GCSE Maths – Algebra

Translations and Reflections (Higher Only)

Worksheet

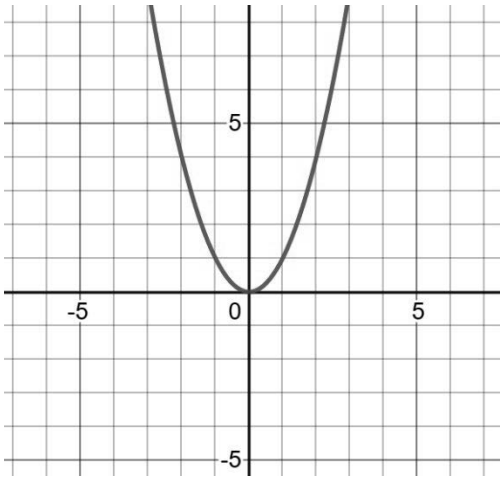
WORKED SOLUTIONS

This worksheet will show you how to work out different types of translations and reflections questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

The graph shows the function $f(x) = x^2$.
Sketch the function $f(x) + 3$.

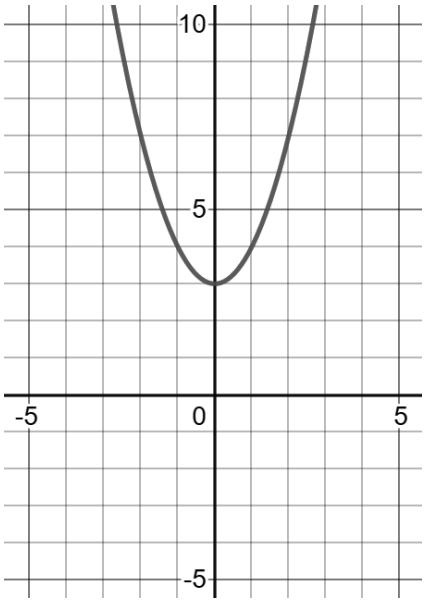


Step 1: Work out how the function will be translated.

Since the translation number $+3$ is outside the bracket, this is a translation in the vertical direction. The curve will be shifted upwards by 3 units.

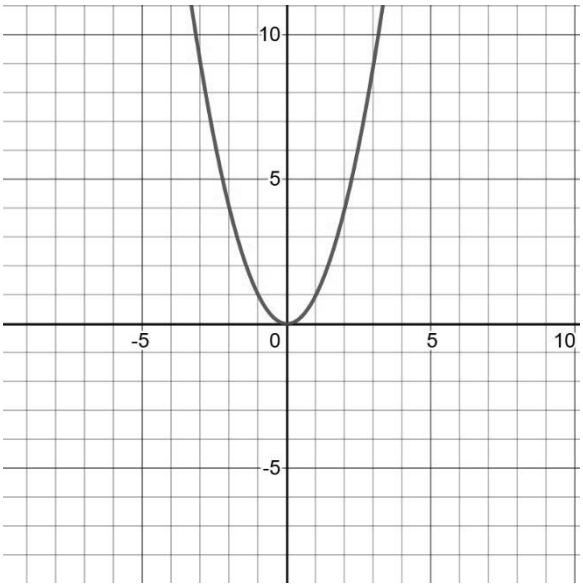
Step 2: Sketch the transformed curve.

Each coordinate is moved up 3 units. In particular, the y-intercept moves from position $(0,0)$ to position $(0,3)$.



Guided Example

The graph shows the function $f(x) = x^2$.
Sketch the function $f(x - 4)$.

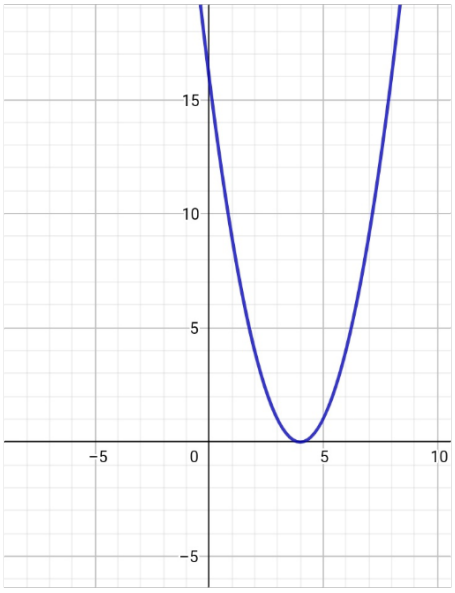


Step 1: Work out how the function will be translated.

Since the -4 is inside the bracket, the graph will be translated in the horizontal direction. $(x-4)$ has a negative value, hence the graph will be shifted to the right by 4 units.

↳ if $(x+4)$, graph shifted to the left.

Step 2: Sketch the transformed curve.

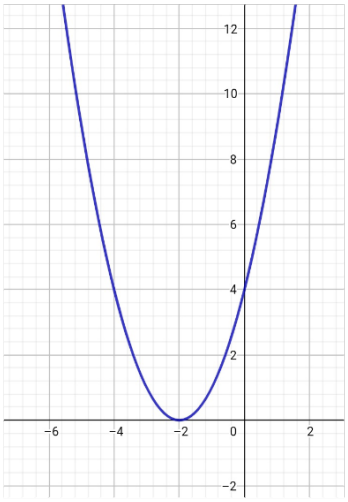


Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. A curve is described by the function $f(x) = x^2$. → the original graph can be referred in the guided example above.
Sketch the following translations:

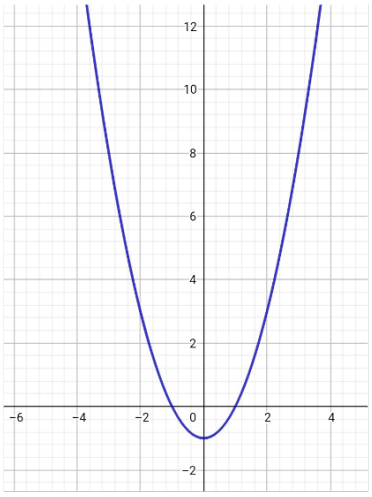
a) $f(x+2)$

The +2 is inside the bracket, so the graph will be translated in the horizontal direction. The +2 has a positive value, hence, the graph is shifted to the left by 2 units. →



b) $f(x)-1$

The −1 is outside the bracket, so the graph will be translated in the vertical direction. Since −1 is a negative value, the graph is shifted downwards by 1 unit. →

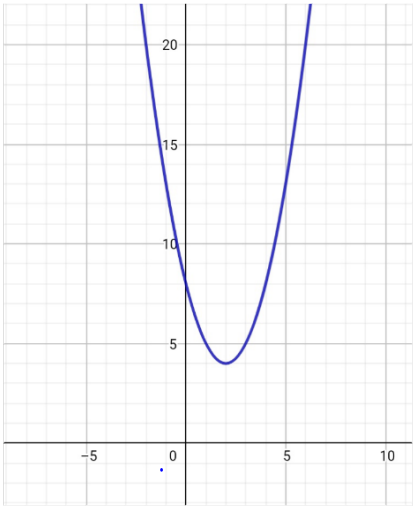


c) $f(x-2)+4$

Both values are present inside and outside the bracket. So, the graph will be translated in both the horizontal and vertical direction. →

Horizontal direction:
$(x-2)$ has a negative value, so, the graph is shifted to the right by 2 units

Vertical direction:
+4 has a positive value, so, the graph is shifted upwards by 4 units

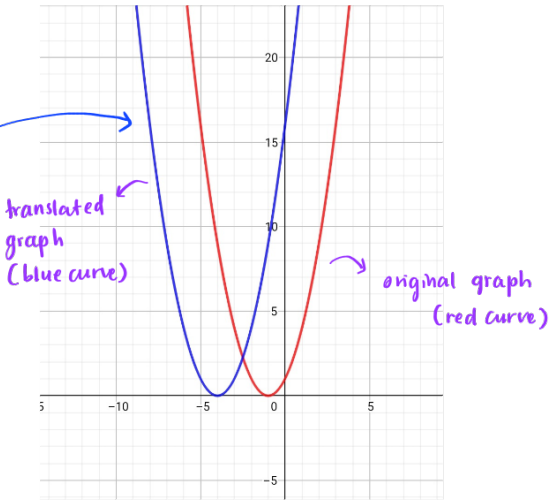


2. A curve is described by the function $f(x) = x^2 + 2x + 1$. Sketch the following translations:

draw the original graph first.

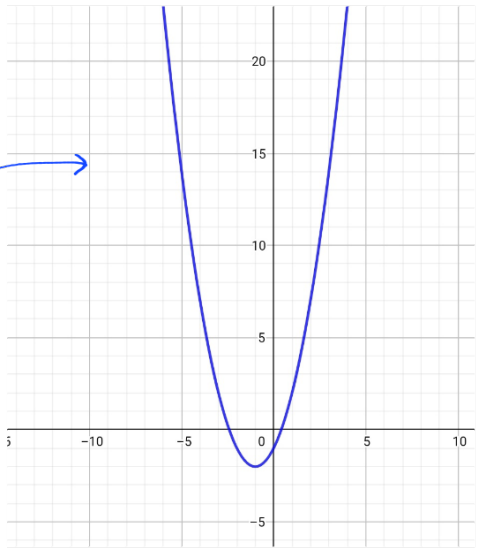
a) $f(x + 3)$

The +3 is inside the bracket, so the graph will be translated in the horizontal direction. The +3 has a positive value, hence, the graph is shifted to the left by 3 units.



b) $f(x) - 2$

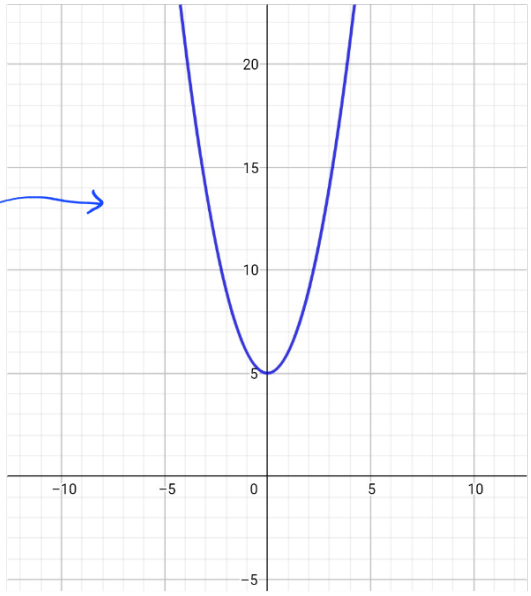
The -2 is outside the bracket, so the graph will be translated in the vertical direction. Since -2 is a negative value, the graph is shifted downwards by 2 units.



c) $f(x - 1) + 5$

Horizontal direction:
$(x-1)$ has a negative value, so, the graph is shifted to the right by 1 unit

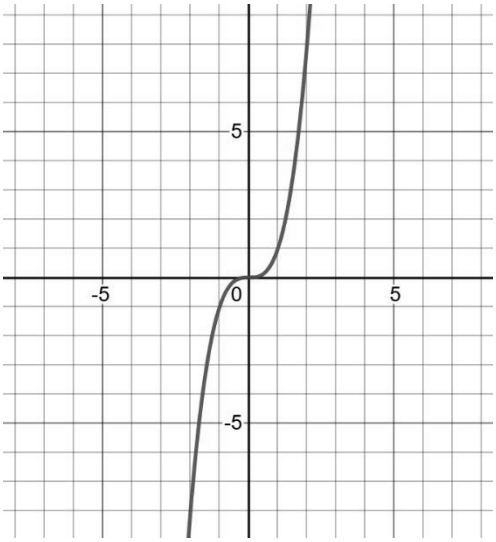
Vertical direction:
+5 has a positive value, so, the graph is shifted upwards by 5 units.



Section B

Worked Example

A curve is given by the function $f(x) = x^3$.
The curve is reflected in the x-axis. Write the equation for the function of the reflection and sketch the new curve.



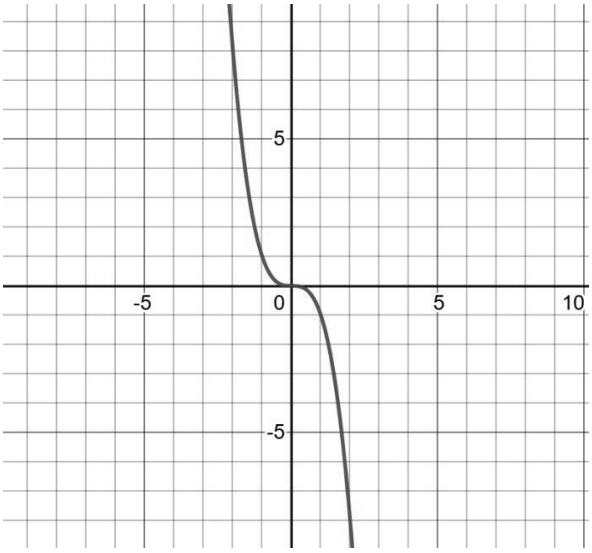
Step 1: Reflection involves introducing a negative sign to the equation for the function. Identify where the negative sign needs to be put in the function equation for the required reflection.

As this is a reflection in the x-axis, the negative sign will be put ***outside*** *the bracket.*

Therefore, the function is written as $-f(x) = -(x^3)$.

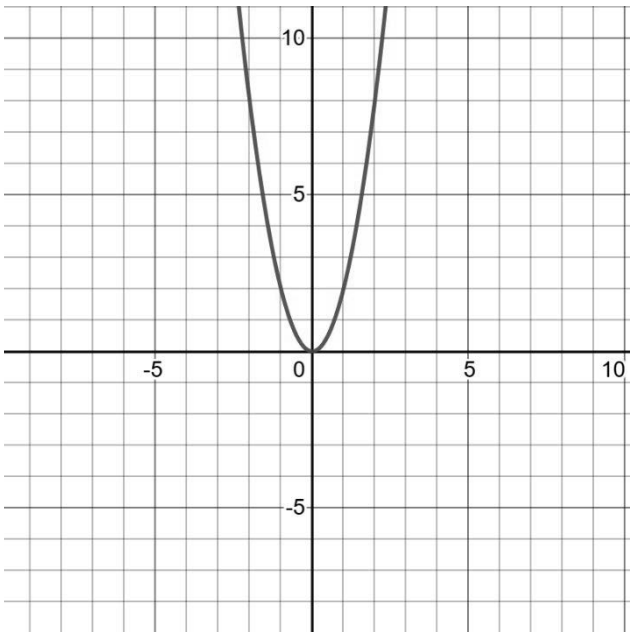
Step 2: Sketch the new curve.

Sketch the new curve by reflecting the given graph in the x-axis.



Guided Example

A curve is given by the function $f(x) = 2x^2$. The curve is reflected in the y-axis. Write the reflection and sketch the new curve.



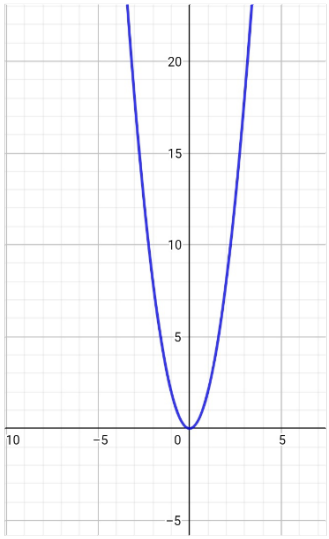
Step 1: Reflection involves introducing a negative sign to the equation for the function. Identify where the negative sign needs to be put in the function equation for the required reflection.

Since the reflection is in the y-axis, the negative sign should be placed inside the bracket:

$f(-x) = 2(-x)^2$

Step 2: Sketch the new curve.

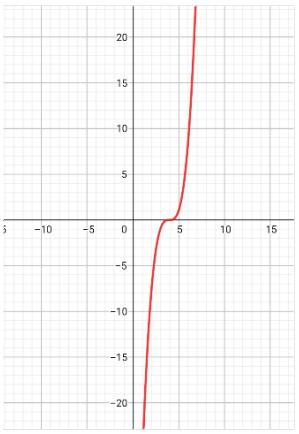
The reflected graph is the same as the original graph since the original graph is already in symmetry in the y-axis.



Now it's your turn!
If you get stuck, look back at the worked and guided examples.

3. A curve is described by the equation $f(x) = (x-4)^3$. Sketch the curve and write the equation of the new function if the curve is reflected in:

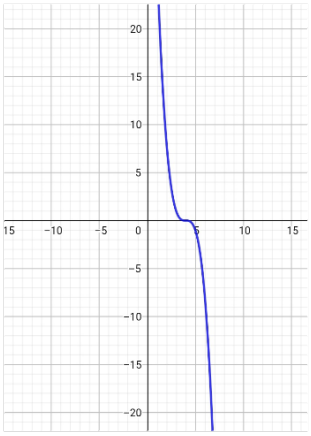
original graph



a) the x-axis

The negative sign should be outside the bracket:

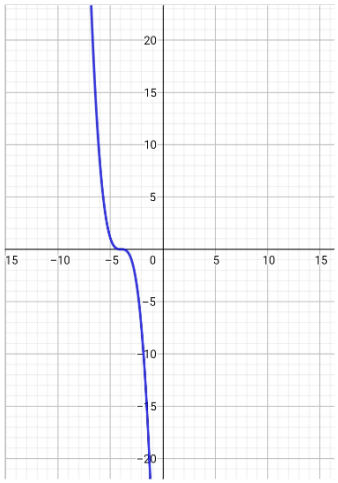
$-f(x) = -(x-4)^3$



b) the y-axis

The negative sign should be inside the bracket:

$f(-x) = ((-x)-4)^3$

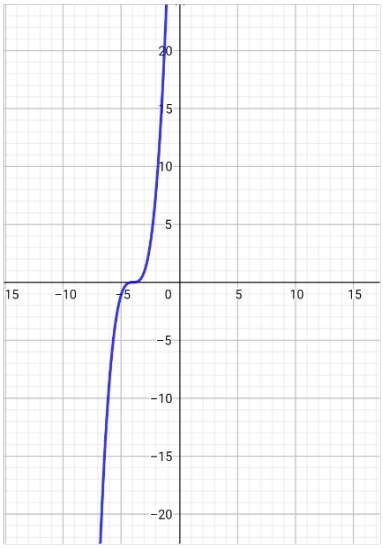


c) both the x-axis and y-axis

The negative sign should be both inside and outside the bracket:

$-f(-x) = -(-x-4)^3$

For easier sketching, you can take the curve in (b), (which is already reflected in the y-axis) and reflect it in the x-axis to get the answer.



Alternatively, you can also use the curve in (a) and reflect it in the y-axis to get the answer.