# Foundation Check In – 3.03 Exact calculations

**Do not use a calculator.**

1. Work out the mean of $\frac{3}{8}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{7}{8}$.

2. A giant tortoise travelled $\frac{2}{9}$ of a mile in 40 minutes. What was the average speed of the giant tortoise in miles per hour?

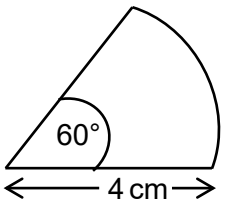
3. What number is half way between $\frac{3}{7}$ and $\frac{5}{9}$?

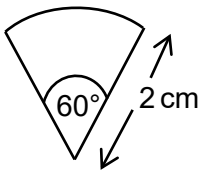
4. A circle of radius 10 cm is split into ten pieces of equal area. What is the exact area of each piece?

5. Simplify $\frac{2}{3}\pi + 4\pi + 1\frac{1}{6}\pi$.

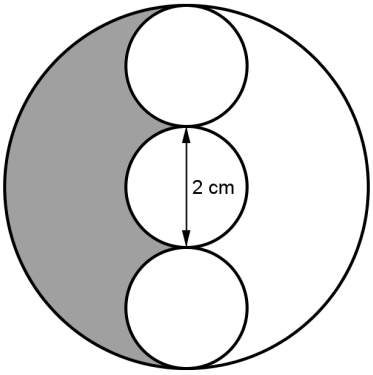
6. Show that $2\frac{1}{2} \times 1\frac{1}{3} \times \frac{3}{4} \times \frac{2}{5} = 1$.

7. Show that the difference in the area of these two sectors is $2\pi$ cm$^2$.

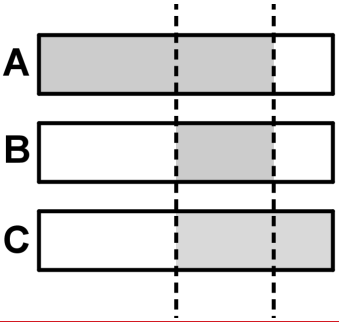


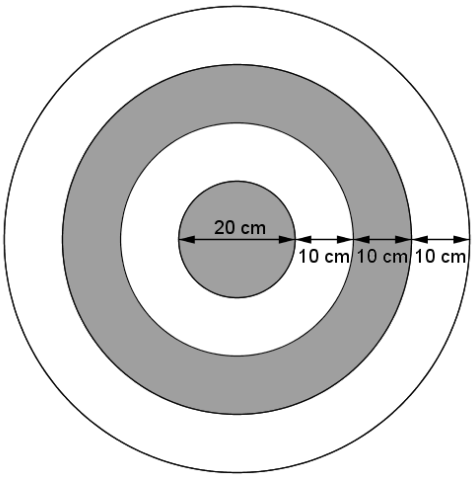

8. The big circle on the right has three circular holes cut out of it. The three holes are congruent and have diameter 2 cm. When the holes are cut, the big circle is split into two parts. Show that the area of the shaded part is $3\pi$ cm$^2$.




9. The three rectangles on the right are identical in size.
   Rectangle A has $\frac{4}{5}$ shaded.
   Rectangle C has $\frac{8}{15}$ shaded.
   Work out the fraction of rectangle B that is shaded.



10. The diagram below shows an archery target that is made up of 4 concentric circles. Calculate the fraction of the total area of the target that is shaded. Give your answer in its simplest form.




**Extension**

Unit fractions are fractions whose numerator is equal to 1. For example, $\frac{1}{8}$ or $\frac{1}{3}$ are unit fractions.

You can make other fractions by adding together unit fractions. For example, $\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$ which shows that $\frac{3}{4}$ can be made by adding two unit fractions.

Can you find ways of making $\frac{3}{4}$ by adding three **different** unit fractions?

## Answers

1. $\frac{13}{8} \div 4 = \frac{13}{8} \times \frac{1}{4} = \frac{13}{32}$

2. Speed $= \frac{2}{9} \div \frac{40}{60} = \frac{2}{9} \times \frac{60}{40} = \frac{2}{9} \times \frac{3}{2} = \frac{1}{3}$ miles per hour.

3. $\frac{\frac{3}{7} + \frac{5}{9}}{2} = \frac{\frac{27}{63} + \frac{35}{63}}{2} = \frac{\frac{62}{63}}{2} = \frac{62}{63} \times \frac{1}{2} = \frac{31}{63}$

4. Area of whole circle is $10^2 \times \pi = 100\pi$, so each piece is $100\pi \div 10 = 10\pi$ cm$^2$.

5. $\frac{2}{3}\pi + 4\pi + 1\frac{1}{6}\pi = \frac{4}{6}\pi + \frac{24}{6}\pi + \frac{7}{6}\pi = \frac{35}{6}\pi$ or $5\frac{5}{6}\pi$

6. $2\frac{1}{2} \times 1\frac{1}{3} \times \frac{3}{4} \times \frac{2}{5} = \frac{5 \times 4 \times 3 \times 2}{2 \times 3 \times 4 \times 5} = 1$

7. $\left(\frac{60}{360} \times \pi \times 4^2\right) - \left(\frac{60}{360} \times \pi \times 2^2\right) = \frac{16}{6}\pi - \frac{4}{6}\pi = \frac{12}{6}\pi = 2\pi$ cm$^2$

8. The big circle has diameter 6 cm, so radius 3 cm. Its area is $9\pi$ cm$^2$. Each small circle has radius 1 cm$^2$, so area $\pi$ cm$^2$. After three holes have been cut out of the big circle, the remaining area is $9\pi - 3\pi = 6\pi$ cm$^2$. The shaded area is half of this i.e. $3\pi$ cm$^2$.

9. The unshaded area of B is equal to the unshaded areas of A and C added together so the shaded area is $1 - \left(1 - \frac{4}{5}\right) - \left(1 - \frac{8}{15}\right) = \frac{15}{15} - \frac{3}{15} - \frac{7}{15} = \frac{5}{15} = \frac{1}{3}$.

   Or shaded area of B = shaded area of C – unshaded area of A, so $\frac{8}{15} - \frac{3}{15} = \frac{5}{15} = \frac{1}{3}$.

   Or shaded area of B = shaded area of A – unshaded area of C, so $\frac{12}{15} - \frac{7}{15} = \frac{5}{15} = \frac{1}{3}$.

10. Total area is $40^2\pi = 1600\pi$ cm$^2$. The inner shaded circle has area $10^2\pi = 100\pi$ cm$^2$. The shaded ring has area $30^2\pi - 20^2\pi = 500\pi$ cm$^2$. So the fraction of the target that is shaded is $\frac{500\pi + 100\pi}{1600\pi} = \frac{600\pi}{1600\pi} = \frac{3}{8}$.

### Extension

There are three ways of making $\frac{3}{4}$ by adding together different unit fractions:

$\frac{3}{4} = \frac{1}{2} + \frac{1}{5} + \frac{1}{20}$, $\frac{3}{4} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12}$ and $\frac{3}{4} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6}$.

# GCSE (9–1) MATHEMATICS

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| | Qu. | Topic | R | A | G |
|---|---|---|---|---|---|
| AO1 | 1 | Add fractions then divide a fraction by an integer | | | |
| AO1 | 2 | Carry out a speed calculation involving fractions | | | |
| AO1 | 3 | Find a fraction half way between two fractions | | | |
| AO1 | 4 | Use multiples of $\pi$ in an exact calculation of the area of a proportion of a circle | | | |
| AO1 | 5 | Simplify an expression involving fractions and $\pi$ | | | |
| AO2 | 6 | Multiply mixed numbers and simplify | | | |
| AO2 | 7 | Use multiples of $\pi$ in an exact calculation involving the areas of two sectors | | | |
| AO2 | 8 | Use multiples of $\pi$ in an exact calculation of the area of a proportion of a compound shape involving circles | | | |
| AO3 | 9 | Solve a problem involving fractions of rectangles | | | |
| AO3 | 10 | Solve a problem involving exact areas of circles | | | |

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