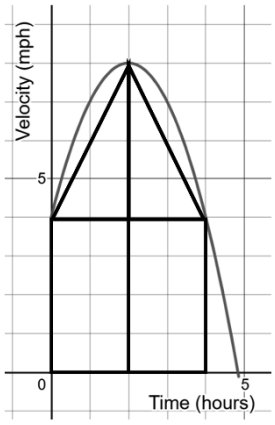
Section E

Worked Example

Below is a velocity-time graph of a car. Estimate the total distance travelled by the car.



Step 1: Draw shapes under the curve to roughly fit the area.



Use squares, rectangles, and triangles to make a rough approximation of the area.

Step 2: Calculate the area of each shape.

$$Rectangles\ Total\ Area = 2 \times (2 \times 4)\text{m}^2 = 2 \times 8\ \text{m}^2 = 16\ \text{m}^2$$

$$Triangles\ Total\ Area = 2 \times (\frac{1}{2} \times 2 \times 4)\text{m}^2 = 2 \times 4\ \text{m}^2 = 8\ \text{m}^2$$

Step 3: Add together the areas of each shape.

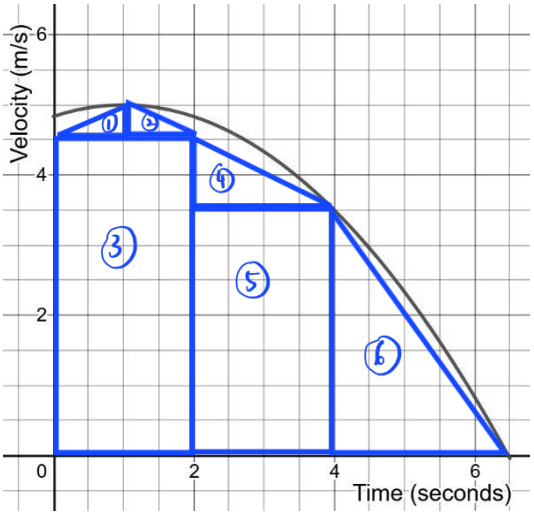
$$Area\ Under\ Curve = 16\ \text{m}^2 + 8\ \text{m}^2 = 24\ \text{m}^2$$

Step 4: Interpret the area in terms of the question context.

The area under the curve is equal to the distance travelled. So, the car travelled $\mathbf{24}$ *miles in total.*

Guided Example

Below is a velocity-time graph for an object. Estimate the total distance travelled by the object.



Step 1: Draw shapes under the curve to roughly fit the area.

Step 2: Calculate the area of each shape.

Area of triangle ① and ② $= 2 \times \frac{1}{2} \times 0.5 \times 1 = 0.5$

Area of rectangle ③ $= 2 \times 4.5 = 9$

Area of triangle ④ $= \frac{1}{2} \times 2 \times 1 = 1$

Area of rectangle ⑤ $= 2 \times 3.5 = 7$

Area of triangle ⑥ $= \frac{1}{2} \times 2.5 \times 3.5 = 4.375$

Step 3: Add together the areas of each shape.

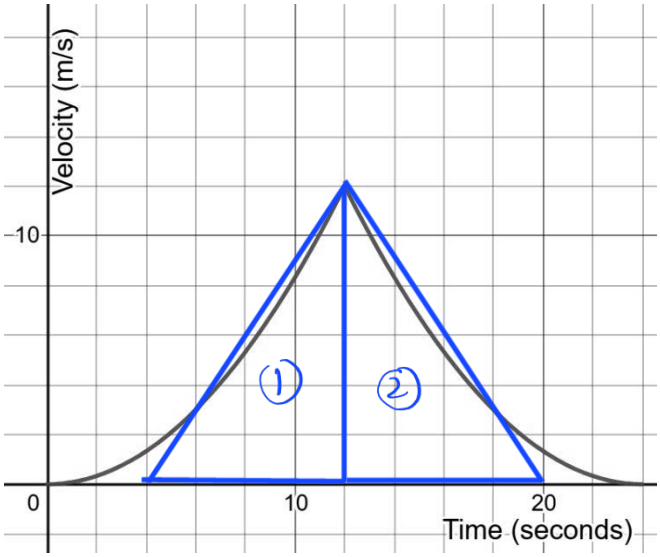
Area under the curve $= 0.5 + 9 + 1 + 7 + 4.375 = 21.875$

Step 4: Interpret the area in terms of the question.

The area under the curve tells us that the total distance travelled is 21.875 m.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

13. Calculate the total distance travelled by the object in this graph:



① Draw shapes under the curve.

Area of triangle ①:

$\frac{1}{2} \times 8 \times 12 = 48$

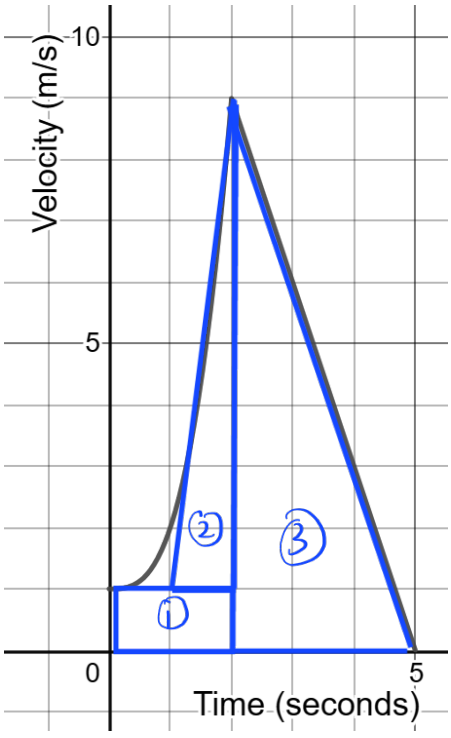
Area of triangle ②:

$\frac{1}{2} \times 8 \times 12 = 48$

Area under the curve $= 48 + 48$
$= 96$

Total distance travelled by the object is 96 m.

14. Calculate the total distance travelled by the object in this graph:



① Draw shapes under the curve

Area for rectangle ① $= 2 \times 1 = 2$

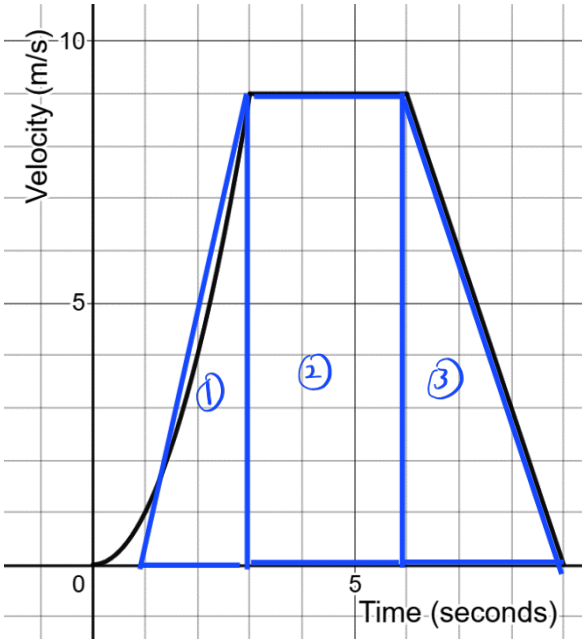
Area for triangle ② $= \frac{1}{2} \times 8 \times 1 = 4$

Area for triangle ③ $= \frac{1}{2} \times 3 \times 9$
$= 13.5$

Area under the curve $= 2 + 4 + 13.5$
$= 19.5$

The total distance travelled by the object is 19.5 m.

15. Find the total distance travelled by the object in this graph:



① Draw shapes under the curve

Area for triangle ① : $\frac{1}{2} \times 2 \times 9 = 9$

Area for rectangle ② : $3 \times 9 = 27$

Area for triangle ③ : $\frac{1}{2} \times 3 \times 9 = 13.5$

Area under the curve :

$9 + 27 + 13.5 = 49.5$ m

The total distance travelled by the object is 49.5 m.