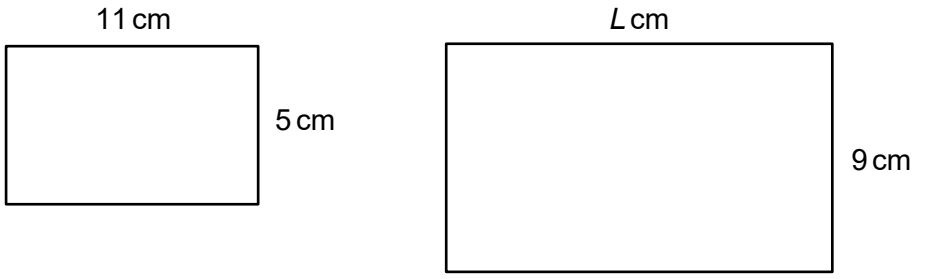
OCR
Oxford Cambridge and RSA

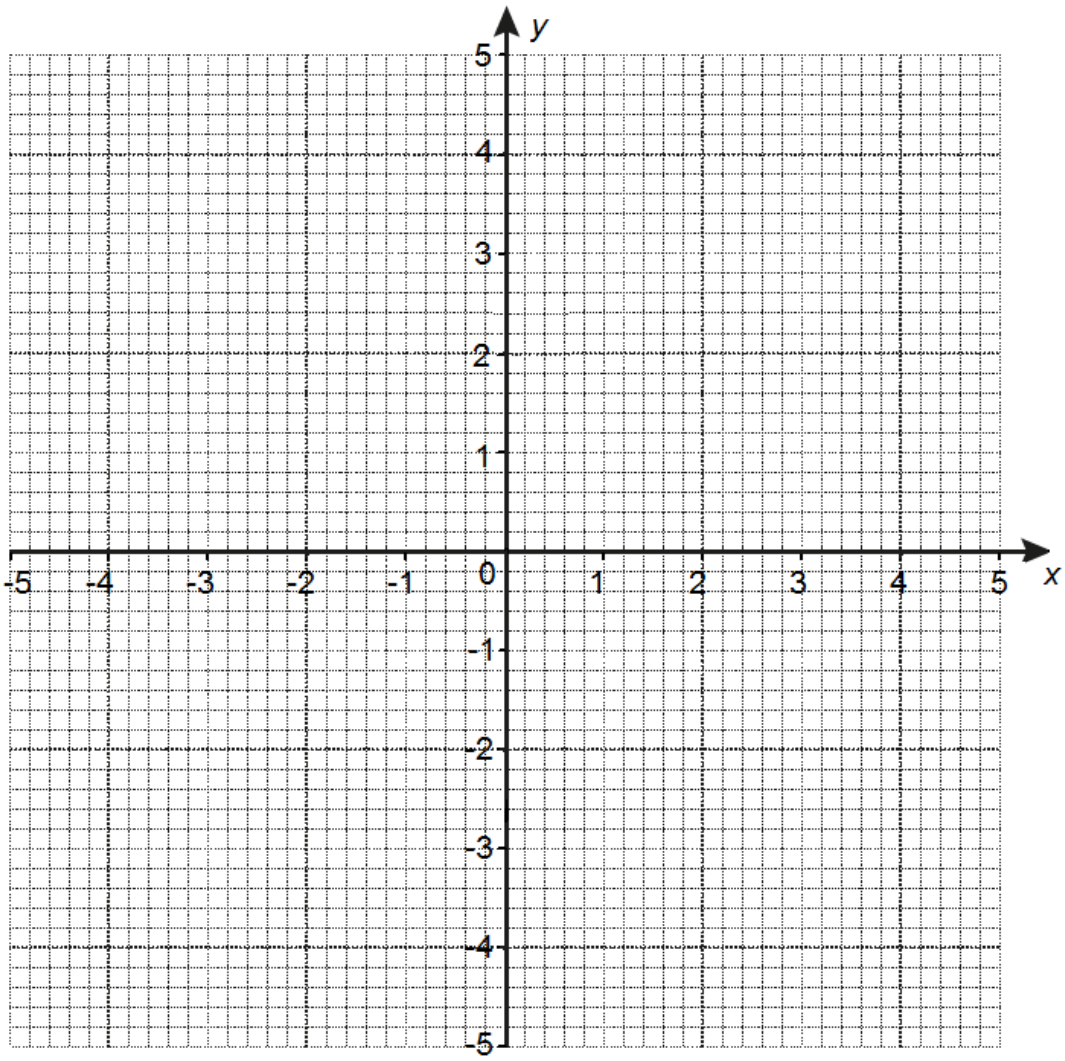
Higher Check In - 9.04 Similarity

1. The two rectangles below are similar. Work out length *L*.

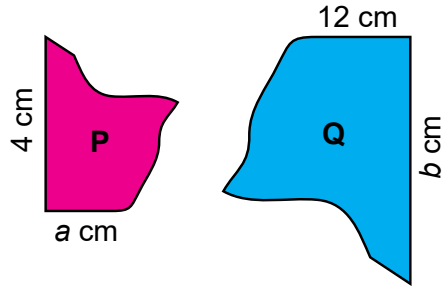


Not to scale

2. On the grid below, plot a triangle with vertices at (1, 4), (2, 2) and (4, 1). Enlarge the triangle using scale factor –2 and centre (1, 1). What are the coordinates of the vertices of the image triangle?



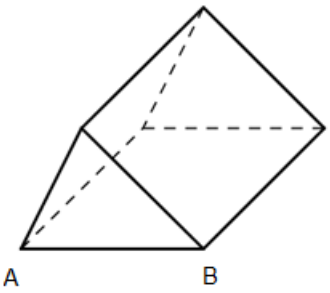
3. Shape **P** is similar to shape **Q**. The area of shape **P** is 9.1 cm^2. The area of shape **Q** is 227.5 cm^2. Work out lengths *a* and *b*.



Not to scale

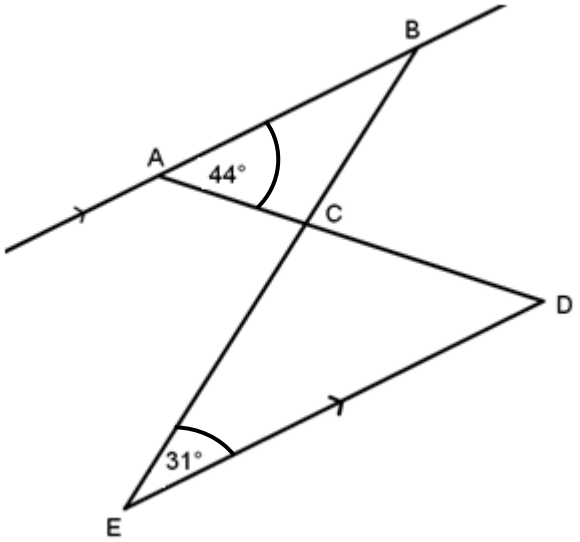
4. A triangle has sides of length 5 cm, 12 cm and 13 cm. Its area is 30 cm^2.
A similar triangle has area 367.5 cm^2. How long are its sides?

5. The triangular prism below has length AB being 4 cm long. Another similar prism has the length corresponding to AB being 9 cm long. The volume of the smaller prism is 30 cm^3. Work out the volume of the larger prism.



6. Owen has a photograph that measures 4 by 6 inches. He takes it to a shop to get it printed onto a large canvas that he can hang on his wall. The finished canvas print measures 16 by 20 inches. Explain why Owen's canvas print is not an enlargement in the mathematical sense of the word. What would the problem be with Owen's picture?

7. Show that triangles ABC and CDE are similar.



Not to scale

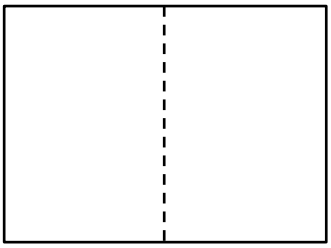
8. A child's snooker table is similar to a full size snooker table. The area of the playing surface is one-quarter the area of the playing surface of a full size snooker table. Francis says that the child's snooker table will be one-eighth the height of a full size snooker table. Show that Francis is incorrect.

9. A piece of A0 paper has area $1\,m^2$. It is rectangular with length 1189 mm. A piece of A4 paper has area $0.0625\,m^2$ and is similar to a piece of A0 paper. Work out the length and width of a piece of A4 paper, giving your answer correct to the nearest millimetre.

10. All spheres are similar. Sphere **A** has a surface area of $36\pi\,cm^2$ and a volume of $36\pi\,cm^3$. Sphere **B** has a surface area of $81\pi\,cm^2$. Work out the volume of sphere **B**.

Extension

The "A series" of paper (like in question 9) has the special property that if you take a piece of paper and cut it into two congruent pieces as below, each of those pieces will be similar to the original piece of paper. For example, a piece of A3 paper cut in half gives two A4 pieces of paper, each of which is similar to the original piece of A3 paper.



Show that the length of a piece of "A series" paper must be $\sqrt{2}$ times the width.

Answers

1. The scale factor is $\frac{9}{5}$ so L is $\frac{9}{5} \times 11 = 19.8\,\text{cm}$.

2. (1, –5), (–1, –1) and (–5, 1).

3. The area scale factor is $\frac{227.5}{9.1} = 25$ and so the linear scale factor is $\sqrt{25} = 5$.

   Therefore $a = \frac{12}{5} = 2.4$ and $b = 4 \times 5 = 20$.

4. The area scale factor is $\frac{367.5}{30} = \frac{49}{4}$ and so the linear scale factor is $\sqrt{\frac{49}{4}} = 3.5$. The shortest side is therefore $5 \times 3.5 = 17.5\,\text{cm}$. The other 2 sides are 42 cm and 45.5 cm.

5. The linear scale factor is $\frac{9}{4}$. The volume scale factor is $\left(\frac{9}{4}\right)^3 = \frac{729}{64}$. The volume of the larger prism is therefore $30 \times \frac{729}{64} = \frac{10935}{32} = 341.7\,\text{cm}^3$ to 1 decimal place.

6. Owen's original photo had sides in the ratio 3 : 2, whereas the canvas is in the ratio 5 : 4. They are not similar, and so the canvas print is not a true enlargement. Either the image will be stretched, or some of the top or bottom will be "cropped". (Alternatively, the short sides have scale factor 4, whereas the long sides have scale factor 3.3&.)

7. The acute angle at D is the same as the 44° angle at A because they are alternate angles. By the same reasoning, the acute angle at B is equal to the 31° angle at E. The vertically opposite angles at C are also equal, and so each triangle has the same 3 angles: they are similar.

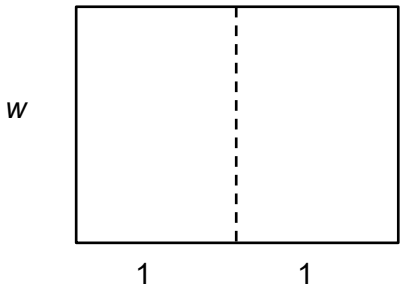
8. If the ratio of areas is $\frac{1}{4}$, then the ratio of lengths (including height) will be $\sqrt{\frac{1}{4}} = 0.5$. The child's snooker table will be half the height of a full size snooker table.

9. The area scale factor is $\frac{1}{16}$ so the linear scale factor is $\sqrt{\frac{1}{16}} = \frac{1}{4}$. The length of a piece of A4 is $\frac{1}{4} \times 1189 = 297\,\text{mm}$. Converting the area of $0.0625\,\text{m}^3$ into $62\,500\,\text{mm}^3$ allows us to work out the width: $62500 \div 297 = 210\,\text{mm}$. The dimensions are 210 by 297 mm.

10. The area scale factor is $\frac{81\pi}{36\pi} = \frac{9}{4}$. Therefore the linear scale factor is $\sqrt{\frac{9}{4}} = \frac{3}{2}$ and the volume scale factor is $\left(\frac{3}{2}\right)^3 = \frac{27}{8}$. The volume of sphere **B** is $\frac{27}{8} \times 36\pi = \frac{243}{2}\pi\,\text{cm}^3$.

Extension

Call the width of the original piece of paper *w* and the length 2. The two smaller pieces of paper therefore have width 1 and length *w*.



The larger and smaller rectangles are similar, so the ratio of their length to width is equal or $\frac{w}{1}=\frac{2}{w}$. This gives $2=w^2$ which, after square rooting, gives $\sqrt{2}=w$. On the smaller piece of paper, the length is $\sqrt{2}$ times its width (since $\sqrt{2}\times 1=\sqrt{2}$). On the larger piece of paper, the length is $\sqrt{2}$ times the width (since $\sqrt{2}\times\sqrt{2}=2$).

GCSE (9–1)

MATHEMATICS

Assessment Objective	Qu.	Topic	R	A	G
AO1	1	Apply similarity to calculate an unknown length in similar rectangles			
AO1	2	Carry out enlargement with a negative scale factor			
AO1	3	Understand the relationship between lengths and areas of similar shapes			
AO1	4	Understand the relationship between lengths and areas of similar shapes			
AO1	5	Understand the relationship between lengths, areas and volumes of similar shapes			
AO2	6	Compare lengths using ratio notation and scale factors			
AO2	7	Prove that two triangles are similar			
AO2	8	Understand the relationship between lengths, areas and volumes of similar shapes			
AO3	9	Solve a problem involving the relationship between lengths and areas of similar shapes			
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