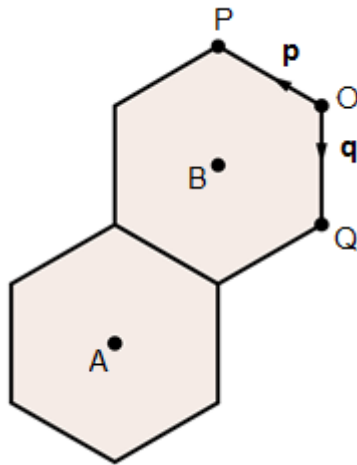


## Higher Check In - 9.03 Plane vector geometry

In questions 1 and 2,  $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ ,  $\mathbf{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$  and  $\mathbf{c} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$ .

1. Work out  $2\mathbf{a} + \mathbf{b} - \mathbf{c}$ .
2. Work out  $\mathbf{b} + \frac{1}{2}(\mathbf{a} + \mathbf{c})$ .
3. Point A has coordinates (7, 4). Point B has coordinates (11, -4). Work out  $\overline{AB}$ .
4. A and B are the centres of the two regular congruent hexagons shown below. Express  $\overline{AB}$  in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .

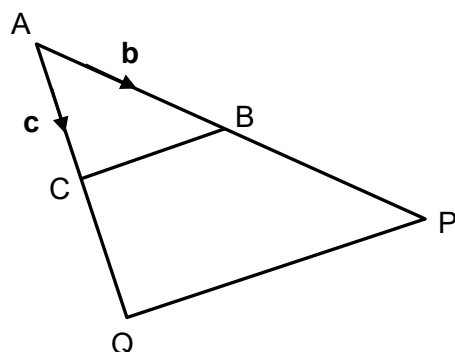


Not to scale

5. A and B are the points such that  $\overline{OA} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$  and  $\overline{OB} = \begin{pmatrix} -7 \\ -5 \end{pmatrix}$ . M is the midpoint of line AB. Find the vector  $\overline{OM}$ .
6. Explain how you can determine that the vectors  $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$  and  $\begin{pmatrix} -2 \\ 3.5 \end{pmatrix}$  are parallel without needing to draw them.
7.  $\overline{OA} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ ,  $\overline{OB} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$  and  $\overline{OC} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ . Show that A, B and C are points on a single straight line.

# GCSE (9–1) MATHEMATICS

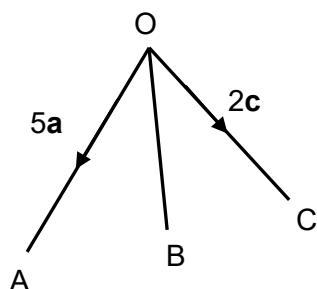
8.  $ABC$  and  $APQ$  are triangles.  $B$  is the midpoint of  $AP$  and  $C$  is the midpoint of  $AQ$ .  
 $\overline{AB} = \mathbf{b}$  and  $\overline{AC} = \mathbf{c}$ . Prove that  $\overline{BC}$  is parallel to  $\overline{PQ}$ .



Not to scale

9.  $\mathbf{a} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} -3 \\ -9 \end{pmatrix}$ . Work out  $\mathbf{c}$  if  $3(\mathbf{a} + \mathbf{c}) = \mathbf{b}$ .

10. On the diagram below,  $\overline{OA} = 5\mathbf{a}$  and  $\overline{OC} = 2\mathbf{c}$ .  $B$  is the point on line  $AC$  such that  $AB : BC = 3 : 2$ . Express  $\overline{OB}$  in terms of  $\mathbf{a}$  and  $\mathbf{c}$ .



Not to scale

## Extension

The diagonals of a parallelogram bisect each other (i.e. the diagonals cross so that they meet at their midpoints: this cuts each diagonal into two parts of equal length). Use vector methods to prove that this is always true for a parallelogram.

# GCSE (9-1) MATHEMATICS

## Answers

$$1. \quad 2\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -3 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -3 \end{pmatrix} = \begin{pmatrix} 6+2-3 \\ -2+5+3 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

$$2. \quad \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \frac{1}{2}\left(\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -3 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$3. \quad \overline{AB} = \begin{pmatrix} 11 \\ -4 \end{pmatrix} - \begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$$

$$4. \quad -\mathbf{p} - 2\mathbf{q}$$

$$5. \quad \overline{AB} = \overline{OB} - \overline{OA} = \begin{pmatrix} -7 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -10 \\ -3 \end{pmatrix}.$$

$$\overline{OM} = \overline{OA} + \frac{1}{2}\overline{AB} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} -5 \\ -1.5 \end{pmatrix} = \begin{pmatrix} -2 \\ -3.5 \end{pmatrix}.$$

Alternatively, the coordinates of A and B could be used to find the midpoint  $(-2, -3.5)$  which could then be converted to a position vector.

$$6. \quad \text{Since } -2\begin{pmatrix} -2 \\ 3.5 \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \end{pmatrix} \text{ one of the vectors is a scalar multiple of the other and therefore they are parallel.}$$

$$7. \quad \overline{AB} = \overline{OB} - \overline{OA} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}. \quad \overline{BC} = \overline{OC} - \overline{OB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}. \text{ Since } \overline{AB} = 2\overline{BC} \text{ then } \overline{AB} \text{ is parallel to } \overline{BC}. \text{ As they share a common point (B), the three points ABC must be on a single straight line.}$$

$$8. \quad \overline{BC} = \overline{BA} + \overline{AC} = -\mathbf{b} + \mathbf{c} = \mathbf{c} - \mathbf{b}. \quad \overline{AP} = 2\mathbf{b} \text{ and } \overline{AQ} = 2\mathbf{c}, \text{ so } \overline{PQ} = \overline{PA} + \overline{AQ} = 2\mathbf{c} - 2\mathbf{b}. \text{ Since } \overline{PQ} = 2\overline{BC} \text{ then } \overline{BC} \text{ is parallel to } \overline{PQ}.$$

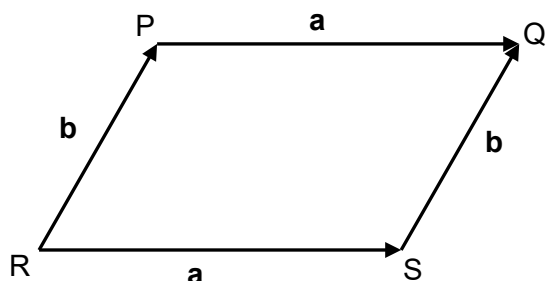
$$9. \quad \text{Dividing } 3\left(\begin{pmatrix} -4 \\ 3 \end{pmatrix} + \mathbf{c}\right) = \begin{pmatrix} -3 \\ -9 \end{pmatrix} \text{ by 3 gives } \begin{pmatrix} -4 \\ 3 \end{pmatrix} + \mathbf{c} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \text{ so } \mathbf{c} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}.$$

$$10. \quad \overline{OA} = 5\mathbf{a}. \quad \overline{AC} = 2\mathbf{c} - 5\mathbf{a}. \quad \overline{AB} = \frac{3}{5}\overline{AC}. \quad \overline{OB} = \overline{OA} + \overline{AB} = 5\mathbf{a} + \frac{3}{5}(2\mathbf{c} - 5\mathbf{a}) = 2\mathbf{a} + \frac{6}{5}\mathbf{c}.$$

# GCSE (9–1) MATHEMATICS

## Extension

The opposite sides of a parallelogram are equal in length and parallel, therefore they can be represented by the same vector.



Not to scale

Diagonal  $\overline{PS} = \mathbf{a} - \mathbf{b}$  and diagonal  $\overline{RQ} = \mathbf{a} + \mathbf{b}$ . Since these diagonals are not parallel, they will cross at a point. If X is the point where they cross, we can describe  $\overline{RX}$  by  $\overline{RX} = r(\overline{RQ}) = r(\mathbf{a} + \mathbf{b}) = r\mathbf{a} + r\mathbf{b}$  and also by  $\overline{RX} = \mathbf{b} + s(\overline{PS}) = \mathbf{b} + s(\mathbf{a} - \mathbf{b}) = \mathbf{b} + s\mathbf{a} - s\mathbf{b}$  where  $r$  and  $s$  are fractions. Since these describe the same journey, they are equal so  $\mathbf{b} + s\mathbf{a} - s\mathbf{b} = r\mathbf{a} + r\mathbf{b}$ .

Comparing the coefficients of **a** on each side we get  $s = r$  (this tells us they cross the same fraction along each diagonal, although we do not yet know what fraction.)

Comparing the coefficients of **b** on each side we get  $1 - s = r$ . Since  $s = r$  we have

$$1 - r = r \text{ so } 1 = 2r \text{ and } r = \frac{1}{2}.$$

Therefore the point where the two diagonals meet is halfway along each diagonal i.e. the diagonals bisect each other.

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# GCSE (9–1)

# MATHEMATICS

Assessment Objective	Qu.	Topic	R	A	G
AO1	1	Carry out arithmetic with vectors			
AO1	2	Carry out arithmetic with vectors			
AO1	3	Find a vector			
AO1	4	Use vectors in geometric arguments			
AO1	5	Use vectors to find a midpoint			
AO2	6	Use vector methods to show two vectors are parallel			
AO2	7	Use vectors to prove three points are on a single straight line			
AO2	8	Use vectors in a geometric proof			
AO3	9	Solve a problem involving vectors			
AO3	10	Use vectors in geometric arguments			

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