GCSE Maths – Algebra

Roots, Intercepts and Turning Points

Worksheet

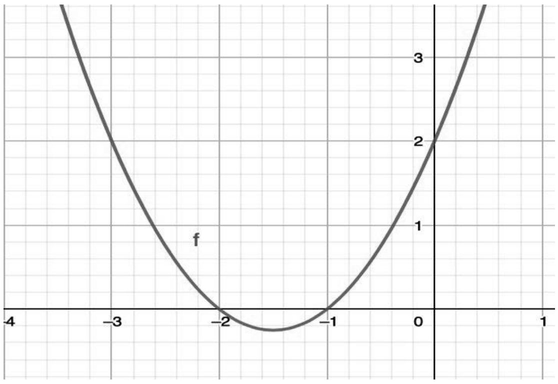
WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions involving roots, intercepts and turning points. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Using the graph, find the number of roots and their values for the equation $f(x) = x^2 + 3x + 2$



Step 1: Look at the graph and identify how many times it crosses the x-axis.

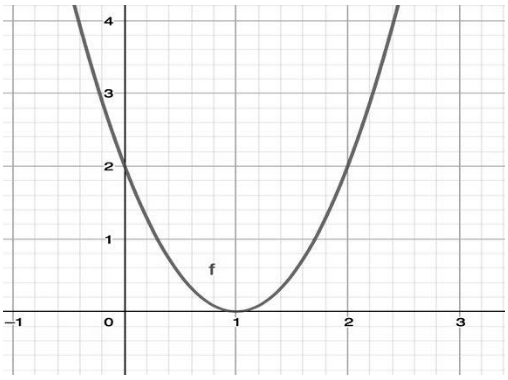
In this example we see the graph crosses at two points. These two points represent the roots of the graph so in this case there are two roots.

Step 2: Look at where the graph crosses the x-axis. These are the roots of the equation.

The graph crosses the x axis at $x = -1$ and $x = -2$ so these are the roots of the equation $f(x) = x^2 + 3x + 2$.

Guided Example

Using the graph, find the number of roots and their values for the equation $f(x) = 2x^2 - 4x + 2$



Step 1: Look at the graph and identify how many times it crosses the x-axis.

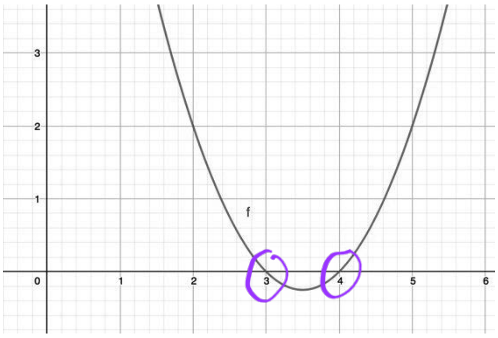
The graph crosses the x-axis only once. Hence, it only has one root.

Step 2: Look at where the graph crosses the x-axis. These are the roots of the equation.

The graph crosses at (1,0) where x = 1. Hence x = 1 is the root of the equation $f(x) = 2x^2 - 4x + 2$.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

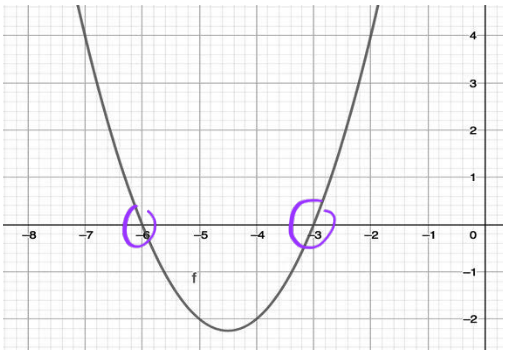
1. Find the number of roots and their values for the equation $f(x) = x^2 - 7x + 12$



The graph crosses at 2 points across the x-axis. Hence, the number of roots are 2. Their values are :

$x = 3$ and $x = 4$

2. Find the number of roots and their values for the equation $f(x) = x^2 + 9x + 18$

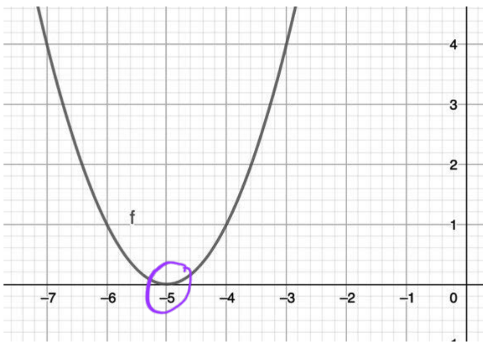


The graph crosses the x-axis at 2 points. Hence, the number of roots for this equation is 2.

The values of the roots are :

$x = -3$ and $x = -6$

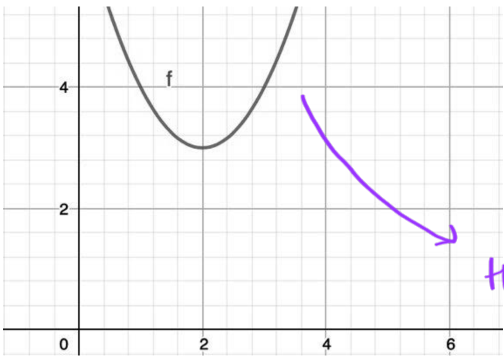
3. Find the number of roots and their values for the equation $f(x) = x^2 + 10x + 25$



The graph only crosses at one point along the x-axis. Hence, the number of root for this equation is 1.

The root is $x = -5$

4. Find the number of roots and their values for the equation $f(x) = x^2 - 4x + 7$



The graph does not cross the x-axis. Hence, the equation has no roots.

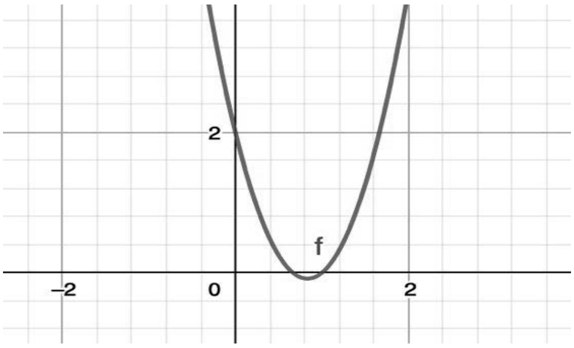
Number of root = 0

the graph does not intercept x-axis. Hence, no real roots.

Section B

Worked Example

Using the graph, find the y-intercept of the curve given by the equation $f(x) = x^2 + 3x + 2$



Step 1: Using the graph, identify where the curve crosses the y-axis.

In this example we see there is one y-intercept at $y = 2$.

Therefore, the y-intercept is $\mathbf{2}$.

Worked Example

Calculate the y-intercept of the curve given by the quadratic equation $f(x) = x^2 + 3x + 2$

Step 1: The y-intercept occurs when $x = 0$. Substitute $x = 0$ into the equation for $y = f(x)$.

$$f(0) = (0)^2 + 3(0) + 2$$

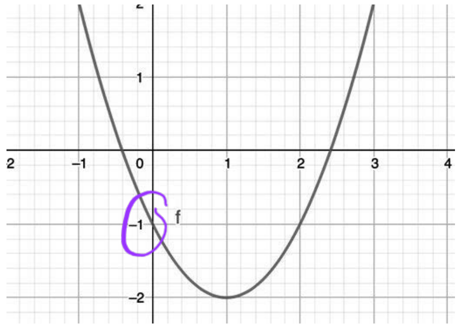
Step 2: Perform calculations to find the value of y when $x = 0$.

$$(0)^2 + 3(0) + 2 = 0 + 0 + 2 = 2$$

Therefore, the y-intercept is $\mathbf{2}$.

Guided Example

Using the graph, find the y-intercept of the equation $f(x) = x^2 - 2x - 1$. Check the solution by substituting a suitable value into the equation.



Step 1: Using the graph, identify the point at which the curve crossed the y-axis.

The graph crosses the y-axis at $y = -1$. Hence the y-intercept is $y = -1$.

Step 2: Substitute $x = 0$ into the equation $y = f(x)$ to check the value you found is correct.

$f(x) = x^2 - 2x - 1$

$f(0) = (0)^2 - 2(0) - 1 = -1$

Therefore, the y-intercept is $y = -1$

Guided Example

Calculate the value of the y-intercept for the curve given by the quadratic equation $f(x) = 5x^2 - 18x + 4$.

Step 1: The y-intercept occurs when $x = 0$. Substitute $x = 0$ into the equation for $y = f(x)$.

$f(x) = 5x^2 - 18x + 4$

$f(0) = 5(0)^2 - 18(0) + 4$

Step 2: Perform calculations to find the value of y when $x = 0$.

$f(0) = 5(0)^2 - 18(0) + 4$

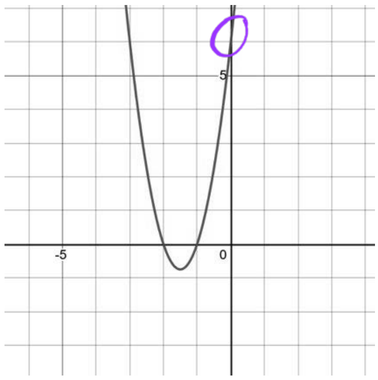
$f(0) = 0 - 0 + 4$

$f(0) = 4$

The y-intercept is $y = 4$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Use the graph to find the y-intercept of the curve given by the equation $f(x) = 9x^2 - 8x + 6$.



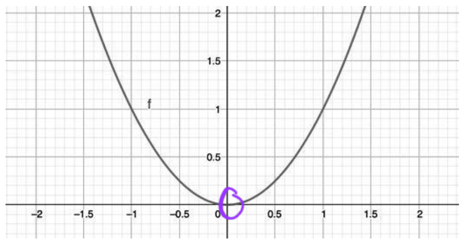
The graph crosses the y-axis at $y=6$. Hence, the y-intercept of the curve is $y=6$.

6. Calculate the y-intercept of the curve given by the equation $f(x) = 7x^2 + 8x - 2$.

$f(x) = 7x^2 + 8x - 2$
$f(0) = 7(0)^2 + 8(0) - 2$
$f(0) = 0 + 0 - 2$
$f(0) = -2$

The y-intercept is $y = -2$

7. Use the graph to find the y-intercept of the curve given by the equation $f(x) = x^2$.



The graph crosses the y-axis at $y=0$. Hence, the y-intercept is $y=0$.

8. Calculate the y-intercept of the curve given by the equation $f(x) = x^2 - 10x + 13$.

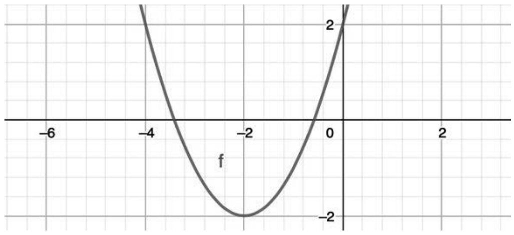
$f(x) = x^2 - 10x + 13$
$f(0) = (0)^2 - 10(0) + 13$
$f(0) = 0 - 0 + 13 = 13$

The y-intercept is $y = 13$

Section C

Worked Example

By looking at the graph, find the turning point of the curve given by the equation $f(x) = x^2 + 4x + 2$



Step 1: Identify the turning point as the minimum point of the graph. The coordinates of this point are the co-ordinates of the turning point.

Here, the minimum point occurs in the bottom left quadrant with coordinates $(-2, -2)$*. This means the turning point is at position* $(-2, -2)$*.*

Worked Example

Calculate the turning point of the curve given by the equation $f(x) = x^2 - 6x + 9$.

Step 1: We want the equation to be in the form $y = (x + a)^2 + b$ as the turning point occurs at position $(-a, b)$. Obtain the equation in this form by completing the square.

$$x^2 - 6x + 9 = (x - 3)^2 - 9 + 9 = (x - 3)^2 + 0$$

Step 2: Using the equation in the form $y = (x + a)^2 + b$, find the position of the turning point.

For an equation in the form $y = (x + a)^2 + b$*, the turning point occurs at* $(-a, b)$*.*

Here, $(x + a)^2 + b = (x - 3)^2 + 0$.

So, $a = -3$ *and* $b = 0$*. Therefore, the turning point occurs at* $(-a, b) = (-(-3), 0) = (\mathbf{3}, \mathbf{0})$.

Guided Example

Calculate the turning point of the curve given by the equation $f(x) = x^2 - 6x + 9$.

Step 1: We want the equation to be in the form $y = (x + a)^2 + b$ as the turning point occurs at position $(-a, b)$. Obtain the equation in this form by completing the square.

$$f(x) = \underbrace{x^2 - 6x + 9}_{\text{completing the square}} = \left(x + \frac{(-6)}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 + 9 = (x-3)^2 - 9 + 9$$
$$= (x-3)^2 + 0$$

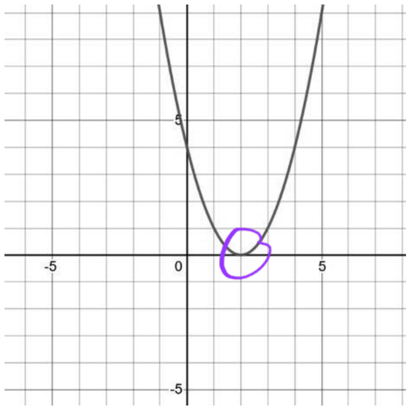
Step 2: Using the equation in the form $y = (x + a)^2 + b$, find the position of the turning point.

$y = (x-3)^2 + 0$ Therefore, the turning point is $(3, 0)$

$a = -3, b = 0$ Turning point $= (-a, b) = (-(-3), 0)$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

9. Use the graph to find the turning points for the curve $y = x^2 - 4x + 4$.



Here, the minimum point of the graph occurs at point (2, 0). Hence, the turning point of the graph is (2, 0).

10. By completing the square, find the turning points of the curve $y = x^2 - 6x + 2$.

$y = x^2 - 6x + 2$ complete the square to change it to form $(x+a)^2 + b$

$y = \left(x + \left(\frac{-6}{2}\right)\right)^2 - \left(\frac{-6}{2}\right)^2 + 2$

$= (x-3)^2 - (-3)^2 + 2 = (x-3)^2 - 9 + 2 = (x-3)^2 - 7$

$a = -3, b = -7$

The turning point is $(3, -7)$

11. By completing the square, find the turning points of the curve $y = x^2 + 3x - 1$.

$y = x^2 + 3x - 1$

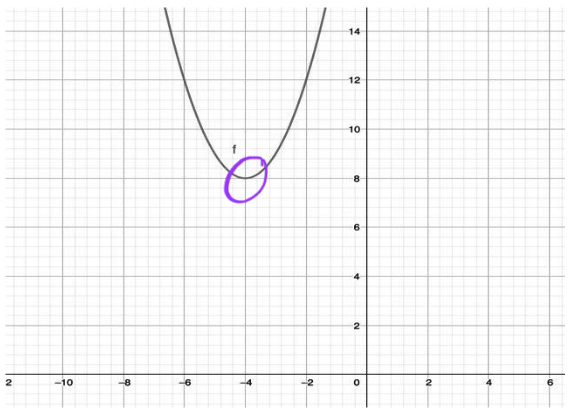
$= \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 1$

$= \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 1$

$y = \left(x + \frac{3}{2}\right)^2 - \frac{13}{4}$ ← $a = \frac{3}{2}, b = -\frac{13}{4}$

The turning point is $\left(-\frac{3}{2}, -\frac{13}{4}\right)$

12. Use the graph to find the turning points for the given curve.



The minimum point of the graph occurs at point (-4, 8). Hence, the turning point for the given curve is (-4, 8).